

# Role of Self-Efficacy and Self-Concept Beliefs in Mathematical Problem Solving: A Path Analysis

Frank Pajares and M. David Miller

Path analysis was used to test the predictive and mediational role of self-efficacy beliefs in mathematical problem solving. Results revealed that math self-efficacy was more predictive of problem solving than was math self-concept, perceived usefulness of mathematics, prior experience with mathematics, or gender ( $N = 350$ ). Self-efficacy also mediated the effect of gender and prior experience on self-concept, perceived usefulness, and problem solving. Gender and prior experience influenced self-concept, perceived usefulness, and problem solving largely through the mediational role of self-efficacy. Men had higher performance, self-efficacy, and self-concept and lower anxiety, but these differences were due largely to the influence of self-efficacy, for gender had a direct effect only on self-efficacy and a prior experience variable. Results support the hypothesized role of self-efficacy in A. Bandura's (1986) social cognitive theory.

Social cognitive theory suggests that self-efficacy, "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (Bandura, 1986, p. 391), strongly influences the choices people make, the effort they expend, and how long they persevere in the face of challenge. According to Bandura (1986), how people behave can often be better predicted by their beliefs about their capabilities than by what they are actually capable of accomplishing, for these beliefs help determine what individuals do with the knowledge and skills they have.

Although researchers have established that self-efficacy is a strong predictor of behavior (Maddux, Norton, & Stoltenberg, 1986), research on the relationship between self-efficacy and academic performance in areas such as mathematics is still limited (Bouffard-Bouchard, 1989). Studies of math self-efficacy have been largely correlational, and researchers have emphasized the need to construct causal models with which to conceptualize and test hypothesized relationships (Hackett & Betz, 1989; Meece, Wigfield, & Eccles, 1990). When causal modeling has been used, most models have excluded key variables identified as influencing math performance (most notably, self-concept), or the theoretical framework used to hypothesize relationships was not based on social cognitive theory. Thus, results have added little to a better understanding of self-efficacy's influence.

Bandura (1986) hypothesized that self-efficacy beliefs mediate the effect of other determinants of performance such as gender and prior experience on subsequent perfor-

mance; that is, when these determinants are controlled, self-efficacy judgments are better predictors of performance. Bandura also argued that constructs such as self-concept, perceived usefulness, and anxiety are "common mechanisms" of personal agency in the sense that they, like self-efficacy beliefs, also influence an outcome. However, these mechanisms are, to a great extent, the result of self-efficacy judgments—their influence is largely due to the confidence with which individuals approach a task. Consequently, although strong correlational relationships are observed between these mechanisms and related outcomes, the relationships are mostly due to the influence of self-efficacy on the common mechanisms. Self-efficacy judgments also mediate the effect of gender and prior experience on the common mechanisms; that is, when gender and prior experience are controlled, self-efficacy is a stronger predictor not only of a related outcome but of common mechanisms such as anxiety, self-concept, and perceived usefulness.

The purpose of this study was to use path analysis techniques to test Bandura's (1986) hypotheses regarding the predictive and mediational role of self-efficacy in the area of mathematics. First, we sought to determine whether the confidence with which students approach the solving of math problems had stronger direct effects on their problem-solving performance than did their math self-concept, math anxiety, perceived usefulness of mathematics, prior experience with mathematics, and gender. Second, we tested whether self-efficacy mediated the effect of gender and prior experience on both the common mechanisms and problem-solving performance.

Of special interest to us was the interplay between self-efficacy and self-concept. The conceptual difference between these two constructs is not always clear in investigations. Reyes (1984), for example, used the terms *math confidence* and *math self-concept* synonymously, and Felson (1984) referred to *academic self-concept* as self-perceptions of ability, suggesting that one reason why these self-percepts affect performance is because of their effect on students' effort, persistence, and anxiety. Others

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have written about self-concept "of ability" and have operationalized it as individuals' ratings of their ability in academic areas, that is, basically as generalized academic self-efficacy (e.g., Bachman & O'Malley, 1986; Feather, 1988).

Self-concept differs from self-efficacy in that self-efficacy is a context-specific assessment of competence to perform a specific task, a judgment of one's capabilities to execute specific behaviors in specific situations. Self-concept is not measured at that level of specificity and includes beliefs of self-worth associated with one's perceived competence. It is clear that beliefs regarding confidence are part of an individual's self-concept, but Bandura (1986) argued that self-concept and self-efficacy represent different phenomena and must not be mistaken for each other.

Shavelson, Hubner, and Stanton (1976) introduced a hierarchical model that differentiated between general, academic, social, emotional, and physical self-concepts. Academic self-concepts were further differentiated as English, history, science, or math self-concepts. This model is now widely accepted and researchers warn that using global indexes instead of the more specific self-appraisals is of limited value. Self-concept judgments in academic endeavors, however, may be subject or course specific, but they are never item or task specific. They are not specific assessments of capability. Compared with self-efficacy judgments, self-concept judgments are more global and less context dependent. The course-specific self-concept question, "Are you a good math student?" taps different cognitive and affective processes than the self-efficacy question, "Can you solve this specific problem?"

