Fun with Mediation

PSYC 943: Fundamentals
of Multivariate Modeling

Lecture 11: November 6, 2013

Today's Lecture

- A brief intro to mediation:
  - Terminology → Mediation = regression with new words
  - Testing significance of indirect effects as evidence for mediation

- Example from last time:
  - Multiple indirect effects in predicting math self-efficacy

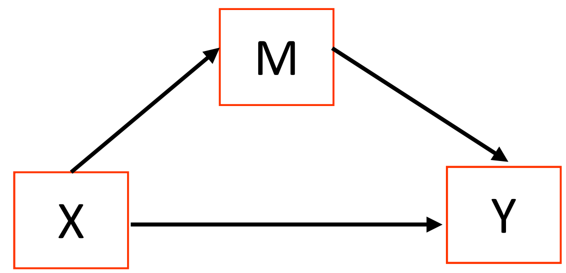
- Complications: when mediators or outcomes are not normal
  - Mediation with other distributions
  - Robust ML to the rescue?
  - Example predicting two binary outcomes

INTRODUCTION TO MEDIATION

Terminology: Mediation ≠ Moderation

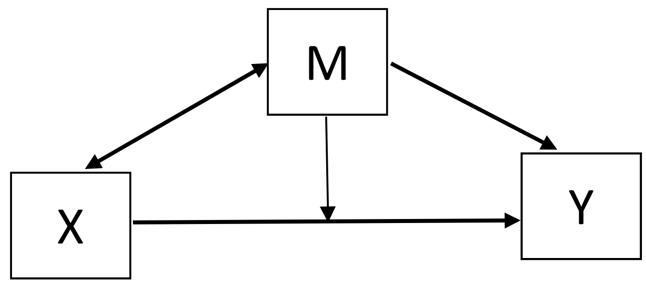
Mediational model:

- X **causes** M, M **causes** Y
- M is an outcome of X but a predictor of Y



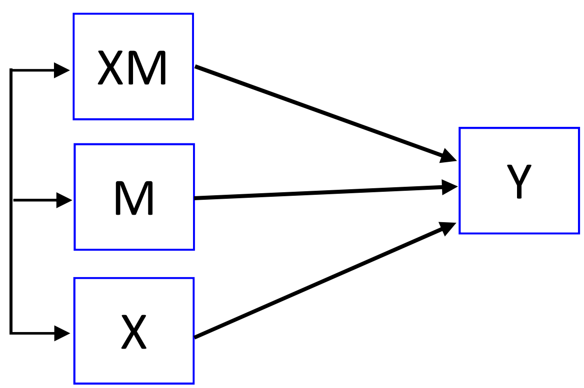
Moderator model:

- M adjusts the size of X→Y relationship



This figure does NOT depict an estimable model.

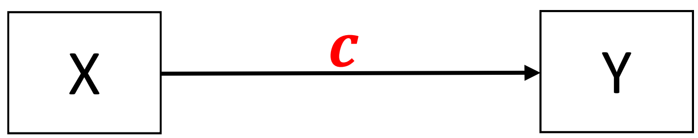
- M is a predictor of Y, and is **correlated** with X
- Moderation is represented by an interaction effect

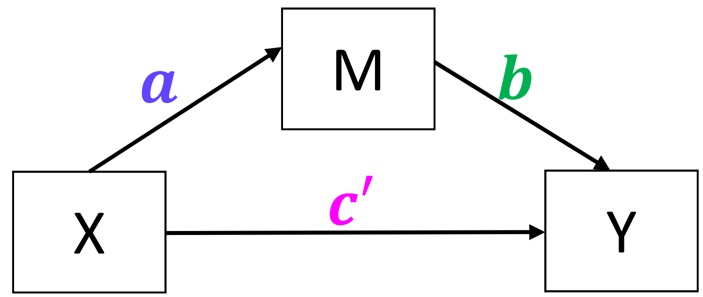


This is what is actually implied by above model.

Terminology: Mediation Effects

c = uncontrolled X to Y path (Y on X;)





The big question in mediation:

- Phrased as usual regression → *Is the effect of X predicting Y still significant after controlling for M?*
- Phrased as "mediation" → *Is the effect of X predicting Y significantly mediated by M?*

- Phrased either way, is $c \neq c'$?

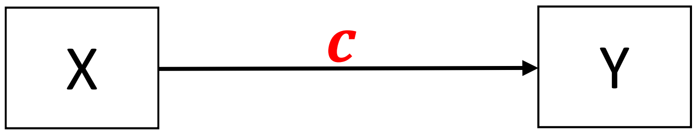
Direct Effects:

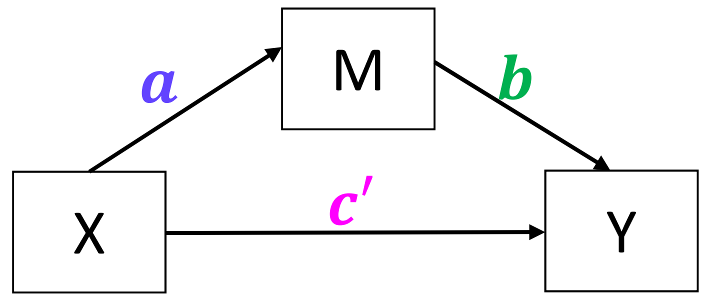
- a = X to M path (M on X;)
- b = M to Y path (Y on M;)
- c' = X to Y path controlled for M (Y on X;)
- $a * b$ = indirect effect of X to Y

- The estimates for $c - c'$ and $a * b$ will be equivalent in MVN observed variables (if same *N*)

Old versus New Rules for Mediation

c = uncontrolled X to Y path
(Y on X;)





- Baron & Kenny (1986, JPSP) rules were standard for a long time…
  - Simulation studies have found these rules to be way too conservative

- Old rule that can now be broken:
  - X must predict Y in the first place (c must be initially significant)
  - When not? Differential power for paths, suppressor effects of mediators
  - Mediation is really about whether $c \neq c'$, not whether each is significant

- Old rules that pry still hold:
  - X must predict M (a must be significant)
  - M must predict Y (b must be significant)

Testing Significance of Mediation

- Need to obtain a SE in order to test if $\boldsymbol{c - c' = 0}$ or if $\boldsymbol{a * b = 0}$
  - For $\boldsymbol{c - c'}$ → "difference in coefficients SE"
  - For $\boldsymbol{a * b}$ → "product of coefficients SE" → we'll start here

