

More Practice with Main Effects and Interactions in General Linear Models

The models for this example come from Hoffman (in preparation) chapter 2. We will be examining the extent to which cognition (as measured by an information test outcome) can be predicted from age (centered at 85 years) grip strength (centered at 9 pounds), sex (with men as the reference group) and dementia status (none, future, current, with none as the reference) in a sample of 550 older adults.

SAS Syntax and Output for Data Manipulation:

```
* Location for SAS files for these models - change this path to run on your computer;
%LET filesave= F:\Example Data\943\Example2;
LIBNAME filesave "&filesave.";
```

```
TITLE "Descriptive Statistics for Example Variables in Original Data";
PROC FREQ DATA=filesave.example2; TABLE sexMW demgroup; RUN;
PROC MEANS DATA=filesave.example2; VAR cognition age grip; RUN;
TITLE;
```

sexmw: Sex (0=Men, 1=Women)				
sexmw	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0	227	41.27	227	41.27
1	323	58.73	550	100.00

demgroup: Dementia Diagnosis (1=None, 2=Future, 3=Current)				
demgroup	Frequency	Percent	Cumulative Frequency	Cumulative Percent
1	399	72.55	399	72.55
2	109	19.82	508	92.36
3	42	7.64	550	100.00

Variable	Label	N	Mean	Std Dev	Minimum	Maximum
cognition	cognition: Cognitive Outcome	550	24.8218182	10.9890273	0	44.0000000
age	age: Age in Years	550	84.9267863	3.4300288	80.0164932	96.9672832
grip	grip: Grip Strength in Pounds	550	9.1127273	2.9829543	0	19.0000000

```
* Import example data into work library and center predictors;
DATA work.example2; SET filesave.example2;
* Center continuous predictors;
age85 = age - 85; LABEL age85= "age85: Age in Years (0=85)";
grip9 = grip - 9; LABEL grip9= "grip9: Grip Strength in Pounds (0=9)";
* Creating manual contrasts between groups (demNF, demNC) to map onto model equation;
IF demgroup=1 THEN DO; demNF=0; demNC=0; END; * None;
ELSE IF demgroup=2 THEN DO; demNF=1; demNC=0; END; * Future;
ELSE IF demgroup=3 THEN DO; demNF=0; demNC=1; END; * Current;
ELSE DO; demNF=.; demNC=.; END; * Otherwise is missing;
LABEL demNF= "demNF: None vs. Future Dementia"
demNC= "demNC: None vs. Current Dementia";
RUN;
```

The accompanying syntax file then has three macro programs I wrote that do the following:

R2Change: calculates an F-test for the improvement in R^2 from nested models

Asymp: calculates the asymptotic covariance matrix of the fixed effects needed for...

Regions: calculates regions of significance for continuous predictor interactions

SAS PROC GLM Syntax and Output for Equation 2.7, in which the i subscript indicates person:

$$\text{Cognition}_i = \beta_0 + \beta_1 (\text{Age}_i - 85) + \beta_2 (\text{Grip}_i - 9) + \beta_3 (\text{SexMW}_i) + e_i$$

```
TITLE1 "Eq 2.7: Model with Age, Grip, Sex (0=Men, 1=Women)";
PROC GLM DATA=work.example2;
* Model Y = fixed effects, options after / print extra output;
  MODEL cognition = age85 grip9 sexMW / SOLUTION SS3 EFFECTSIZE;
* Save pieces of output for use in macros (optional);
  ODS OUTPUT FitStatistics=work.Rsq2_7;
* Intercepts per sex (hold age=85, grip=9);
  ESTIMATE "Intercept for Men"   intercept 1 sexMW 0;
  ESTIMATE "Intercept for Women" intercept 1 sexMW 1;
* Need both RUN and QUIT, shut off title;
RUN; QUIT; TITLE1;
```

The EFFECTSIZE option provides additional output related to η^2 and ω^2 (not shown to save space). The ODS OUTPUT is saving the R^2 for use later.

ANOVA Test of Significance for Model R^2

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	6574.67775	2191.55925	20.04	<.0001
Error	546	59721.86043	109.38070		
Corrected Total	549	66296.53818			

Model R^2

R-Square	Coeff Var	Root MSE	cognition Mean
0.099171	42.13439	10.45852	24.82182

ANOVA Table of F-tests for Fixed Effects (*won't be uniquely useful, so is omitted from now on*)

Source	DF	Type III SS	Mean Square	F Value	Pr > F
age85	1	1172.922323	1172.922323	10.72	0.0011
grip9	1	1179.415578	1179.415578	10.78	0.0011
sexmw	1	1609.324132	1609.324132	14.71	0.0001

ESTIMATE Statement Results: Model-Implied Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept for Men	26.9594307	0.73887292	36.49	<.0001
Intercept for Women	23.1606507	0.60034925	38.58	<.0001

Solution for Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept	26.95943069	0.73887292	36.49	<.0001
age85	-0.43377191	0.13246378	-3.27	0.0011
grip9	0.54600194	0.16627659	3.28	0.0011
sexmw	-3.79877994	0.99035915	-3.84	0.0001

Interpret each fixed effect:

Intercept $\beta_0 =$

Main effect of Age $\beta_1 =$

Main effect of Grip Strength $\beta_2 =$

Main effect of Sex $\beta_3 =$

The intercept for women is calculated from:

SAS PROC GLM Syntax and Output for Equation 2.8, adding 2 contrasts for dementia group:

$$\text{Cognition}_i = \beta_0 + \beta_1 (\text{Age}_i - 85) + \beta_2 (\text{Grip}_i - 9) + \beta_3 (\text{SexMW}_i) + \beta_4 (\text{DemNF}_i) + \beta_5 (\text{DemNC}_i) + e_i$$