Self-concept theorists have argued that an individual's self-concept mediates the influence of other determinants on subsequent performance and is the stronger predictor of that performance when those determinants are controlled. Social cognitive theorists propose that these are functions of self-efficacy. A substantive question of our study, then, is whether individuals' beliefs of self-worth as math students are more predictive of solving math problems (and mediate the effect of their gender and prior experience) than their beliefs of their capability to solve the specific problems required to assess just what sort of students they are.

As regards anxiety, Bandura (1986) contended that it is only when people cannot predict or exercise control over events that they have reason to fear them. Anxiety is generally determined by the confidence individuals bring to a task. Efficacy beliefs predict "how well people cope with threats and how much fear arousal they experience" (p. 321). Self-efficacy will retain predictiveness of performance even when the effects of anxiety are controlled, whereas the effect of anxiety should dissipate when self-efficacy percepts are controlled.

## Overview of Research Findings

### *Math Self-Efficacy*

Confidence in learning mathematics, conceptual forerunner to math self-efficacy, has consistently been found to

predict math-related performance (Hackett, 1985). Early on, confidence was globally assessed by asking students general questions about their perceived math abilities. Math self-efficacy has more recently been assessed as individuals' judgments of their capabilities to solve specific math problems, perform math-related tasks, or succeed in math-related courses (see Betz & Hackett, 1983).

Most researchers investigating the relationship between math self-efficacy and performance have reported a strong correspondence. Collins (1982) found that, when prior performance was controlled, children with high self-efficacy outperformed children with low self-efficacy in the completion of novel math problems, showed greater effort, and persisted longer in reworking incorrect problems. Siegel, Galassi, and Ware (1985) found that a model that included self-efficacy accounted for a larger portion of the variance in math performance than did a model with anxiety and aptitude as the independent measures.

Bandura (1986) cautioned that, because judgments of self-efficacy are task specific, different ways of assessing confidence will differently correspond to the assessed performance. Self-efficacy must be specifically rather than globally assessed, must correspond directly to the criterial performance task, and must be measured as closely as possible in time to that task. These guidelines are seldom followed, and so the mismeasurement of self-efficacy is a recurring theme in educational research, often producing poorly defined constructs, confounded relationships, ambiguous findings, and uninterpretable results.

For example, Benson (1989) found that the path from what she called math self-efficacy to performance was not significant, whereas that between math self-concept and performance was. However, self-efficacy was assessed with three global items dealing with expected success in the class (e.g., "No matter how hard I study, I will not do well in this class"). Self-concept was assessed with seven items specific to feelings of math self-worth (e.g., "I feel insecure in a math class"). Performance was the mid-term exam grade in a statistics course. Benson concluded that additional studies were required to verify the relationship between the two constructs and to explore why self-efficacy did not influence performance or statistical test anxiety. One answer may be that comparing confidence to succeed in a class with a statistics mid-term grade is not likely to produce the sort of correspondence that social cognitive theory hypothesizes. The more appropriately assessed feelings of math-specific self-concept, on the other hand, should, and did, have a stronger correspondence with a math-related measure.

Cooper and Robinson (1991) found that a regression model with math anxiety, American College Test—Quantitative (ACT-Q) scores, and prior math experience revealed that self-efficacy did not account for a significant portion of the variance in performance. However, the researchers compared self-efficacy to succeed in math *courses* with scores on a performance measure that consisted of *solving problems*. Norwich (1987) reported that when the effects of math self-concept were controlled, self-efficacy did not predict math performance. Only prior math performance and math self-concept contributed significant vari-

ance. However, Norwich used hierarchical regression and entered variables according to their assumed causal influences from a self-concept perspective, with self-concept entered first and self-efficacy last. Results may have been different if the order had been entered with causal assumptions hypothesized by social cognitive theory.

Recently, Randhawa, Beamer, and Lundberg (1993) found that *generalized* math self-efficacy mediated the effect of various math attitudes on math achievement. However, their generalized math self-efficacy was the composite score of the three subscales of the Mathematics Self-Efficacy Scale (MSES)—judgments of capability to solve math problems, complete math-related tasks, and succeed in math-related courses. The criterial task was a mixture of the composite of teacher-assigned grades in algebra and scores on an algebra achievement test. The achievement test portion of the outcome measure was conceptually related only to the problems scale, although problems on the test differed markedly from those presented on the self-efficacy assessment; the teacher-assigned grades bore little relation to any of the self-efficacy judgments. Consequently, although the math attitude measures had a strong direct effect on self-efficacy, they also had a stronger direct effect on performance than did self-efficacy. Bandura (1986) wrote that “it is no more informative to speak of self-efficacy in global terms than to speak of nonspecific social behavior” (p. 411). The choice of a generalized math self-efficacy score by Randhawa et al. is theoretically at odds with Bandura’s warning, as was their hypothesis that math attitudes similar to self-concept and anxiety are causally predominant over self-efficacy judgments.

### *Math Self-Concept*

Although sometimes confounded by imprecise definitions and varying measurements, findings consistently show that math self-concept is related to math performance (see Marsh, Walker, & Debus, 1991). However, few studies have compared the effects of self-efficacy and self-concept on performance. Marsh et al. (1991) compared the direct effect of achievement on the math self-concept and self-efficacy of fifth graders and reported a stronger direct effect on self-concept than on self-efficacy. Such a hypothesized relationship, however, begged the question of which self-belief had the stronger influence on achievement.