- Use "multivariate delta method" (second-derivative approximation shown here) to get SE for product of two random variables $\boldsymbol{a * b}$
  - $SE_{a*b} = \sqrt{a^2 SE_b^2 + b^2 SE_a^2 + SE_a^2 SE_b^2}$
  - An equivalent formula to calculate SE_{a*b} that may have less rounding error because it avoids squaring $\boldsymbol{a}$ and $\boldsymbol{b}$ is $SE_{a*b} = \dfrac{ab\sqrt{t_a^2 + t_b^2 + 1}}{t_a t_b}$
  - This is known as the "Sobel test" and can be calculated by hand using the results of a simultaneous path model or separate regression models, and is also provided through MODEL INDIRECT or MODEL CONSTRAINT in Mplus

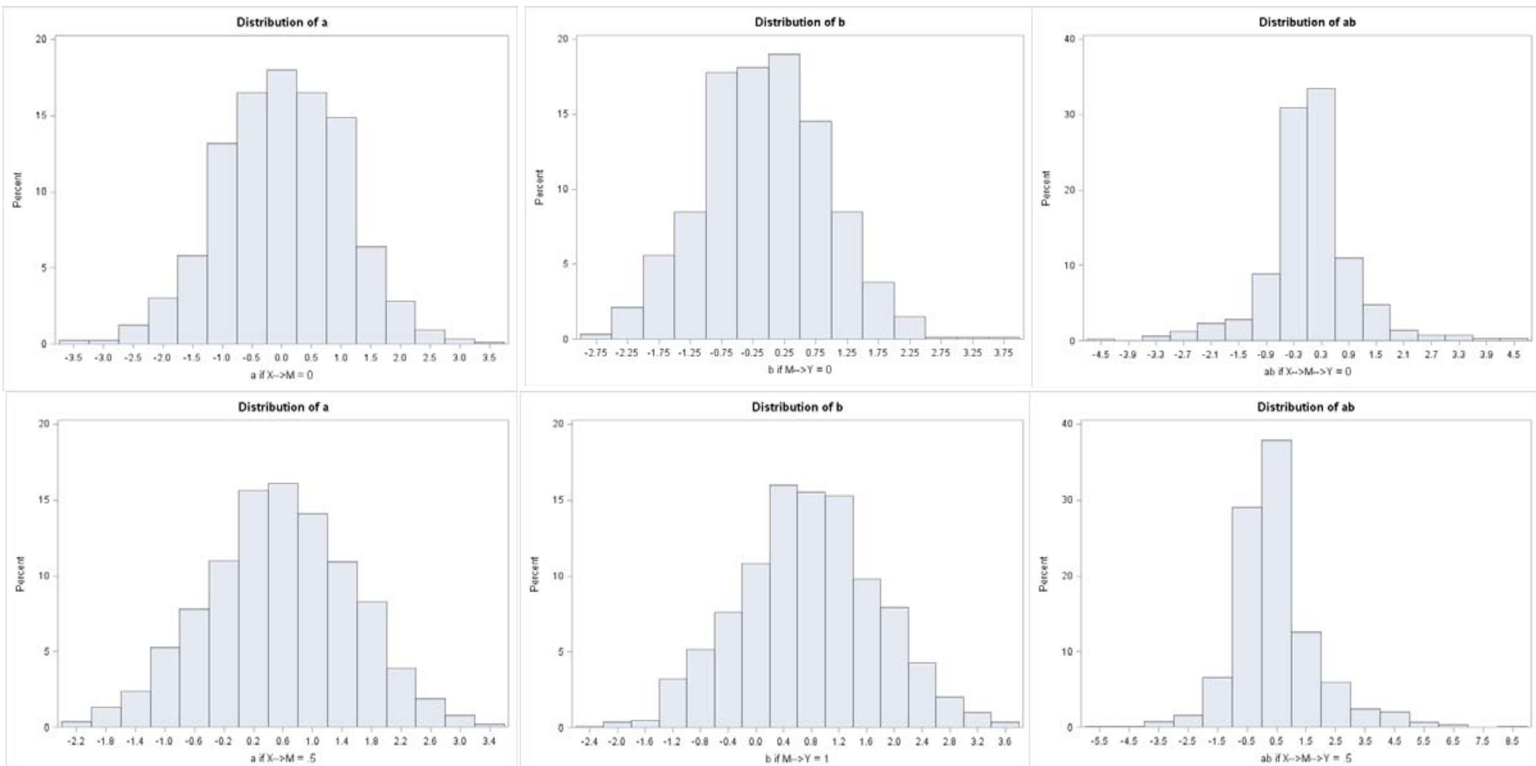
Testing Significance of Mediation

- One problem: we *shouldn't* use this SE for usual significance test
  - So, nope: $t_{indirect} = \frac{a*b}{SE_{a*b}}$ or $95\%\ CI = a * b \pm 1.96 * SE_{a*b}$
  - Why? Although the estimates for $\boldsymbol{a}$ and $\boldsymbol{b}$ will be normally distributed, the estimate of their product won't be, especially if a and b are near 0

Distribution of $\boldsymbol{a}$ | Distribution of $\boldsymbol{b}$ | Distribution of $\boldsymbol{a} * \boldsymbol{b}$

$\boldsymbol{a}$ = 0
$\boldsymbol{b}$ = 0

$\boldsymbol{a}$ = .50
$\boldsymbol{b}$ = .75



Testing Significance of Mediation

- So what do we do? Another idea based on same premise:
  - For $\boldsymbol{a} * \boldsymbol{b}$ → find "distribution of the product SE" → $\boldsymbol{z_a} * \boldsymbol{z_b} = \frac{\boldsymbol{a}}{\boldsymbol{SE_a}} * \frac{\boldsymbol{b}}{\boldsymbol{SE_b}}$ in which the sampling distribution does not have a tractable form, but tables of critical values have been derived through simulation for the single mediator case (but may not generalize to more complex models)
  - Implemented in PRODCLIN program for use with SAS, SPSS, and R

- A better solution: **bootstrap the data** to find the empirical SE and asymmetric CI for the indirect effect
  - Bootstrap = draw n samples with replacement from your **data**, re-estimate mediation model and calculate $\boldsymbol{a} * \boldsymbol{b}$ within each bootstrap sample
  - Point estimate of $\boldsymbol{a} * \boldsymbol{b}$ is mean or median over n bootstrap samples
  - SE_{a*b} is standard deviation of estimated $\boldsymbol{a} * \boldsymbol{b}$ over n bootstrap samples
  - 95% CI can be computed as estimates at the 2.5 and 97.5 percentiles
  - Typically at least 500 or 1000 n bootstrap samples are used