We can use the model equation to calculate the **dementia group means** for predicted cognition:

$$\begin{aligned} \text{Cognition for None} &= \beta_0 \\ \text{Cognition for Future} &= \beta_0 + \beta_4 \\ \text{Cognition for Current} &= \beta_0 + \beta_5 \end{aligned}$$

We can determine the **differences between the dementia group means** as follows:

$$\begin{aligned} \text{None vs. Future} &= \text{Future} - \text{None} = (\beta_0 + \beta_4) - (\beta_0) = \beta_4 \\ \text{None vs. Current} &= \text{Current} - \text{None} = (\beta_0 + \beta_5) - (\beta_0) = \beta_5 \\ \text{Future vs. Current} &= \text{Current} - \text{Future} = (\beta_0 + \beta_5) - (\beta_0 + \beta_4) = \beta_5 - \beta_4 = -\beta_4 + \beta_5 \end{aligned}$$

These values are then requested via the ESTIMATE statements below...

```
TITLE1 "Eq 2.8: Model adding Dementia Group (demNF, demNC) as manual contrasts";
PROC GLM DATA=work.example2;
* Model Y = fixed effects, options after / print extra output;
  MODEL cognition = age85 grip9 sexMW demNF demNC / SOLUTION SS3 EFFECTSIZE;
* Request group means (hold age=85, grip=9, men);
  ESTIMATE "Intercept for None"    intercept 1 demNF 0 demNC 0;
  ESTIMATE "Intercept for Future"  intercept 1 demNF 1 demNC 0;
  ESTIMATE "Intercept for Current" intercept 1 demNF 0 demNC 1;
* Request group differences;
  ESTIMATE "None vs. Future"      demNF 1 demNC 0;
  ESTIMATE "None vs. Current"     demNF 0 demNC 1;
  ESTIMATE "Future vs. Current"   demNF -1 demNC 1;
* Save pieces of output for use in macros (optional);
  ODS OUTPUT FitStatistics=work.Rsq2_8;
* Need both RUN and QUIT, shut off titles;
RUN; QUIT; TITLE1;
```

ANOVA Test for Model R²

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	18385.97930	3677.19586	41.75	<.0001
Error	544	47910.55888	88.07088		
Corrected Total	549	66296.53818			

R-Square	Coeff Var	Root MSE	cognition Mean
0.277329	37.80790	9.384609	24.82182

ESTIMATES: Requested Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept for None	29.2643254	0.69850792	41.90	<.0001
Intercept for Future	23.5423544	1.07852978	21.83	<.0001
Intercept for Current	12.7845121	1.53019333	8.35	<.0001
None vs. Future	-5.7219710	1.01907848	-5.61	<.0001
None vs. Current	-16.4798133	1.52275357	-10.82	<.0001
Future vs. Current	-10.7578423	1.70795708	-6.30	<.0001

Solution for Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept	29.26432541	0.69850792	41.90	<.0001
age85	-0.40573396	0.11889717	-3.41	0.0007
grip9	0.60422556	0.14977568	4.03	<.0001
sexmw	-3.65737421	0.89143262	-4.10	<.0001
demNF	-5.72197100	1.01907848	-5.61	<.0001
demNC	-16.47981327	1.52275357	-10.82	<.0001

I wrote a macro program to calculate the **R² change F-test** from adding these 2 fixed effects for dementia group (i.e., a df=2 test rather than for each separate contrast), which is called below:

```
/* To use R2Change Macro, enter:
   N=                Total Sample Size
   DataL=            Name of dataset that stores R2 for larger model (on ODS)
   NpredL=           Number of predictors in larger model
   DataS=            Name of dataset that stores R2 for smaller model (on ODS)
   NpredS=           Number of predictors in smaller model */

* Call macro to test R2 change from Eq 2_8 vs. Eq 2_7;
%R2Change(N=550, DataL=work.Rsq2_8, NpredL=5, DataS=work.Rsq2_7, NpredS=3);
```

Which then generates this output, indicating that the overall df=2 effect of group is significant

R2 Change F-Test for work.Rsq2_8 (L) vs. work.Rsq2_7 (S)

Rsquare	RsquareS	Rsquare	Npred	Npred	DFnum	DFdenom	Fvalue	Pvalue
L		Diff	L	S				
0.27733	0.099171	0.17816	5	3	2	544	67.0557	0

SAS PROC GLM Syntax and Output for Equation 2.9, adding interaction of age*grip:

$$\text{Cognition}_i = \beta_0 + \beta_1 (\text{Age}_i - 85) + \beta_2 (\text{Grip}_i - 9) + \beta_3 (\text{SexMW}_i) + \beta_4 (\text{DemNF}_i) + \beta_5 (\text{DemNC}_i) + \beta_6 (\text{Age}_i - 85)(\text{Grip}_i - 9) + e_i$$

We can use the model equation to calculate the **simple effect of age** at any level of *grip strength* (age as the effect, grip strength as the moderator):

$$\text{Simple Effect of Age} = \beta_1 + \beta_6(\text{Grip}_i - 9)$$

Likewise, we can use the model equation to calculate the **simple effect of grip strength** at any level of *age* (grip strength as the effect, age as the moderator):