Few researchers have investigated the relationship between math self-concept and math self-efficacy, and none from the perspective of social cognitive theory. Relich (1983), cited in Marsh (1990), assessed math self-concept, math achievement, performance on a division task, and self-efficacy for the division task. Achievement correlated equally strongly with self-efficacy (globally assessed) and self-concept. Specific performance on the division task, however, was more strongly correlated with specifically assessed self-efficacy than with math self-concept. These results provide support for the task-specific nature of self-efficacy measurement. Other findings are contradictory.

Norwich (1987) found that self-concept was not related to self-efficacy when students were either familiar or unfamiliar with a task. Marsh et al. (1991) compared the math self-concept and self-efficacy of elementary school students and reported correlations as low as .18.

### *Math Anxiety*

As early as 1957, Dreger and Aiken suspected that individuals suffered from “number anxiety,” and various studies have since demonstrated a negative correlation between math anxiety and math performance (see Schwarzer, Seipp, & Schwarzer, 1989, for meta-analysis). In most cases, however, math anxiety is not a powerful predictor when variables such as self-efficacy, self-concept, prior experience, and perceived usefulness are controlled (Llabre & Suarez, 1985; Meece et al., 1990).

Hackett (1985) investigated the effects of math self-efficacy on math anxiety using path analyses with relationships hypothesized from social cognitive theory and found that self-efficacy had a strong direct effect. Self-efficacy also had a stronger direct effect on choice of math-related careers than did anxiety and an even stronger total effect. Math self-efficacy was also a stronger predictor of math anxiety than either prior high school math experience or gender.

### *Perceived Usefulness of Mathematics*

Fennema and Sherman (1976) incorporated perceived usefulness into their Mathematics Attitude Scales, and researchers have used these and other scales to demonstrate that perceived usefulness is consistently related to math performance (e.g., Armstrong, 1985). As was the case with math confidence, correlations were generally moderate. As expected, students’ perceived usefulness of mathematics is also related to the confidence they express in their ability (Hackett & Betz, 1989; Lent, Lopez, & Bieschke, 1991).

### *Gender*

Literature on the relationship between gender and math performance is abundant. Early findings showed that children did not differ in their math performance during elementary school but that differences began to appear in middle school and increased with time and schooling (see Fennema & Sherman, 1978). Data from the National Assessment of Educational Progress, Differential Aptitude Test national norming groups, and Preliminary Scholastic Aptitude Test—Mathematics show declines over the last two decades in gender differences in quantitative tasks. Only at the highest levels of math achievement do men continue to outperform women (Feingold, 1988).

Some researchers have suggested that gender differences in mathematics stem from sociocultural factors and urge investigators to expand collection of data such as students’ confidence (Ethington, 1989; Hart, 1990). Fennema and

Sherman (1978) found that when affective variables such as confidence were included in a model, performance differences disappeared. This led them to suspect that affective variables were the source of sex differences. Subsequent researchers have reported that, when differences in preparation and confidence are controlled, fewer differences on math achievement are found (e.g., Lapan, Boggs, & Morrill, 1989). Social cognitive theory suggests that self-efficacy percepts mediate the effect of gender on math performance.

The relationship between gender and math self-efficacy has not been explored as thoroughly as that between gender and math performance. Early studies suggested that boys were more confident in their math skills (Fennema & Sherman, 1978). Recent findings suggest that boys and girls report equal confidence during the elementary years but, by middle and high school, boys grown more confident (Pintrich & De Groot, 1990).

### *Prior Experience with Mathematics*

Percepts of ability are formed as individuals attempt and complete tasks. However, Bandura (1986) argued that people are more influenced by how they interpret their experience than by their attainments per se. For this reason, self-efficacy beliefs usually predict future behavior better than does past experience. Prior experience influences subsequent behavior largely through its effect on self-efficacy beliefs, and these can influence performance "independent of past behavior" (p. 424). The few studies on math performance to include math background report a relationship (e.g., Cooper & Robinson, 1991; Hackett, 1985). Support for Bandura's contention also comes from studies that explore the role of these variables from an expectancy-value orientation (Meece et al., 1990; Pokay & Blumenfeld, 1990). Researchers who have explored the relationship between prior experience and math self-efficacy have reported both significant correlations and direct effects (e.g., Cooper & Robinson, 1991; Hackett, 1985).

## Method

### *Participants and Setting*

Participants consisted of 350 undergraduates (229 women and 121 men) from a large public university in the South, the majority of whom were enrolled in classes in the College of Education. Participation was voluntary and no remuneration was provided, but most instructors provided course credit. Of the total sample, 137 were education majors, and 213 represented a variety of majors throughout the university.

### *Measures*

**Math self-efficacy.** The problems scale of the Mathematics Confidence Scale (MCS) was created by Dowling (1978), who developed the instrument to assess the math confidence of college students. Problems on the MCS represent three components of mathematics (arithmetic, algebra, and geometry), three levels of cognitive demand (computation, comprehension, and application),

and two problem contexts (real and abstract). (Sample item: "There are three numbers. The second is twice the first and the first is one-third of the other number. Their sum is 48. Find the largest number.") Dowling reported a correlation of .57 between the MCS and the confidence scale of Fennema and Sherman's (1976) Mathematics Attitude Scales. Because the original 10-point Likert scale was considered cumbersome and unnecessarily over-specific, Langenfeld and Pajares (1992) conducted a study with 145 undergraduates and using a 5-point scale. Coefficient alpha revealed that the instrument had strong internal consistency (.91).