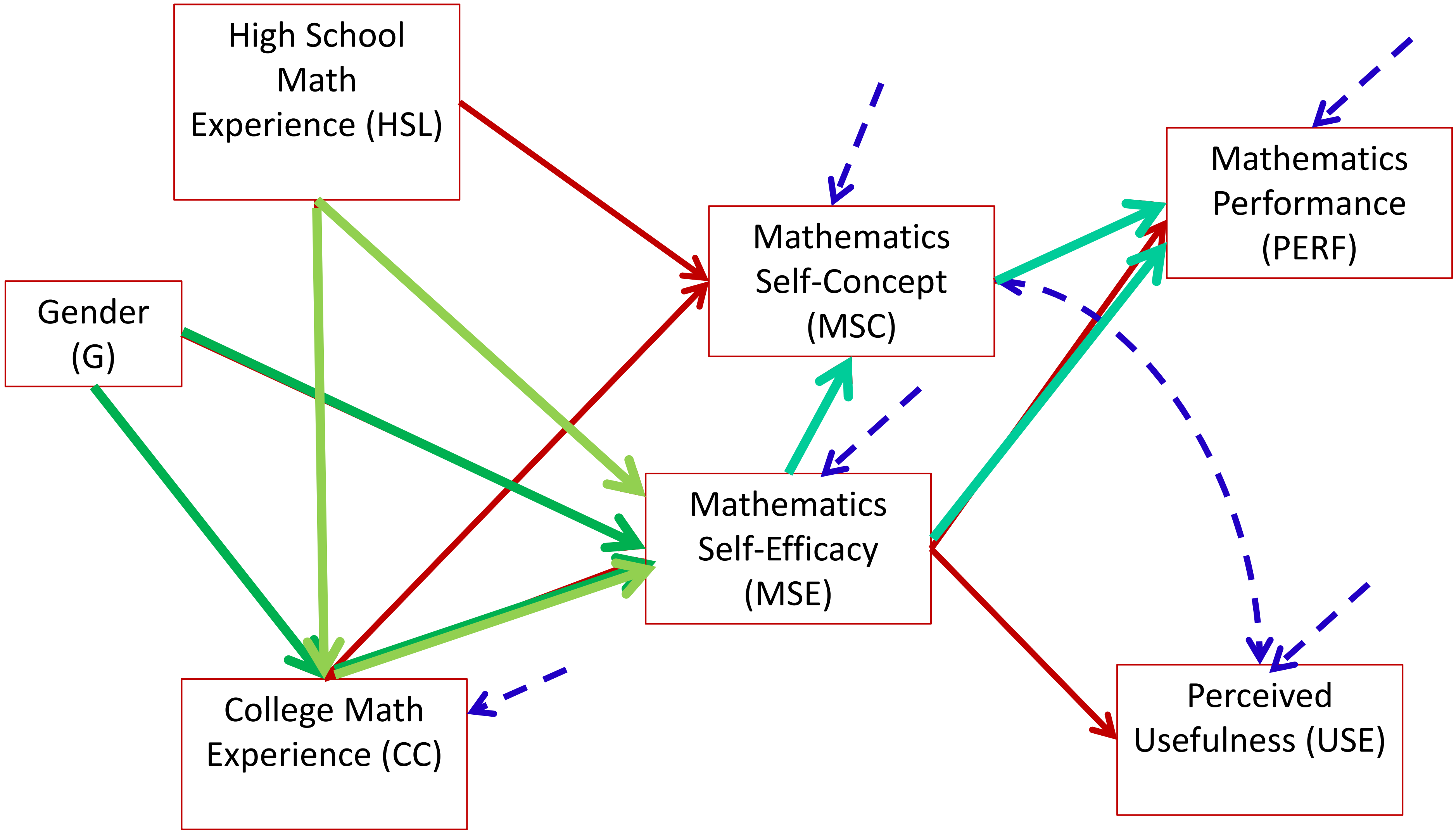
Testing Significance of Mediation

- There are multiple kinds of bootstrap CIs possible in testing the significance of the $a * b$ indirect effect within MVN data
  - Regular bootstrap CI = "**percentile**" (as just described)
    - In Mplus, OUTPUT: CINTERVAL(bootstrap);
  - **Bias-corrected bootstrap** CI = shifts CIs so that median is sample estimate
    - In Mplus, OUTPUT: CINTERVAL(BCbootstrap); **** Supposed to be best one*
  - Accelerated bootstrap CI = ???
    - Not given in Mplus (as far as I know)

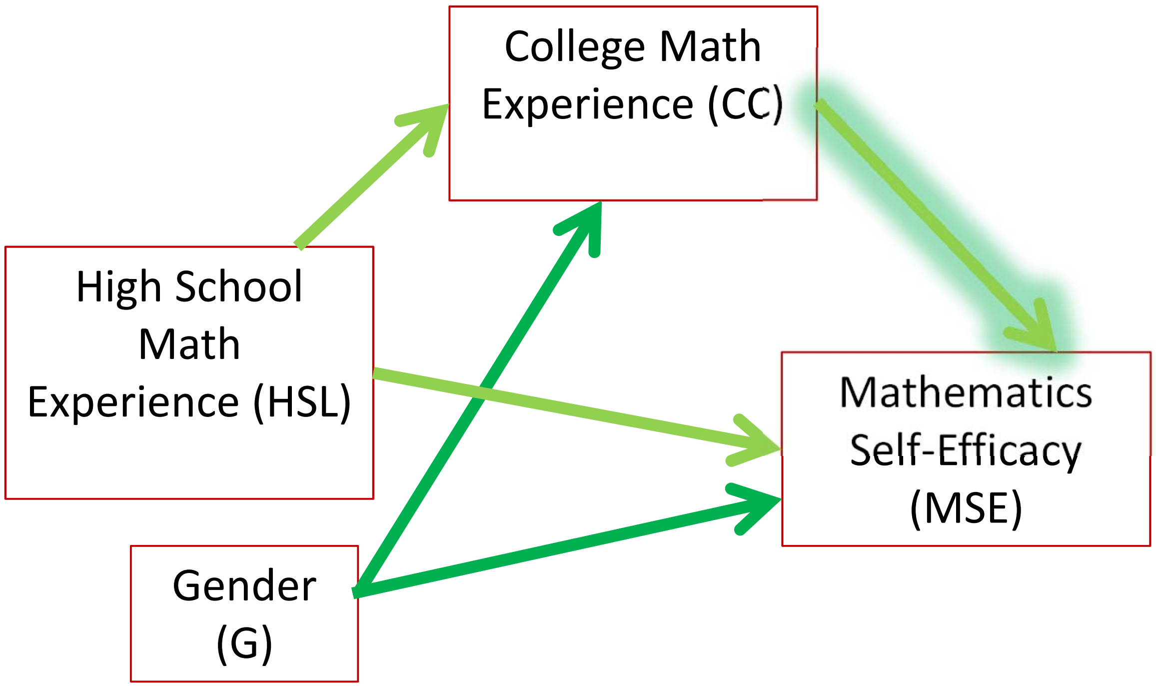
- For not simply MVN data (i.e., non-normal mediators or outcomes, multilevel data), a different bootstrap approach can be used
  - *Parametric*, *Monte Carlo*, or *empirical-M* bootstrap → Draw repeatedly from a and b parameter distributions instead of the data, then compute point estimates, SE, and CIs from those distributions
  - See http://www.quantpsy.org/medn.htm for online calculators