$$\text{Simple Effect of Grip} = \beta_2 + \beta_6(\text{Age}_i - 85)$$

Examples of these are requested via the ESTIMATE statements...

```
TITLE1 "Eq 2.9: Model with Age by Grip Interaction";
PROC GLM DATA=work.example2;
* Model Y = fixed effects, options after / print extra output;
MODEL cognition = age85 grip9 sexMW demNF demNC age85*grip9
/ SOLUTION SS3 EFFECTSIZE INVERSE;
* Request simple effects at other values of age and grip;
ESTIMATE "Age Effect for Grip=6" age85 1 age85*grip9 -3;
ESTIMATE "Age Effect for Grip=9" age85 1 age85*grip9 0;
ESTIMATE "Age Effect for Grip=12" age85 1 age85*grip9 3;

ESTIMATE "Grip Effect for Age=80" grip9 1 age85*grip9 -5;
ESTIMATE "Grip Effect for Age=85" grip9 1 age85*grip9 0;
ESTIMATE "Grip Effect for Age=90" grip9 1 age85*grip9 5;
* Save pieces of output for use in macros (optional);
ODS OUTPUT FitStatistics=work.Rsq2_9 InvXPX=work.Inv2_9
OverallANOVA=work.Ano2_9 ParameterEstimates=work.Fix2_9;
* Need both RUN and QUIT, shut off title;
RUN; QUIT; TITLE1;
```

The INVERSE option is needed for a macro program that calculates regions of significance for the interaction.

The ODS command is saving (in order): R², matrix inverse of XTX, ANOVA R² table, and the solution for fixed effects for the macro programs that follow.

ANOVA Test for Model R²

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	19185.04106	3197.50684	36.85	<.0001
Error	543	47111.49712	86.76150		
Corrected Total	549	66296.53818			

R-Square	Coeff Var	Root MSE	cognition Mean
0.289382	37.52580	9.314586	24.82182

ESTIMATES: Requested Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Age Effect for Grip=6	-0.70301601	0.15336958	-4.58	<.0001
Age Effect for Grip=9	-0.33396058	0.12035656	-2.77	0.0057
Age Effect for Grip=12	0.03509486	0.18715387	0.19	0.8513
Grip Effect for Age=80	0.00432624	0.24733508	0.02	0.9861
Grip Effect for Age=85	0.61941863	0.14874241	4.16	<.0001
Grip Effect for Age=90	1.23451102	0.25540829	4.83	<.0001

Solution for Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept	29.40780315	0.69490615	42.32	<.0001
age85	-0.33396058	0.12035656	-2.77	0.0057
grip9	0.61941863	0.14874241	4.16	<.0001
sexmw	-3.45563720	0.88727488	-3.89	0.0001
demNF	-5.92254309	1.01363159	-5.84	<.0001
demNC	-16.30040485	1.51254730	-10.78	<.0001
age85*grip9	0.12301848	0.04053626	3.03	0.0025

Interpret these fixed effects:

Simple main effect of Age $\beta_1 =$

Simple main effect of Grip Strength $\beta_2 =$

Interpret Age by Grip Strength $\beta_6 \rightarrow$ Age as Simple Effect, Grip as Moderator via $\beta_1 + \beta_6(\text{Grip}_i - 9)$:

Interpret Age by Grip Strength $\beta_6 \rightarrow$ Grip as Simple Effect, Age as Moderator via $\beta_2 + \beta_6(\text{Age}_i - 85)$:

Calculate predicted values for “fake people” to plot interaction:

$$\text{Predicted Cognition}_i = \beta_0 + \beta_1 (\text{Age}_i - 85) + \beta_2 (\text{Grip}_i - 9) + \beta_6 (\text{Age}_i - 85)(\text{Grip}_i - 9)$$

$$\text{Grip Strength} = 12, \text{Age} = 80: 29.41 - 0.33(-5) + 0.62(3) + 0.12(-5)(3) = 31.12$$

$$\text{Grip Strength} = 12, \text{Age} = 85: 29.41 - 0.33(0) + 0.62(3) + 0.12(0)(3) = 31.27$$

$$\text{Grip Strength} = 12, \text{Age} = 90: 29.41 - 0.33(5) + 0.62(3) + 0.12(5)(3) = 31.42$$

$$\text{Grip Strength} = 9, \text{Age} = 80: 29.41 - 0.33(-5) + 0.62(0) + 0.12(-5)(0) = 31.06$$