**Perceived usefulness of mathematics.** The measure of perceived usefulness was adapted from a 20-item instrument created by Shell, Murphy, and Bruning (1989) in which students were asked to rate the importance of reading or writing skills for achieving various life goals. The items include the domains of employment, social activities, family life, education, and citizenship. We replaced the words *reading* or *writing* with *mathematics*. (Sample item: "How important is skill in mathematics for graduating from college?") Shell et al. (1989) reported Cronbach's alpha of .93 and positive and above .40 item/total correlations for all items on the original instrument. Langenfeld and Pajares (1992) also obtained .93, with the instrument reworded to reflect perceived usefulness of mathematics.

**Math anxiety.** The Mathematics Anxiety Scale (MAS) was adapted by Betz (1978) from the anxiety scale of the Fennema-Sherman Mathematics Attitudes Scales to create an instrument more appropriate to college students. The MAS consists of 10 items—five positively worded and five negatively worded. (Sample item: "I get really uptight during math tests.") Scoring of the negative items is reversed so that a high score indicates low anxiety. Betz reported a split-half reliability coefficient of .92. Correlations of about .70 have been reported between the MAS and the 98-item Mathematics Anxiety Rating Scale (Cooper & Robinson, 1991). Dew, Galassi, and Galassi (1983) reported Cronbach's alpha of .72 and 2-week test-retest reliability of .87. Hackett and Betz (1989) reported Cronbach's alpha values ranging from .86 to .90. Frary and Ling (1983) subjected the items to factor analysis and found that they loaded highly (.89) on the factor they defined as math anxiety.

**Math self-concept.** The Self Description Questionnaires (SDQ) comprise academic and course-specific self-concept items. We used the math scale of the SDQIII, a 180-item self-concept measure consisting of 13 scales and developed specifically to assess self-concept of older adolescents and college students (Marsh, 1992). (Sample item: "Mathematics makes me feel inadequate.") Marsh and O'Neill (1984) conducted two validation studies of the SDQIII and reported high reliabilities for the 13 factors. Factor loadings for the math scale items ranged from .74 to .91. In two studies, they reported coefficient alphas of .93 and .95 for the math scale. They also reported that the scale correlated strongly with criterion measures such as school certificate (achievement) scores in mathematics (.58), students' self-descriptions (.74), and ratings by others (.77).

**Prior experience.** Frary and Ling (1983) defined prior high school math experience as the maximum math level attained in high school courses, whereas other researchers have asked students to self-report the number of years during which they had taken high school mathematics (e.g., Hackett & Betz, 1989). However, some students can take 4 years of mathematics and never progress beyond geometry; others can take more than one math course in 1 year. It is logically more likely for a student who has progressed through Advanced Placement Calculus in 4 years of high school to have developed stronger self-efficacy and prior knowledge than another student who only progressed through Algebra I in those

same 4 years. Consequently, we operationalized prior high school math experience as did Frary and Ling.

College math experience has been operationalized as (a) the number of semesters in which mathematics was taken, (b) the number of math courses taken, or (c) the number of semester credits obtained. The first is shortsighted—students may take one or more courses during a semester. The number of college math courses is so varied, however, that assessing college math experience the way one might assess high school experience is not feasible. Thus, earned semester credits makes the most sense.

**Math performance.** The Mathematics Problems Performance Scale (MPPS), like the math self-efficacy scale, was developed by Dowling (1978). Consistent with Bandura's (1986) guidelines, the problems on which performance was assessed were the same as those on which confidence was measured. The MPPS is an 18-item, multiple-choice instrument constructed with mid-range difficulty items from the National Longitudinal Study of Mathematical Abilities (NLSMA), developed specifically for use with college students, and composed of three nonorthogonal subscales consisting of math components, cognitive demand, and problem context (see Romberg & Wilson, 1969). Dowling (1978) reported KR 20 coefficient of reliability of .788 and mean item difficulty of .291. For an item to be included in the final instrument, the following criteria had to be met: (a) percentage correct between .30 and .70, (b) point-biserial correlation coefficient greater than .50, (c) discrimination index greater than .40, and (d) significant corrected phi coefficient ( $p < .01$ ).

### Procedure and Data Collection

The instruments were group administered in individual classes. Students were first asked to complete the self-efficacy, perceived usefulness, self-concept, and anxiety measures. They were informed they would be asked to solve the problems on which their confidence was being assessed. After these forms were collected, the performance measure was administered. Bandura (1986) suggested that efficacy and performance be assessed within as close a time period as possible and that efficacy assessment precede performance assessment. All instruments were administered during one class period. This also ensured that absences would not create a situation whereby some students completed one measure but not another. Pilot testing had demonstrated that the class periods provided ample time in which to complete all measures.

### Data Analysis

Path analysis techniques are used to examine the direct and indirect effects between variables, although they are not without

their critics (see Freedman, 1987). Cook and Campbell (1979) suggested that they are especially appropriate when "theoretical, empirical, and commonsense knowledge of a problem" (p. 307) provides a defensible mapping of the latent variables present and their probable causal links. Path analysis is, therefore, especially appropriate in an investigation in which the tenets of social cognitive theory and previous findings are such that hypothesized relationships have strong theoretical and empirical support.