PREVIOUS EXAMPLE: INDIRECT EFFECTS

Final Example Model: Examining Mediation Effects



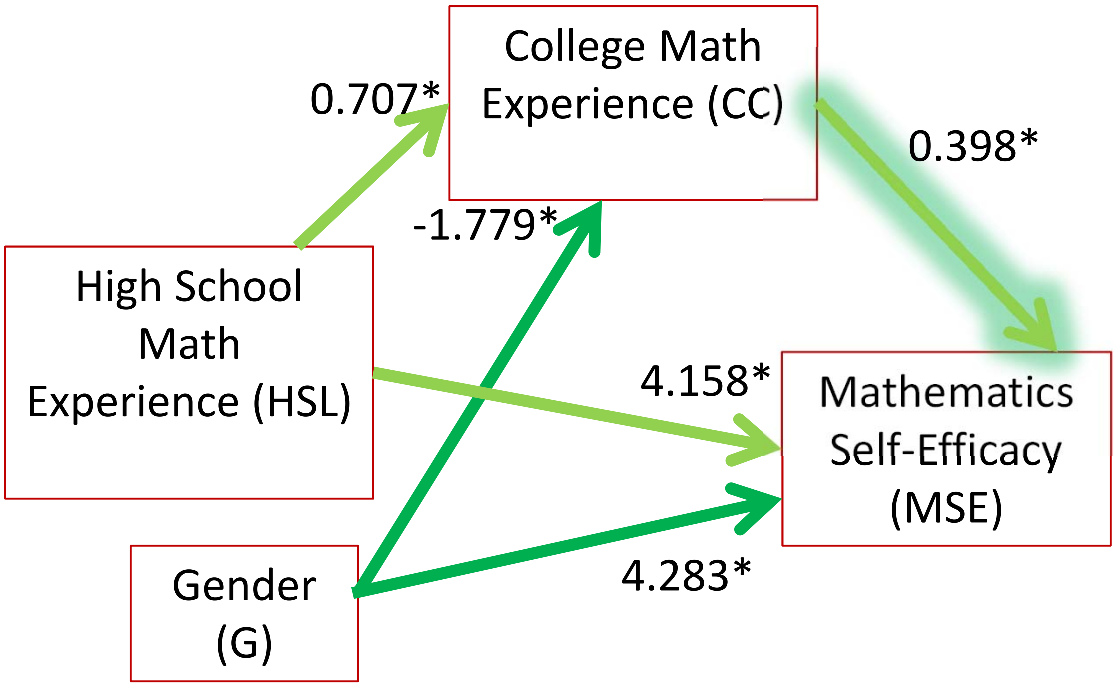
MSE Indirect Effects, Isolated



- Two potential pathways (indirect effects) from high school math and gender through college math to predict math self-efficacy

```
ANALYSIS:
    ESTIMATOR = ML;

MODEL:
    cc   ON hsl gender;
    mse  ON hsl gender cc;

    msc  ON hsl cc mse;
    use  ON mse;
    perf ON mse msc;
    hsl;
    perf WITH use@0;
    msc WITH use;

MODEL INDIRECT:
    mse IND hsl;
    mse IND gender;

OUTPUT:
    STDYX STDY
    CINTERVAL;
```

MSE *Direct* Effects Solutions using ML



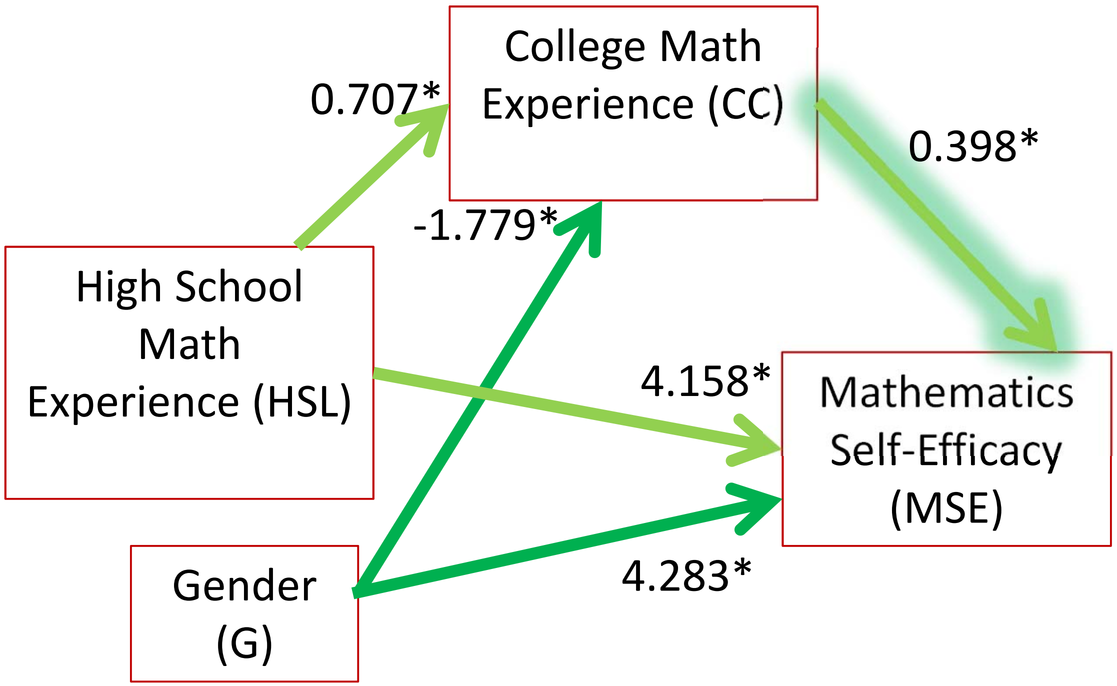
STANDARDIZED MODEL RESULTS

	StdYX Estimate	StdY Estimate
CC ON		
HSL	0.158	0.158
GENDER	-0.143	-0.301
MSE ON		
HSL	0.466	0.466
GENDER	0.172	0.363
CC	0.199	0.199