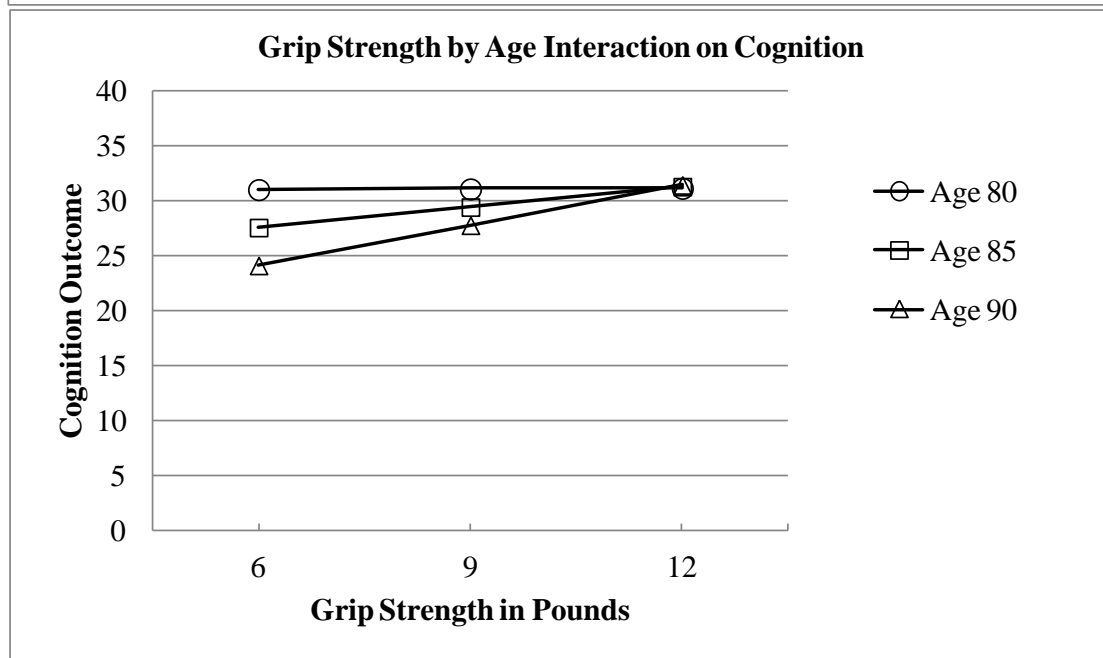
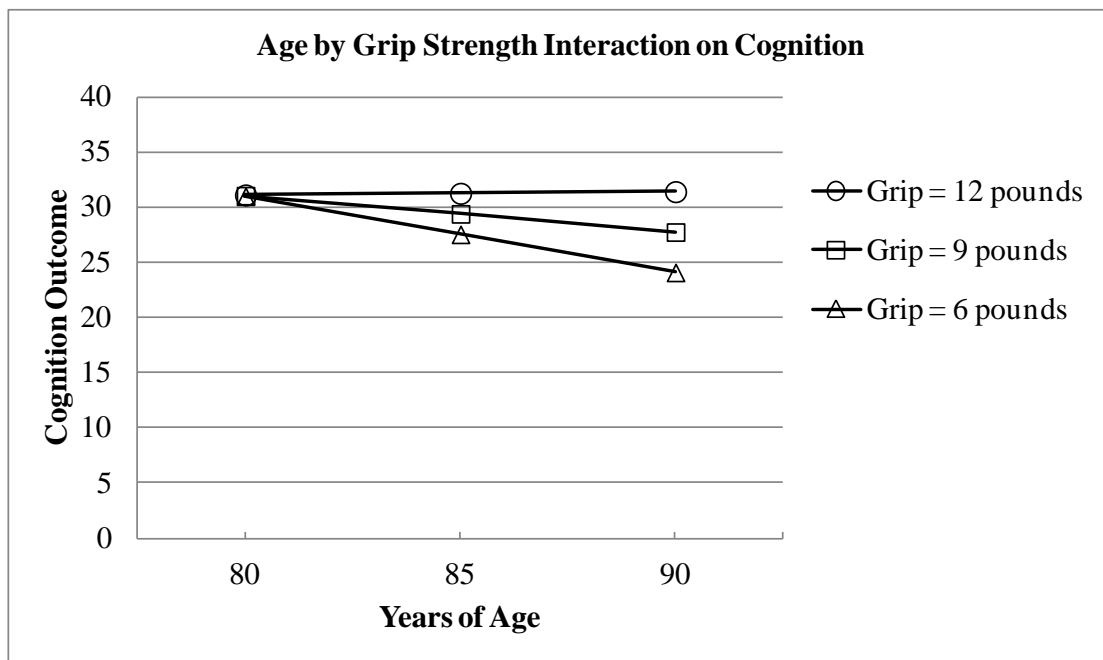
$$\text{Grip Strength} = 9, \text{Age} = 85: 29.41 - 0.33(0) + 0.62(0) + 0.12(0)(0) = 29.41$$

$$\text{Grip Strength} = 9, \text{Age} = 90: 29.41 - 0.33(5) + 0.62(0) + 0.12(5)(0) = 27.76$$

$$\text{Grip Strength} = 6, \text{Age} = 80: 29.41 - 0.33(-5) + 0.62(-3) + 0.12(-5)(-3) = 31.00$$

$$\text{Grip Strength} = 6, \text{Age} = 85: 29.41 - 0.33(0) + 0.62(-3) + 0.12(0)(-3) = 27.55$$

$$\text{Grip Strength} = 6, \text{Age} = 90: 29.41 - 0.33(5) + 0.62(-3) + 0.12(5)(-3) = 24.10$$



I wrote another macro program to create the **asymptotic covariance matrix**, which has the variances and covariances among the fixed effects, which is called below:

```
/* To Use Asymp Macro, enter:
    DV=           Name of dependent variable
    AnoData=      Name of dataset that stores MSE for model (on ODS)
    InvData=      Name of dataset that stores X'X inv matrix for model (on ODS)
    Npred=       Number of predictors in model */

* Call macro to make asymptotic covariance matrix;
%Asymp(DV=cognition, AnoData=work.Ano2_9, InvData=work.Inv2_9, Npred=6);
```

The asymptotic covariance matrix is needed in order to calculate what are known as **regions of significance**, which describe the values of the moderator as which the simple main effects turn on or off (for significance). I wrote a macro program to do this as well, which is called twice below:

```
/* To use Regions Macro, enter:
    FixData=      Name of dataset that stores fixed effects for model (on ODS)
    InvData=      Name of dataset that stores X'X inv matrix for model (on ODS)
    Pred=         Case-sensitive name of predictor effect regions are for
    Mod=          Case-sensitive name of moderator effect (for region values)
    ModCenter=    Centering point of moderator predictor
    Interact=     Case-sensitive name of interaction effect */

* Call macro to calculate regions of significance for age effect;
%Regions(FixData=work.Fix2_9, InvData=work.Inv2_9, Pred=age85, Mod=grip9, ModCenter=9,
        Interact=age85*grip9);

* Call macro to calculate regions of significance for grip effect;
%Regions(FixData=work.Fix2_9, InvData=work.Inv2_9, Pred=grip9, Mod=age85, ModCenter=85,
        Interact=age85*grip9);
```

These macro programs then provide the following output for this model:

The effect of age85 will be significant at centered values of grip9 BELOW the lower bound and ABOVE the upper bound, which translate to these uncentered lower and upper bounds.