The path model tested was as follows: Gender was hypothesized to influence all variables; level of high school mathematics would mediate the influence of gender and influence the remaining variables; math experience in college would mediate the influence of gender and high school math experience and influence the remaining variables; math self-efficacy would mediate these prior influences on both the performance task and the three common mechanisms—math self-concept, math anxiety, and perceived usefulness; the common mechanisms were hypothesized to influence performance directly.

## Results

Table 1 presents the means, standard deviations, and Pearson product-moment correlations for all variables in the study. Students averaged a high school math level close to analytical geometry and trigonometry (or pre-calculus), a high level explained by the fact that the sample consisted of university students. Participants had completed an average of 10.3 college credits in math courses. The importance of these variables rested on the fact that social cognitive theory predicts that, although an individual's self-efficacy beliefs are partly based on such prior experience, these beliefs are more strongly predictive of subsequent performance.

Math self-efficacy scores ranged from 36 to 90. Possible scores ranged from 18 to 90; hence, although some students expressed maximum confidence in their problem-solving ability, even the least confident averaged 2 on the 5-point Likert scale. A mean score of 4.09 (*much confidence*) per item suggested that students were largely confident about their ability to solve the problems. The phenomenon of scoring above the midpoint of the Likert scale was not as pronounced with math self-concept or anxiety and did not occur with perceived usefulness. Self-concept scores ranged from 10 to 80 (the minimum and maximum possible), with a mean of 5.0 on the 8-point Likert scale, indicating that a positive self-concept statement is *more true than false*.

Table 1  
Means, Standard Deviations, and Zero-Order Correlations of Variables in the Path Analysis

Variable	<i>M</i>	<i>SD</i>	Gender	HSL	CC	USE	MSC	MAS	MSE	PERF
HSL	4.9	1.2	.11*	—						
CC	10.3	6.0	-.07	.15**	—					
USE	50.9	15.2	.05	.12*	.06	—				
MSC	49.7	16.6	.13*	.48***	.25***	.40***	—			
MAS	31.8	10.9	.15**	.44***	.20***	.32***	.87***	—		
MSE	73.6	10.5	.24***	.47***	.23***	.19***	.61***	.56***	—	
PERF	14.1	2.8	.17***	.44***	.23***	.14***	.54***	.51***	.70***	—

Note. HSL = high school level; CC = college credits earned; USE = perceived usefulness of mathematics; MSC = math self-concept; MAS = math anxiety; MSE = math self-efficacy; PERF = math performance.

\*  $p < .05$ . \*\*  $p < .01$ . \*\*\*  $p < .0001$ .

Anxiety scores also ranged from the minimum of 10 to the maximum of 50, with a mean of 3.2 on the 5-point Likert scale. Recall that the anxiety measure scored higher for low anxiety, hence 3.2 represents anxiety slightly lower than *undecided*. Perceived usefulness scores ranged from 21 (minimum possible = 20) to 97 (maximum possible = 100), with a mean of 2.5 on the 5-point Likert scale. Students found mathematics more than *a little important* but less than *moderately important*.

Correlations between all independent variables and both self-efficacy and performance were significant. Magnitudes were consistent with those of previous investigations. The .87 correlation between self-concept and anxiety, however, created a problem of multicollinearity. Although the MAS and the math scale of the SDQIII have been used extensively and purport to measure different constructs, they have not previously been used in the same study. Because effects from the common mechanisms would in part be a function of multicollinearity, we decided to remove anxiety from the path analysis, a choice guided by our primary interest in the interplay between self-concept and self-efficacy. Anxiety was included, however, in subsequent analyses of gender differences. Further analyses showed that no other correlation was high enough to create an instability in the parameter estimates of the path analysis. Consequently, all other common mechanisms were maintained in the model.

Table 2 provides a decomposition of effects from the path analysis. The independent variables accounted for 52% of

the variability in problem-solving performance,  $F(6, 343) = 61.80, p < .0001$ . Each of the regression models was also significant. Figure 1 illustrates the path analysis model with nonsignificant paths removed. Note that this is the full model and not a reduced model recomputed with nonsignificant relationships removed. Figure 1 is provided for ease of interpretation and to show the residual path coefficients (R). These coefficients represent factors that affect a specific variable but that are not measured or accounted for in the model—the square root of the unexplained variation in the dependent variable.

Of all path coefficients from the independent variables to performance, only those from math self-efficacy ( $\beta = .545, t = 10.87, p < .0001$ ), math self-concept ( $\beta = .163, t = 3.07, p < .005$ ), and high school level ( $\beta = .099, t = 2.22, p < .05$ ) were significant. The magnitude of the self-efficacy/performance path coefficient is such, however, that the answer to the substantive question of the study is readily apparent. The effects of gender, high school level, and college credits on math self-efficacy were all positive and significant, as hypothesized. Table 3 provides an overview of direct and indirect effects on math performance and math self-efficacy.