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
CC ON				
HSL	0.707	0.255	2.775	0.006
GENDER	-1.779	0.686	-2.595	0.019
MSE ON				
HSL	4.158	0.434	9.589	0.000
GENDER	4.283	1.180	3.631	0.000
CC	0.398	0.101	3.937	0.000

MSE *Indirect* Effects Solutions using ML: Sobel Test



Indirect Effects: a*b

HSL = 0.707 * 0.398 = 0.281

Gender = -1.779 * 0.398 = -0.707

Total Effects: direct + indirect

HSL = 4.158 + 0.281 = 4.439

Gender = 4.238 + -0.707 = 3.576

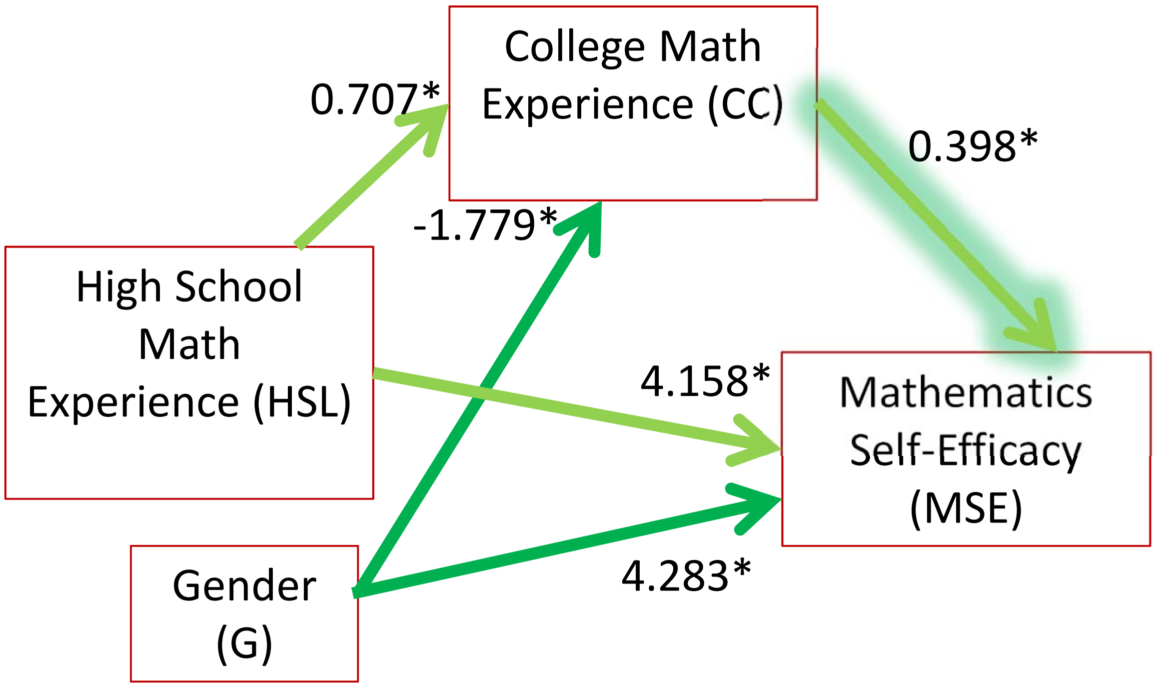
TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Effects from HSL to MSE				
Total	4.439	0.437	10.159	0.000
Specific indirect				
MSE CC HSL	0.281	0.121	2.324	0.020
Effects from GENDER to MSE				
Total	3.576	1.189	3.008	0.003
Specific indirect				
MSE CC GENDER	-0.707	0.329	-2.148	0.032

Conclusion:
The effects of high school math and gender on college math are *partially** responsible for the effects of high school math and gender on math self-efficacy.

* See Preacher & Kelly (2011) for a discussion of how to (and how not to) assess mediation effect size

MSE *Indirect* Effects: Bootstrapping to Double-Check




```
ANALYSIS:
    ESTIMATOR = ML;
    BOOTSTRAP = 1000;

MODEL:
    cc   ON hsl gender;
    mse  ON hsl gender cc;

    msc  ON hsl cc mse;
    use  ON mse;
    perf ON mse msc;
    hsl;
    perf WITH use@0;
    msc WITH use;

MODEL INDIRECT:
    mse IND hsl;
    mse IND gender;

OUTPUT:
    STDYX STDY
    CINTERVAL(BCBOOTSTRAP);
```

Normal-distribution 95% CI for indirect effects:

- HSL: Est = 0.281, CI = 0.044 to 0.518
- Gender: Est = -0.707, CI = -1.352 to -0.062

- Let's make sure the results are robust to an assumption of a normal distribution for the indirect effect by bootstrapping the data →

MSE *Direct* Effects Solutions: Regular ML vs. Bootstrap

MODEL RESULTS UNDER REGULAR ML

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
CC ON				
HSL	0.707	0.255	2.775	0.006
GENDER	-1.779	0.686	-2.595	0.019
MSE ON				
HSL	4.158	0.434	9.589	0.000
GENDER	4.283	1.180	3.631	0.000
CC	0.398	0.101	3.937	0.000

MODEL RESULTS USING BOOTSTRAPPING

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
CC ON				
HSL	0.707	0.246	2.871	0.004
GENDER	-1.779	0.695	-2.558	0.011
MSE ON				
HSL	4.158	0.412	10.086	0.000
GENDER	4.283	1.130	3.792	0.000
CC	0.398	0.109	3.645	0.000

MSE *Indirect* Effects Solutions: Regular ML vs. Bootstrap

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS: ML

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Effects from HSL to MSE				
Total	4.439	0.437	10.159	0.000
Specific indirect				
MSE CC HSL	0.281	0.121	2.324	0.020
Effects from GENDER to MSE				
Total	3.576	1.189	3.008	0.003
Specific indirect				
MSE CC GENDER	-0.707	0.329	-2.148	0.032

Normal distribution
95% CI for indirect effects:

HSL: CI = 0.281 ± 1.96*SE
CI = 0.044 to 0.518

Gender: -0.707 ± 1.96*SE
CI = -1.352 to -0.062

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS: BOOTSTRAP

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Effects from HSL to MSE				
Total	4.439	0.428	10.378	0.000
Specific indirect				
MSE CC HSL	0.281	0.119	2.352	0.019
Effects from GENDER to MSE				
Total	3.576	1.171	3.054	0.002
Specific indirect				
MSE CC GENDER	-0.707	0.358	-1.976	0.048

Empirical distribution
95% CI for indirect effects:

HSL: CI = 0.098 to 0.597
-0.316, +0.183 around Est

Gender: CI = -1.631 to -0.169
-0.539, +0.923 around Est

COMPLICATIONS

Mediation with Non-Normal Variables

- All the path models we've shown you so far assume every variable in the likelihood* is multivariate normal
  - * In the likelihood → is predicted by something or has an estimated mean, variance, or covariance with another variable (i.e., the missing data trick)
  - In reality, one may have non-normal (NN) mediators or outcomes…

- Estimation gets tricky, because there is no closed-form ML anymore
  - NN outcomes → fit link function to Y, requires numeric integration
    - Becomes exponentially more complex with more non-normal variables
  - NN mediators → fit link function M, but estimation is even trickier
    - In Mplus, requires Monte Carlo integration (re-sampling approach)

- Interpretation gets tricky, because the paths are of different kinds
  - For example, X → M → binary Y: X → regular M, M → logit Y
  - For example, X → binary M → Y: X → logit M, regular M → Y
  - Oh, and there are no standard absolute model fit statistics in ML (no observed covariance matrix to compare the model predictions to)

Robust Estimators for Not-Quite-Normal Variables

- In some cases it is clear that a link function is needed:
  - Binary or ordinal variables (fewer than 5 categories, usually)

- In other cases a link function might be preferable to use, but practically impossible to do in complex models
  - Count data or skewed continuous data
  - Weighted least squares estimators are sometimes used in this case, but they assume MCAR and use only a second-order summary of the data

- For not-quite-normal data, robust ML may be a reasonable solution
  - Still full-information ML (uses all data, not a summary thereof)
  - Corrects standard errors for multivariate non-normality

Robust ML for Non-Normal Data

- **MLR in Mplus**: ≈ Yuan-Bentler T_2 (permits MCAR or MAR missing)
  - Same estimates and LL, corrected standard errors for all model parameters
- **χ^2-based fit statistics** are adjusted based on an estimated **scaling factor:**
  - Scaling factor = 1.000 = perfectly multivariate normal = same as ML
  - Scaling factor > 1.000 = leptokurtosis (too-fat tails; fixes too big χ^2)
  - Scaling factor < 1.000 = platykurtosis (too-thin tails; fixes too small χ^2)
- **SEs** computed with Huber-White 'sandwich' estimator → uses an information matrix from the variance of the partial first derivatives to correct the information matrix from the partial second derivatives
  - Leptokurtosis (too-fat tails) → increases information; fixes too small SEs
  - Platykurtosis (too-thin tails) → lowers information; fixes too big SEs
- In **SAS**: use "EMPIRICAL" option in PROC MIXED line
  - SEs are computed the same way but for fixed effects only, but can be unstable in unbalanced data, especially in small samples
  - SAS does not provide the needed scaling factor to adjust -2ΔLL test (not sure if this is a problem if you just use the fixed effect *p*-values)

Scaled Likelihood Ratio Test for use with MLR

- Likelihood ratio test has a few extra steps:

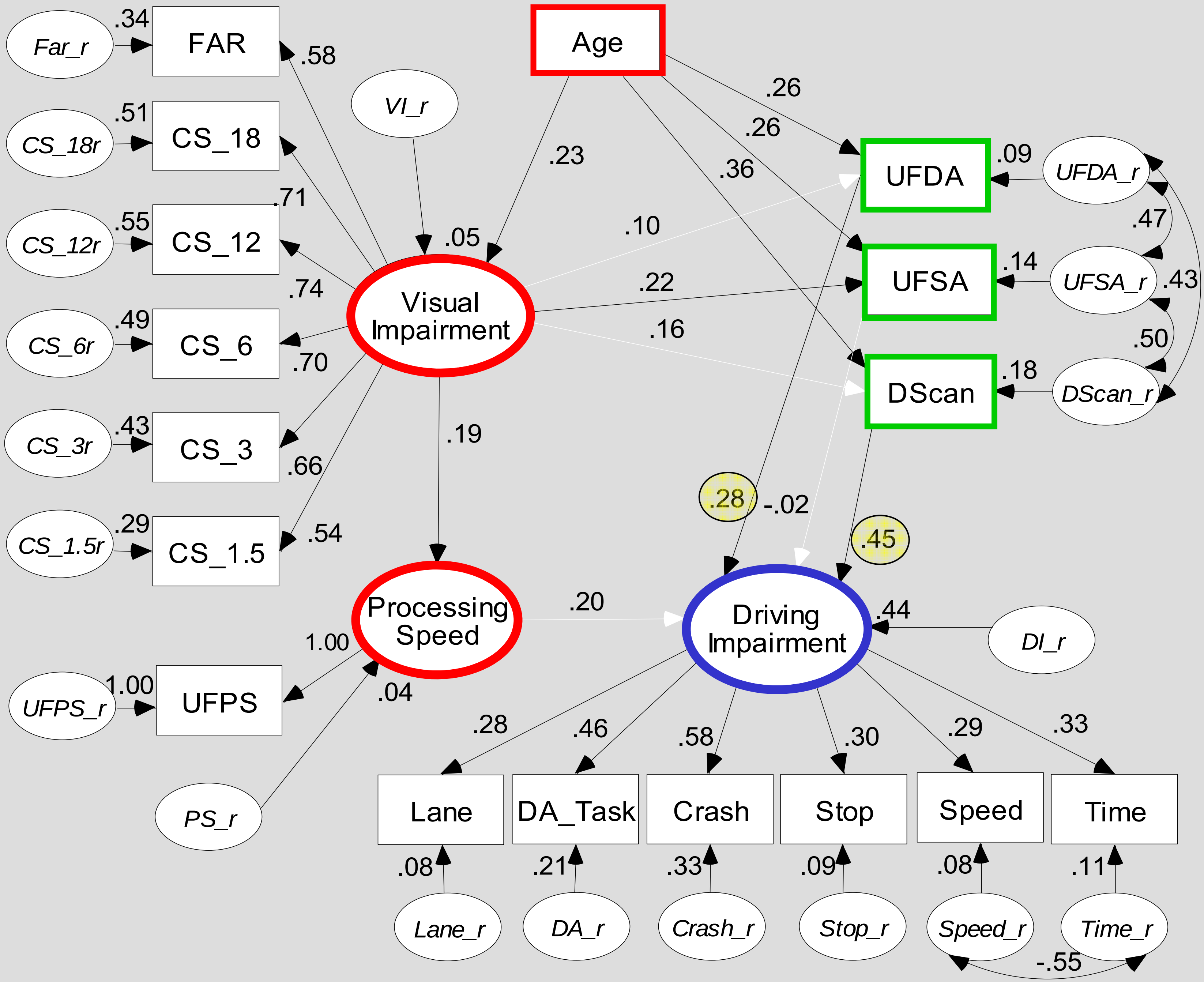
1. Calculate **-2ΔLL** = $-2*(LL_{fewer} – LL_{more})$
2. Calculate **difference scaling correction** =

$$\frac{(\#parms_{fewer}*scale_{fewer}) – (\#parms_{more}*scale_{more})}{(\#parms_{fewer} – \#parms_{more})}$$