Centered	Centered	Uncentered	Uncentered
Lower	Upper	Lower	Upper
0.66813	9.48171	9.66813	18.4817

The effect of grip9 will be significant at centered values of age85 BELOW the lower bound and ABOVE the upper bound, which translate to these uncentered lower and upper bounds.

Centered	Centered	Uncentered	Uncentered
Lower	Upper	Lower	Upper
-14.8144	-2.28566	70.1856	82.7143

To better understand what this is telling us, I requested the simple main effects of each predictor at these lower and upper moderating values by adding to the PROC GLM syntax from before:

```
ESTIMATE "Age Effect for Grip=9.66813"      age85 1 age85*grip9 0.66813;
ESTIMATE "Age Effect for Grip=18.4817"      age85 1 age85*grip9 9.66813;

ESTIMATE "Grip Effect for Age=70.1856"      grip9 1 age85*grip9 -14.8144;
ESTIMATE "Grip Effect for Age=82.7143"      grip9 1 age85*grip9 -2.28566;
```

ESTIMATES: Requested Fixed Effects		Standard		
Parameter	Estimate	Error	t Value	Pr > t
Age Effect for Grip=9.66813	-0.25176824	0.12845343	-1.96	0.0505
Age Effect for Grip=18.4817	0.85539806	0.43199146	1.98	0.0482
Grip Effect for Age=70.1856	-1.20302631	0.61378846	-1.96	0.0505
Grip Effect for Age=82.7143	0.33824021	0.17257180	1.96	0.0505

Accordingly, the effect of age will be significantly negative below grip strength = 9.66 pounds, nonsignificant between 9.66 and 18.48 pounds, and significantly positive after 18.48 pounds. Likewise, the effect of grip strength will be significantly negative below age = 70.19 years, nonsignificant between age = 70.19 and 82.71 years, and significantly positive after age = 82.71 years.

SAS PROC GLM Syntax and Output for Equation 2.8, adding 2 contrasts for dementia group:

$$\begin{aligned} \text{Cognition}_i = & \beta_0 + \beta_1 (\text{Age}_i - 85) + \beta_2 (\text{Grip}_i - 9) + \beta_3 (\text{SexMW}_i) \\ & + \beta_4 (\text{DemNF}_i) + \beta_5 (\text{DemNC}_i) + \beta_6 (\text{Age}_i - 85)(\text{Grip}_i - 9) \\ & + \beta_7 (\text{SexMW}_i)(\text{DemNF}_i) + \beta_8 (\text{SexMW}_i)(\text{DemNC}_i) + e_i \end{aligned}$$

We can use the model equation to calculate the **sex and dementia group means** for cognition:

	<u>For Men:</u>	<u>For Women:</u>
Cognition for None =	β_0	$\beta_0 + \beta_3$
Cognition for Future =	$\beta_0 + \beta_4$	$\beta_0 + \beta_4 + \beta_3 + \beta_7$
Cognition for Current =	$\beta_0 + \beta_5$	$\beta_0 + \beta_5 + \beta_3 + \beta_8$

We can determine the **differences between the sex and dementia group means** as follows:

For Men:

None vs. Future =	Future – None =	$(\beta_0 + \beta_4) - (\beta_0)$	= β_4
None vs. Current =	Current – None =	$(\beta_0 + \beta_5) - (\beta_0)$	= β_5
Future vs. Current =	Current – Future =	$(\beta_0 + \beta_5) - (\beta_0 + \beta_4)$	= $-\beta_4 + \beta_5$

For Women:

None vs. Future =	Future – None =	$(\beta_0 + \beta_4 + \beta_3 + \beta_7) - (\beta_0 + \beta_3)$	= $\beta_4 + \beta_7$
None vs. Current =	Current – None =	$(\beta_0 + \beta_5 + \beta_3 + \beta_8) - (\beta_0 + \beta_3)$	= $\beta_5 + \beta_8$
Future vs. Current =	Current – Future =	$(\beta_0 + \beta_5 + \beta_3 + \beta_8) - (\beta_0 + \beta_4 + \beta_3 + \beta_7)$	= $-\beta_4 + \beta_5 - \beta_7 + \beta_8$

These values are then requested via the ESTIMATE statements below...

```
TITLE1 "Eq 2.13: Add Sex by Dementia Interaction";
PROC GLM DATA=work.example2;
* Model Y = fixed effects, options after / print extra output;
  MODEL cognition = age85 grip9 sexMW demNF demNC age85*grip9 sexMW*demNF sexMW*demNC
    / SOLUTION SS3 EFFECTSIZE;
* Save pieces of output for use in macros (optional);
  ODS OUTPUT FitStatistics=work.Rsq2_13;

* Request group means (hold age=85, grip=9);
  ESTIMATE "Intercept: None Men"      intercept 1 demNF 0 demNC 0 sexMW 0 sexMW*demNF 0 sexMW*demNC 0;
  ESTIMATE "Intercept: Future Men"    intercept 1 demNF 1 demNC 0 sexMW 0 sexMW*demNF 0 sexMW*demNC 0;
  ESTIMATE "Intercept: Current Men"   intercept 1 demNF 0 demNC 1 sexMW 0 sexMW*demNF 0 sexMW*demNC 0;
  ESTIMATE "Intercept: None Women"    intercept 1 demNF 0 demNC 0 sexMW 1 sexMW*demNF 0 sexMW*demNC 0;
  ESTIMATE "Intercept: Future Women"  intercept 1 demNF 1 demNC 0 sexMW 1 sexMW*demNF 1 sexMW*demNC 0;
  ESTIMATE "Intercept: Current Women" intercept 1 demNF 0 demNC 1 sexMW 1 sexMW*demNF 0 sexMW*demNC 1;

* Request simple effects of dementia per sex;
  ESTIMATE "None vs. Future: Men"     demNF 1 demNC 0 sexMW*demNF 0 sexMW*demNC 0;
  ESTIMATE "None vs. Current: Men"     demNF 0 demNC 1 sexMW*demNF 0 sexMW*demNC 0;
  ESTIMATE "Future vs. Current: Men"    demNF -1 demNC 1 sexMW*demNF 0 sexMW*demNC 0;
  ESTIMATE "None vs. Future: Women"    demNF 1 demNC 0 sexMW*demNF 1 sexMW*demNC 0;
  ESTIMATE "None vs. Current: Women"    demNF 0 demNC 1 sexMW*demNF 0 sexMW*demNC 1;
  ESTIMATE "Future vs. Current: Women" demNF -1 demNC 1 sexMW*demNF -1 sexMW*demNC 1;

* Request simple effects of sex per dementia group;
  ESTIMATE "Men vs. Women: None"       sexMW 1 sexMW*demNF 0 sexMW*demNC 0;
  ESTIMATE "Men vs. Women: Future"     sexMW 1 sexMW*demNF 1 sexMW*demNC 0;
  ESTIMATE "Men vs. Women: Current"    sexMW 1 sexMW*demNF 0 sexMW*demNC 1;

* Need both RUN and QUIT, shut off title;
RUN; QUIT; TITLE1;
```