As hypothesized, self-efficacy had stronger direct effects on performance than did any of the variables in the study. Moreover, both its direct and total effects were significantly stronger than those of the other variables. Math self-concept and high school level each had modest direct effects. Re-

Table 2  
*Decomposition of Effects From the Path Analysis*

Effect	(Intercept) Parameter estimate	Standardized estimate	<i>t</i>	<i>R</i> <sup>2</sup>
On high-school level	(4.85)			
of gender	0.29	.114	2.14*	.01
On college credits	(6.82)			
of gender	-1.06	-.089	-1.58	.03
of high school level	0.79	.159	3.00**	
On math self-efficacy	(51.05)			
of gender	4.60	.209	4.57***	.29
of high school level	3.57	.419	9.07***	
of college credits	0.32	.185	4.03***	
On perceived usefulness	(30.59)			
of gender	0.34	.011	0.20	.04
of high school level	0.57	.046	0.77	
of college credits	0.05	.019	0.34	
of math self-efficacy	0.23	.158	2.51**	
On math self-concept	(-24.47)			
of gender	-0.35	-.010	-0.24	.43
of high school level	3.40	.251	5.44***	
of college credits	0.29	.106	2.50**	
of math self-efficacy	0.74	.466	9.65***	
On math performance	(1.16)			
of gender	0.07	.011	0.29	.52
of high school level	0.22	.099	2.22*	
of college credits	0.02	.052	1.33	
of math self-efficacy	0.14	.545	10.87***	
of perceived usefulness	-0.01	-.042	-1.02	
of math self-concept	0.03	.163	3.07**	

Note. *N* = 350.

\*  $p < .05$ . \*\*  $p < .01$ . \*\*\*  $p < .0001$ .

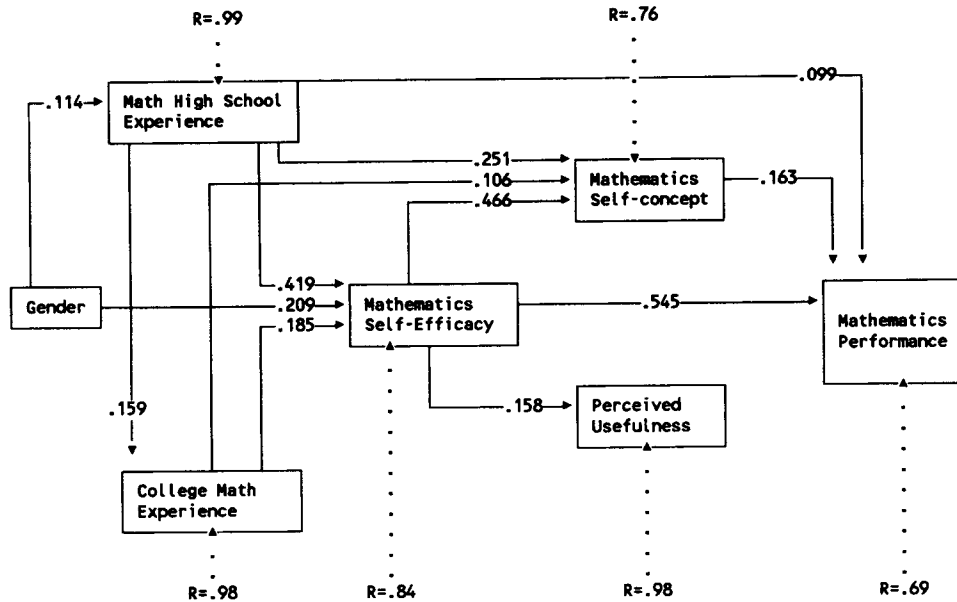


Figure 1. Significant path coefficients between all variables in the study. Dotted lines indicate path coefficients for residuals (R).

sults from the path analysis suggest that the relationship between performance and both self-concept and perceived usefulness was largely a result of noncausal covariation, largely due to the effect of problems self-efficacy. It is clear that self-efficacy affected performance almost exclusively directly rather than indirectly through the mediated variables. Moreover, prior experience and gender affected performance largely through their influence on self-efficacy. Their indirect effects on performance were strong, as were their direct effects on self-efficacy. The indirect effects of these variables on performance were, in fact, also largely a result of self-efficacy. The interactive effects of gender were tested and found nonsignificant.

#### Gender Differences

Had the data been examined by looking only at correlational relationships and mean differences, past findings reporting gender differences in math-related constructs would have been supported. Results of independent samples *t* tests, adjusted using the Dunn procedure ( $\alpha = .005$ ), are presented in Table 4.

Men reported higher math self-efficacy than did women. Consistent with prior findings, women expressed higher levels of math anxiety. Men also had a higher average score on the performance measure. These differences are noteworthy, considering that men and women did not differ in

Table 3  
Direct and Indirect Effects on Math Self-Efficacy and Mathematics Problem-Solving Performance

Effect	<i>r</i>	Direct effect	Indirect effect	Total effect	Noncausal covariation
On math performance					
of math self-concept	.540*	.163*	.000	.163*	.377
of perceived usefulness	.140*	-.042	.000	-.042	.182
of math self-efficacy	.697*	.545*	.083	.625*	.072
of college credits	.232*	.052	.115*	.167*	.065
of high school level	.438*	.099*	.276*	.375*	.063
of gender	.171*	.011	.121*	.132*	.039
On math self-efficacy					
of college credits	.234*	.185*	.000	.185*	.049
of high school level	.470*	.419*	.029	.448*	.022
of gender	.245*	.209*	.031	.240*	.005

\*  $p < .05$ .