3. Calculate **rescaled difference** = −2ΔLL / scaling correction
4. Calculate **Δdf** = $\#parms_{more} – \#parms_{fewer}$
5. **Compare rescaled difference to χ^2 with df = Δdf**
   - Add 1 parameter? $LL_{diff} > 3.84$, add 2: $LL_{diff} > 5.99$…
   - Absolute values of LL are meaningless (is relative fit only)
   - Process generalizes to many other kinds of models

- I built a spreadsheet to do this for you (see webpage)

EXAMPLE:
PREDICTING BINARY OUTCOME



Hoffman, McDowd, Atchley, & Dubinsky (2005, *Psychology and Aging*)

Hoffman & McDowd (2010, *Psychology and Aging*)

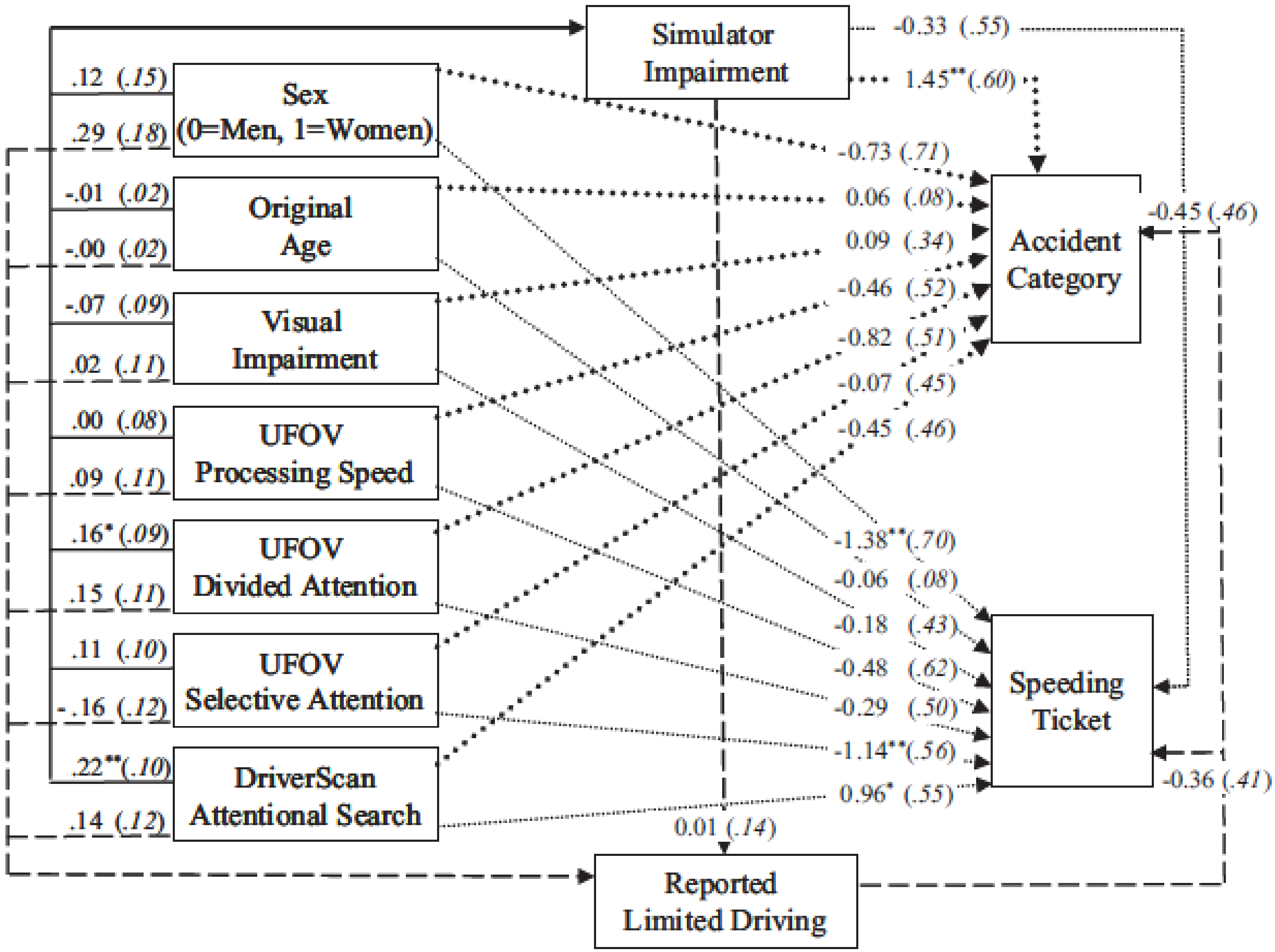
- Follow-up data from 114/152 persons from dissertation sample
  - 91 reported no accident since then, 9 reported no-fault accident
  - 14 reported at least partially-at-fault accident
  - 14 reported a speeding ticket
  - Tendency to limit driving (mean of 4 Likert items on 1-5 scale, 0 = 2)
  - Only 3 persons no longer drove

- No differences found between completers/non-completers in sex, age, visual impairment, UFOV, DriverScan, or simulator impairment

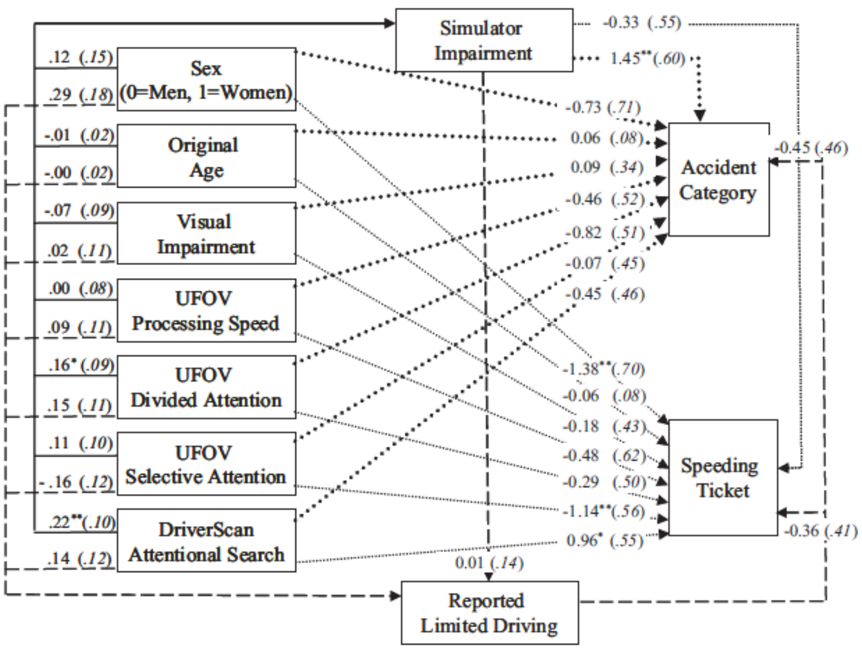
- Model: Predict accidents and speeding tickets (binary outcomes)