ANOVA Test for Model R²

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	19785.46147	2473.18268	28.77	<.0001
Error	541	46511.07671	85.97242		
Corrected Total	549	66296.53818			

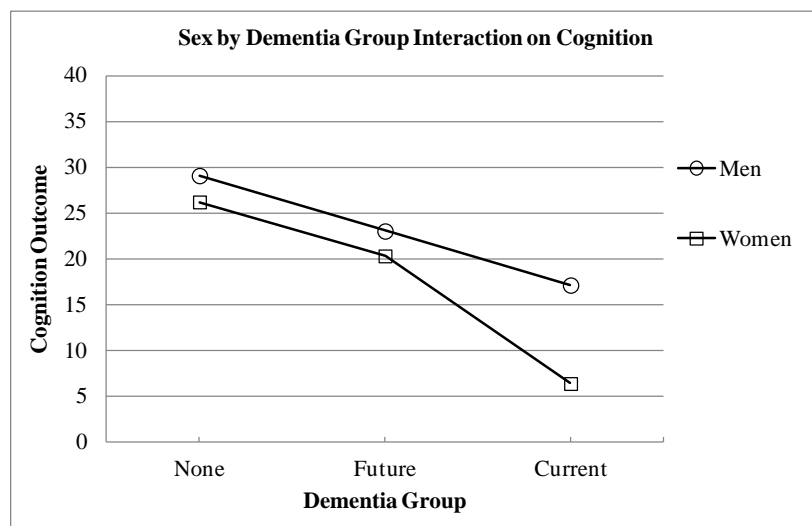
R-Square	Coeff Var	Root MSE	cognition Mean
0.298439	37.35476	9.272131	24.82182

ESTIMATES: Requested Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept: None Men	29.0701463	0.74849920	38.84	<.0001
Intercept: Future Men	23.0142449	1.49276013	15.42	<.0001
Intercept: Current Men	17.0994158	2.14021810	7.99	<.0001
Intercept: None Women	26.1945523	0.63883397	41.00	<.0001
Intercept: Future Women	20.3029208	1.11863329	18.15	<.0001
Intercept: Current Women	6.3487222	1.94788049	3.26	0.0012
None vs. Future: Men	-6.0559015	1.63512607	-3.70	0.0002
None vs. Current: Men	-11.9707305	2.24495370	-5.33	<.0001
Future vs. Current: Men	-5.9148291	2.58676242	-2.29	0.0226
None vs. Future: Women	-5.8916315	1.27776082	-4.61	<.0001
None vs. Current: Women	-19.8458301	2.02858306	-9.78	<.0001
Future vs. Current: Women	-13.9541986	2.23891711	-6.23	<.0001
Men vs. Women: None	-2.8755941	1.01123720	-2.84	0.0046
Men vs. Women: Future	-2.7113241	1.87406883	-1.45	0.1485
Men vs. Women: Current	-10.7506936	2.89932314	-3.71	0.0002

Solution for Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept	29.07014634	0.74849920	38.84	<.0001
age85	-0.33479877	0.11988755	-2.79	0.0054
grip9	0.61789286	0.14807943	4.17	<.0001
sexmw	-2.87559408	1.01123720	-2.84	0.0046
demNF	-6.05590147	1.63512607	-3.70	0.0002
demNC	-11.97073055	2.24495370	-5.33	<.0001
age85*grip9	0.12215159	0.04035286	3.03	0.0026
sexmw*demNF	0.16426999	2.07047524	0.08	0.9368
sexmw*demNC	-7.87509954	3.02453647	-2.60	0.0095



	None	Future	Current
Men	29.07	23.01	17.10
Women	26.19	20.29	6.34

The overall model improvement for the $df=2$ sex*group interaction is significant according to an R^2 change test:

```
* Call macro to test R2 change from Eq 2_13 vs. Eq 2_9;
%R2Change(N=550, DataL=work.Rsq2_13, NpredL=8, DataS=work.Rsq2_9, NpredS=6);
```

R2 Change F-Test for work.Rsq2_13 (L) vs. work.Rsq2_9 (S)

Rsquare L	Rsquare S	Rsquare Diff	Npred L	Npred S	DFnum	DFdenom	Fvalue	Pvalue
0.29844	0.28938	.009056588	8	6	2	541	3.49194	0.031130