Table 4  
Mean Gender Differences in Math-Related Constructs

Construct	Women	Men	Difference	<i>t</i>	Prob >   <i>t</i>
High school	5.1	4.8	0.3	-2.12	.0348
College credits	10.6	9.8	0.8	1.19	.2366
Self-efficacy	71.8	77.1	-5.3	-4.89*	.0001
Self-concept	48.2	52.6	-4.4	-2.55	.0113
Math anxiety	30.6	34.1	-3.5	-3.10*	.0021
Perceived usefulness	50.3	52.0	-1.7	-1.02	.3102
Performance	13.7	14.7	-1.0	-3.29*	.0011

Note. Ns: women = 229; men = 121. For the analysis, women were entered as 0 and men were entered as 1.

\*  $p \leq .005$ .

prior experience. According to the path analysis, differences in performance were due to a difference in math self-efficacy. Differences in math self-concept and perceived usefulness did not reach significance.

### The Question of Overconfidence

Past findings suggest that students generally overestimate their math performance capability (see Hackett & Betz, 1989). We defined overestimation as marking an item 4 or 5 on the self-efficacy Likert scale (*much confidence* or *complete confidence*) and then incorrectly answering that item; underestimation was defined as marking an item 1 or 2 (*no confidence at all* or *very little confidence*) and getting the item correct. An answer of 3 was considered as *uncertain* and did not play a role in this assessment. Two types of congruence were possible. *Congruence—no errors* occurred when the student correctly predicted each subsequent response. *Congruence—equal errors* took place when the student overestimated and underestimated equally; that is, the student made one or more errors of overestimation and an equal number of errors of underestimation (see Table 5).

Note that 57% of the students overestimated their performance and 20% underestimated it. Of 350 students, only 25 correctly predicted their responses to all 18 math problems. The magnitude of overconfidence was also greater than that of underconfidence. Students overestimated by an average of 1.91 problems; they underestimated by an average of 1.06. Students in the overestimation and underestimation groups, however, erred equally; that is, the 57% that overestimated did so on 2.7 problems, whereas the 20% that underestimated did so on 2.67 problems. A chi-square test demonstrated that there was no significant relationship be-

tween gender and level of confidence, critical  $\chi^2$  (2,  $N = 350$ ) = 5.99,  $\alpha = .05$ ; observed  $\chi^2$  (2,  $N = 350$ ) = .74. These results are similar to those reported by Hackett and Betz (1989).

### Discussion

The purpose of this study was to discover whether self-efficacy beliefs play the mediational role ascribed to them by Bandura (1986) and social cognitive theory, and whether these beliefs are stronger predictors of performance than are other presumed determinants and common mechanisms. We focused on the influence of self-efficacy on mathematics because our interest in self-efficacy was founded on a broader interest in education and the academic performance of students. In this context, the solving of math problems afforded a clearer and more reliable assessment than was possible in other academic contexts, but results would nevertheless inform social cognitive theory and its claims about self-efficacy in general.

The important finding to emerge was that students' judgments about their capability to solve math problems were more predictive of their ability to solve those problems than were other variables found by previous research also to be strongly related to math performance. Self-efficacy also mediated the effect of gender and prior experience on math self-concept, perceived usefulness of mathematics, and math problem-solving performance. Tests of mean differences showed that men and women differed in performance, self-efficacy, and self-concept, but results of the path analysis suggest that these differences were mediated by differences in the students' self-efficacy perceptions, for only between gender and self-efficacy was there a significant path coefficient. The effects of gender on self-concept and performance were largely indirect and mediated by self-efficacy. That is, the poorer performance and lower self-concept of the female students were largely due to lower judgments of their capability. A similar phenomenon occurred with prior experience, both in high school and college.

It should come as no surprise that what people believe they can do predicts what they can actually do and affects how they feel about themselves as does of that task. How could it be otherwise? And yet, as we noted earlier, re-

Table 5  
Overestimation, Underestimation, and Congruence

Measure	Women		Men		Total	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Overestimation	130	57	70	58	200	57
Underestimation	48	21	21	17	69	20
Congruence—total	51	21	30	25	81	23
Congruence—no errors	13	6	12	10	25	7
Congruence—equal errors	38	17	18	15	56	16



searchers with other theoretical perspectives have long argued that academic performance is better explained by factors such as prior experience or beliefs of self-worth. The clear implication to emerge from our findings is that researchers and school practitioners should be looking to students' beliefs about their capabilities as important mediators and predictors of performance. It is not within the scope of this investigation to outline the ways through which social cognitive theorists suggest that both self-efficacy and academic performance can be enhanced. Instead, the reader is directed to the work of Schunk and associates, who provide insights as to how this can best be accomplished (see Schunk, 1989, 1991).