- Original analysis used ML with MonteCarlo Integration
  - I'll use MLR to demonstrate here → MVN then assumed for continuous mediators of simulator driving impairment and limiting driving

Path Model Predicting Driving Outcomes



Mplus Code for Direct and Indirect Effects

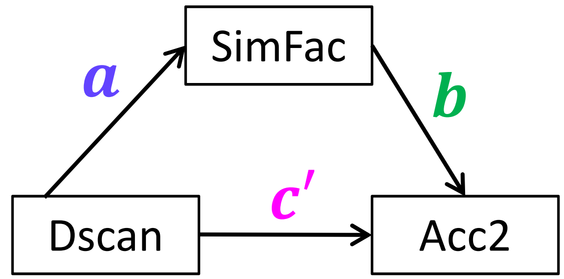



```
TITLE:   Path Analysis Dissertation Follow-up
DATA:    FILE = driver.dat;
VARIABLE:
! List of variables in data file
NAMES = PartID sex age75 cs_1_5 cs_3 cs_6
cs_12 cs_18 far near zufov1 zufov2 zufov3
Dscan lane da_task crash stop speed time
simfac part visfac attfac limit4 ticket2
speed2 follow attr nacc2 jacc2 acc2;
! Variables to be analyzed in this model
USEVARIABLE = sex age75 visfac zufov1 zufov2
zufov3 Dscan simfac limit4 speed2 acc2;
! Missing data identifier
MISSING = .;
! Categorical outcomes
CATEGORICAL = acc2 speed2;
ANALYSIS: ! Estimation options
ESTIMATOR = MLR; INTEGRATION = MONTECARLO;
OUTPUT: STDYX;
```

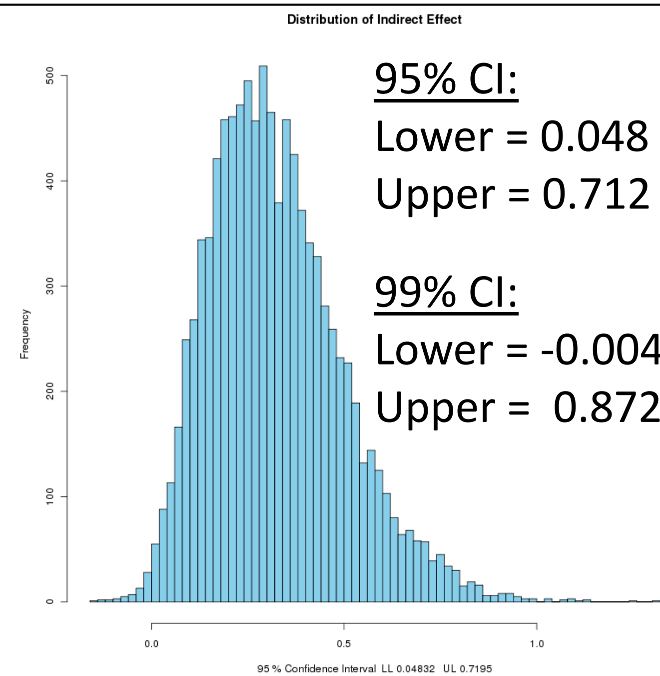
```
MODEL:
simfac ON sex age75 visfac zufov1 zufov2 zufov3 Dscan               (sim1-sim7);
limit4 ON sex age75 visfac zufov1 zufov2 zufov3 Dscan simfac        (lim1-lim8);
acc2   ON sex age75 visfac zufov1 zufov2 zufov3 Dscan simfac limit4 (acc1-acc9);
speed2 ON sex age75 visfac zufov1 zufov2 zufov3 Dscan simfac limit4 (spd1-spd9);

MODEL CONSTRAINT:        ! Like ESTIMATE in SAS
NEW(DStoAcc);            ! List names of estimated effects on NEW
DStoAcc = sim7 * acc8;   ! Indirect effect of Dscan --> Sim --> Acc
```

Mplus Output for Direct and Indirect Effects (Truncated)

```
MODEL FIT INFORMATION
Number of Free Parameters                        39
Loglikelihood
          H0 Value                         -356.400
          H0 Scaling Correction Factor       1.0066
            for MLR
Information Criteria
          Akaike (AIC)                      790.799
          Bayesian (BIC)                    907.953
          Sample-Size Adjusted BIC          784.529
            (n* = (n + 2) / 24)


MODEL RESULTS
                                                      Two-Tailed
                 Estimate       S.E.  Est./S.E.    P-Value
SIMFAC     ON
   DSCAN            0.216      0.081      2.661      0.008

ACC2       ON
   DSCAN           -0.477      0.320     -1.491      0.136
   SIMFAC           1.497      0.532      2.813      0.005

New/Additional Parameters
   DSTOACC          0.323      0.160      2.026      0.043
```



Then used Monte Carlo resampling to assess empirical distribution of indirect effect via this web utility: http://www.quantpsy.org/medn.htm



95% CI:
Lower = 0.048
Upper = 0.712

99% CI:
Lower = -0.004
Upper = 0.872

Summary

- Path models are a very useful way to examine many different multivariate hypotheses simultaneously:
  - Unique direct and indirect effects (“mediation”)
  - Differences in effect size (via model constraints)
  - Relationships among mediators or outcomes

- Good fit is a pre-requisite to actually interpreting the model results, but good fit does *not* mean it is a good model
  - Good fit = model reproduces the covariance matrix of the endogenous variables (but it does not indicate how big or small those relationships are)
  - However – when all possible relationships among variables are estimated (either as covariances or direct regressions), fit is perfect
    - We used to call this “regression” or in PROC MIXED, “ unstructured R matrix”

- Endogenous variables can have any distribution, but…
  - Estimation is much easier if they are MVN (use robust ML if not)
  - Absolute model fit is not provided by most software