SAS PROC GLM Syntax and Output for Equation 2.18, adding age*sex, grip*sex, age*grip*sex:

$$\begin{aligned} \text{Cognition}_i = & \beta_0 + \beta_1 (\text{Age}_i - 85) + \beta_2 (\text{Grip}_i - 9) + \beta_3 (\text{SexMW}_i) + \beta_4 (\text{DemNF}_i) + \beta_5 (\text{DemNC}_i) + \beta_6 (\text{Age}_i - 85)(\text{Grip}_i - 9) \\ & + \beta_7 (\text{SexMW}_i)(\text{DemNF}_i) + \beta_8 (\text{SexMW}_i)(\text{DemNC}_i) + \beta_9 (\text{Age}_i - 85)(\text{SexMW}_i) + \beta_{10} (\text{Grip}_i - 9)(\text{SexMW}_i) \\ & + \beta_{11} (\text{Age}_i - 85)(\text{Grip}_i - 9)(\text{SexMW}_i) + e_i \end{aligned}$$

```
TITLE1 "Eq. 2.18: Model Three-Way Interaction of Sex by Age by Grip";
PROC GLM DATA=work.example2;
* Model Y = fixed effects, options after / print extra output;
  MODEL cognition = age85 grip9 sexMW demNF demNC age85*grip9 sexMW*demNF sexMW*demNC
    sexMW*age85 sexMW*grip9 sexMW*age85*grip9 / SOLUTION SS3 EFFECTSIZE;
* Save pieces of output for use in macros (optional);
  ODS OUTPUT FitStatistics=work.Rsq2_18;

* Request simple effects of Age;
  ESTIMATE "Age for Grip=6, Men"      age85 1 age85*grip9 -3 sexMW*age85 0 sexMW*age85*grip9 0;
  ESTIMATE "Age for Grip=9, Men"      age85 1 age85*grip9 0 sexMW*age85 0 sexMW*age85*grip9 0;
  ESTIMATE "Age for Grip=12, Men"     age85 1 age85*grip9 3 sexMW*age85 0 sexMW*age85*grip9 0;

  ESTIMATE "Age for Grip=6, Women"    age85 1 age85*grip9 -3 sexMW*age85 1 sexMW*age85*grip9 -3;
  ESTIMATE "Age for Grip=9, Women"    age85 1 age85*grip9 0 sexMW*age85 1 sexMW*age85*grip9 0;
  ESTIMATE "Age for Grip=12, Women"   age85 1 age85*grip9 3 sexMW*age85 1 sexMW*age85*grip9 3;

* Request simple effects of Grip;
  ESTIMATE "Grip for Age=80, Men"     grip9 1 age85*grip9 -5 grip9*sexMW 0 sexMW*age85*grip9 0;
  ESTIMATE "Grip for Age=85, Men"     grip9 1 age85*grip9 0 grip9*sexMW 0 sexMW*age85*grip9 0;
  ESTIMATE "Grip for Age=90, Men"     grip9 1 age85*grip9 5 grip9*sexMW 0 sexMW*age85*grip9 0;

  ESTIMATE "Grip for Age=80, Women"   grip9 1 age85*grip9 -5 grip9*sexMW 1 sexMW*age85*grip9 -5;
  ESTIMATE "Grip for Age=85, Women"   grip9 1 age85*grip9 0 grip9*sexMW 1 sexMW*age85*grip9 0;
  ESTIMATE "Grip for Age=90, Women"   grip9 1 age85*grip9 5 grip9*sexMW 1 sexMW*age85*grip9 5;
```

Simple main effects can be calculated as:

Simple main effect =
 what it is (main effect)
 + what modifies it (2-ways)
 + what modifies it (3-way)

```

* Request simple effects of Sex per Dementia, hold Age=85, Grip=9;
ESTIMATE "Sex for None"      sexMW 1 sexMW*demNF 0 sexMW*demNC 0 sexMW*age85 0 sexMW*grip9 0 sexMW*age85*grip9 0;
ESTIMATE "Sex for Future"    sexMW 1 sexMW*demNF 1 sexMW*demNC 0 sexMW*age85 0 sexMW*grip9 0 sexMW*age85*grip9 0;
ESTIMATE "Sex for Current"   sexMW 1 sexMW*demNF 0 sexMW*demNC 1 sexMW*age85 0 sexMW*grip9 0 sexMW*age85*grip9 0;

* Request simple effects of Dementia per Sex, not conditional on age or grip;
ESTIMATE "None vs. Future: Men" demNF 1 demNC 0 sexMW*demNF 0 sexMW*demNC 0;
ESTIMATE "None vs. Current: Men" demNF 0 demNC 1 sexMW*demNF 0 sexMW*demNC 0;
ESTIMATE "Future vs. Current: Men" demNF -1 demNC 1 sexMW*demNF 0 sexMW*demNC 0;

ESTIMATE "None vs. Future: Women" demNF 1 demNC 0 sexMW*demNF 1 sexMW*demNC 0;
ESTIMATE "None vs. Current: Women" demNF 0 demNC 1 sexMW*demNF 0 sexMW*demNC 1;
ESTIMATE "Future vs. Current: Women" demNF -1 demNC 1 sexMW*demNF -1 sexMW*demNC 1;

* Request simple two-way interactions within Sex*Dementia Group;
ESTIMATE "Sex by None vs. Future" sexMW*demNF 1 sexMW*demNC 0;
ESTIMATE "Sex by None vs. Current" sexMW*demNF 0 sexMW*demNC 1;
ESTIMATE "Sex by Future vs. Current" sexMW*demNF -1 sexMW*demNC 1;

* Request simple two-way interactions of age*grip;
ESTIMATE "Age by Grip: Men" age85*grip9 1 age85*grip9*sexMW 0;
ESTIMATE "Age by Grip: Women" age85*grip9 1 age85*grip9*sexMW 1;

* Request simple two-way interactions of sex*age;
ESTIMATE "Sex by Age: Grip=6" sexMW*age85 1 age85*grip9*sexMW -3;
ESTIMATE "Sex by Age: Grip=9" sexMW*age85 1 age85*grip9*sexMW 0;
ESTIMATE "Sex by Age: Grip=12" sexMW*age85 1 age85*grip9*sexMW 3;

* Request simple two-way interactions of sex*grip;
ESTIMATE "Sex by Grip: Age=80" sexMW*grip9 1 age85*grip9*sexMW -5;
ESTIMATE "Sex by Grip: Age=85" sexMW*grip9 1 age85*grip9*sexMW 0;
ESTIMATE "Sex by Grip: Age=90" sexMW*grip9 1 age85*grip9*sexMW 5;

* Need both RUN and QUIT, shut off title;
RUN; QUIT; TITLE1;

* Call macro to test R2 change from Eq 2_18 vs. Eq 2_13;
%R2Change(N=550, DataL=work.Rsq2_18, NpredL=11, DataS=work.Rsq2_13, NpredS=8);
    
```