Hackett and Betz (1989) suggested that "mathematics teachers should pay as much attention to [students'] self-evaluations of competence as to actual performance" (p. 271). We dare not go so far, but it seems clear that assessing students' self-efficacy can provide classroom teachers with additional insights about their students' subsequent performance—insights beyond those obtainable by simply assessing prior knowledge. If self-efficacy is an important predictor of performance and is a primary cause of feelings of self-worth and perceived usefulness, then efforts to identify, understand, and alter inaccurate judgments should prove beneficial. Moreover, if self-efficacy beliefs are major mediators of behavior and behavior change, then counseling interventions designed to change behavior are useful to the degree that they increase the self-efficacy beliefs related to the behavior in question. The math competence of many undergraduates, for example, may tell us very little about their math self-efficacy, and it is the latter factor that will be critical in their choice of math-related decisions such as pursuing math courses, majors, or careers. Avoidance of math courses has its roots in elementary or middle school and generally begins in high school. It is likely that children's judgments of their competence and potential are largely responsible for this avoidance (see Dweck & Leggett, 1988; Pintrich & De Groot, 1990). If self-efficacy assessments were to begin early in a student's academic career, inaccurate perceptions could also be identified early and appropriate interventions undertaken.

As social cognitive theory predicts, most students in this study overestimated their performance. Bandura argued that some overestimation of capability is useful—it provides needed effort and persistence. Many of the students, however, underestimated their capability, and this is seldom a desirable state of affairs. Students who lack confidence in skills they possess are not likely to engage in tasks in which those skills are required, and they will exert less effort and persistence in the face of difficulty. In the area of mathematics, researchers have demonstrated that self-efficacy strongly influences the choice of majors and career decisions of college students (see Lent & Hackett, 1987). In many cases, inaccurate perceptions of mathematics capability, and not poor preparation or lack of skill, are responsible for avoidance of math-related courses and careers.

It is important to know more about how students develop inaccurate self-efficacy beliefs in the first place. How is it that, in spite of success experiences and clearly demonstra-

ble skills, some students develop a profound lack of confidence in their abilities? Lent and Hackett (1987) suggested that this phenomenon is responsible for women failing to pursue math courses, math-related majors, and careers. Research is needed on the process whereby children develop these beliefs. Dweck and Leggett (1988) were rightly concerned that fixed entity views of ability are developed early and tend to last in the absence of intervention techniques. It would be useful to know more about how these beliefs are acquired, why they persevere, and how they can be altered.

Because interventions designed to alter self-efficacy are often desirable and beneficial, the nature of these interventions should be further explored. Classroom teachers may well be impressed by the force of theory arguing that self-efficacy beliefs are important determinants of performance, but they are apt to be more interested in ways to alter these beliefs when they are inaccurate and debilitating to children. More research is also needed on the effect of these interventions on subsequent performance and self-efficacy as well as on anxiety and other self-beliefs such as attributions and self-concept.

The reader will note that ability was not used as a control variable in the path model. Several researchers discourage the inclusion of an ability measure in path analytic models of math performance. Dew, Galassi, and Galassi (1984) warned that ability assessments such as the SAT-Q are confounded by attitudinal and anxiety elements. Thus, variance accounted for by such measures could well include self-efficacy and anxiety components. This confounding has especially plagued math anxiety research (Llabre & Suarez, 1985). Lent et al. (1991) found that ability did not contribute significant incremental variance to the prediction of math course interest after controlling for self-efficacy. Hackett and Betz (1989) reported a significant relationship between globally assessed math confidence and ACT-Q scores but found that only self-efficacy was a significant predictor of choice of math-related major. They concluded that "the cognitive information tapped by the assessment of math self-efficacy should, theoretically, encompass the information an individual has derived from his or her own past performance, making such achievement information as ACT scores redundant if one possesses information about self-efficacy" (p. 270). Recent similar investigations have excluded an ability measure (e.g., Randhawa et al., 1993), and it was not used in our study. We would be remiss, however, if we did not acknowledge that its exclusion may have influenced the effects found, and we recommend that a future model include an ability measure with an eye to testing our findings.

The academic success associated with reported high school level or number of credits earned in mathematics was also not part of the assessment. Our research focus was exposure to course content rather than success. Level of high school mathematics and number of credits earned provide strong and defensible measures of students' prior experience with mathematics, and our operationalization of this variable is consistent with previous research in the field (Frary & Ling, 1983; Hackett & Betz, 1989). Success variables such as grade point average would shift the emphasis

from course content exposure to previous performance attainments. Future studies, however, might be designed to explore the influence of academic experience controlling for some measure of success.

In addition, researchers might also explore the roles that prerequisite prior knowledge and knowledge of strategies to solve problems play in self-efficacy judgments as well as how these judgments influence variables such as goal setting and attainment. The high average score also suggests that, in spite of Dowling's (1978) efforts, a more challenging performance measure may be more appropriate in academic settings similar to ours. Results of this investigation also demonstrate that analyses such as path analyses or structural equation modeling are required if substantive questions are to be more clearly answered. Had we used simpler correlational or multiple regression analyses, the plethora of significant relationships may have led to conclusions that would have been both unclear and misleading.

Our findings strengthen Bandura's (1986) claim that self-efficacy beliefs are key arbiters of human agency and also lend support to researchers who contend that student motivation may be better explained by these beliefs than by other cognitive or affective processes (see Schunk, 1989, 1991). Social cognitive theory offers a promising avenue through which to better understand this motivation, an avenue that can help researchers and school practitioners not only to understand the process itself but to inform them about how they might pursue the important work of building competence and confidence.

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