Simple two-way interactions can be calculated using the same logic as simple main effects:

Effect = what it is + what modifies it

Simple two-way interaction =
 what it is (2-way)
 + what modifies it (3-way)

Fortunately, the overall model did not improve, according to an R^2 change F-test... but output is provided for your reference!

R2 Change F-Test for work.Rsq2_18 (L) vs. work.Rsq2_13 (S)

Rsquare L	Rsquare S	Rsquare Diff	Npred L	Npred S	DFnum	DFdenom	Fvalue	Pvalue
0.30257	0.29844	.004126581	11	8	3	538	1.06108	0.36516

ANOVA Test for Model R²

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	20059.03951	1823.54905	21.22	<.0001
Error	538	46237.49868	85.94331		
Corrected Total	549	66296.53818			

R-Square	Coeff Var	Root MSE	cognition Mean
0.302565	37.34844	9.270561	24.82182

ESTIMATES: Requested Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Age for Grip=6, Men	-1.1881602	0.36585649	-3.25	0.0012
Age for Grip=9, Men	-0.4982870	0.20625046	-2.42	0.0160
Age for Grip=12, Men	0.1915861	0.23463810	0.82	0.4146
Age for Grip=6, Women	-0.6182774	0.16936809	-3.65	0.0003
Age for Grip=9, Women	-0.4032146	0.18061946	-2.23	0.0260
Age for Grip=12, Women	-0.1881517	0.31031688	-0.61	0.5446
Grip for Age=80, Men	-0.4105136	0.43682207	-0.94	0.3478
Grip for Age=85, Men	0.7392750	0.23641834	3.13	0.0019
Grip for Age=90, Men	1.8890635	0.45758439	4.13	<.0001
Grip for Age=80, Women	0.1807631	0.32937371	0.55	0.5834
Grip for Age=85, Women	0.5392011	0.19200934	2.81	0.0052
Grip for Age=90, Women	0.8976392	0.36218725	2.48	0.0135
Sex for None	-2.9734897	1.03636525	-2.87	0.0043
Sex for Future	-2.5766063	1.89856249	-1.36	0.1753
Sex for Current	-11.1191918	2.91755246	-3.81	0.0002
None vs. Future: Men	-6.1654275	1.63793371	-3.76	0.0002
None vs. Current: Men	-11.7837134	2.24789958	-5.24	<.0001
Future vs. Current: Men	-5.6182859	2.59176746	-2.17	0.0306
None vs. Future: Women	-5.7685441	1.28133333	-4.50	<.0001
None vs. Current: Women	-19.9294155	2.02920586	-9.82	<.0001
Future vs. Current: Women	-14.1608714	2.24315626	-6.31	<.0001
Sex by None vs. Future	0.3968833	2.07957735	0.19	0.8487
Sex by None vs. Current	-8.1457021	3.02832114	-2.69	0.0074
Sex by Future vs. Current	-8.5425855	3.42768268	-2.49	0.0130
Age by Grip: Men	0.2299577	0.07594863	3.03	0.0026
Age by Grip: Women	0.0716876	0.05760755	1.24	0.2139
Sex by Age: Grip=6	0.5698828	0.40315818	1.41	0.1581
Sex by Age: Grip=9	0.0950725	0.27415806	0.35	0.7289
Sex by Age: Grip=12	-0.3797378	0.38903933	-0.98	0.3295
Sex by Grip: Age=80	0.5912767	0.54708369	1.08	0.2803
Sex by Grip: Age=85	-0.2000738	0.30456726	-0.66	0.5115
Sex by Grip: Age=90	-0.9914243	0.58357783	-1.70	0.0899

Solution for Fixed Effects

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept	28.95584314	0.79724317	36.32	<.0001
age85	-0.49828704	0.20625046	-2.42	0.0160
grip9	0.73927496	0.23641834	3.13	0.0019
sexmw	-2.97348968	1.03636525	-2.87	0.0043
demNF	-6.16542746	1.63793371	-3.76	0.0002
demNC	-11.78371340	2.24789958	-5.24	<.0001
age85*grip9	0.22995771	0.07594863	3.03	0.0026
sexmw*demNF	0.39688334	2.07957735	0.19	0.8487
sexmw*demNC	-8.14570211	3.02832114	-2.69	0.0074
age85*sexmw	0.09507247	0.27415806	0.35	0.7289
grip9*sexmw	-0.20007382	0.30456726	-0.66	0.5115
age85*grip9*sexmw	-0.15827010	0.09532484	-1.66	0.0974 → This was a close one! ☺