Assessment of Model Fit

Lecture #6

ICPSR Item Response Theory Workshop

Lecture Overview

- An overview of model fit for IRT Models using Mplus

- Model fit is used to help:
  - Determine if a model fits the data well enough in an absolute sense to use the examinee estimates

  - Select best model among competing models

- Fraction subtraction data will be used to illustrate model fit in practice

ASSESSMENT OF MODEL FIT

Assessing Model Fit

- There is no one best way to assess fit in IRT Models

- Techniques typically used can be put into several general categories:
  - Absolute fit
    - Model based hypothesis tests (if available)
  - Relative fit
    - Information criteria
  - Item fit
    - Univariate
    - Bivariate

- Topics discussed here will mainly focus on fit statistics available in Mplus

Overall Model Fit: Chi-Squared Test

- For small numbers of items (10-15), the traditional Chi-Squared test of model fit can be used
  - Test is invalid for too many items – sparse data

- Mplus gives this automatically
  - Omits when data are sparse

  - Can omit extreme cells from an analysis
    - Misleading

Relative Model Fit: Information Criteria

- Used when comparing between two models
  - 1PL v. 2PL
- Mplus reports:
  - AIC and BIC
  - Sample size adjusted BIC
- All can be used
  - Smallest value is best
- Here, 2PL Model is Preferred using AIC/BIC

- Fraction Subtraction

1PL:

```
MODEL FIT INFORMATION

Number of Free Parameters                       21

Loglikelihood

          H0 Value                       -4797.178

Information Criteria

          Akaike (AIC)                    9636.355
          Bayesian (BIC)                  9726.322
          Sample-Size Adjusted BIC        9659.661
            (n* = (n + 2) / 24)
```

2PL:

```
MODEL FIT INFORMATION

Number of Free Parameters                       40

Loglikelihood

          H0 Value                       -4640.159

Information Criteria

          Akaike (AIC)                    9360.319
          Bayesian (BIC)                  9531.684
          Sample-Size Adjusted BIC        9404.711
            (n* = (n + 2) / 24)
```

Chi-Squared (Deviance) Test

- The 1PL and 2PL are nested models
  - Can use deviance test to statistically test for fit

- Change in -2*loglikelihood:

-2*(-4797.178 – -4640.159) = 314.038

- Change in DF:

40-21 = 19

- Chi-Square p-value < 0.0001

- Conclusion: 2PL is preferred statistically

- Fraction Subtraction

1PL:

MODEL FIT INFORMATION	
Number of Free Parameters	21
Loglikelihood	
H0 Value	-4797.178
Information Criteria	
Akaike (AIC)	9636.355
Bayesian (BIC)	9726.322
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	9659.661

2PL:

MODEL FIT INFORMATION	
Number of Free Parameters	40
Loglikelihood	
H0 Value	-4640.159
Information Criteria	
Akaike (AIC)	9360.319
Bayesian (BIC)	9531.684
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	9404.711

Item Fit Statistics

- The TECH10 option reports a degree of misfit for each
  - Item individually (Univariate)
  - Pair of two items (Bivariate)

- Uses Chi-Squared test for misfit
  - Values for each item are distributed as Chi-square with 1 df (for binary items)

- Misfitting items can be investigated
  - Items can be removed
  - Multidimensional model may be used

Item Fit Statistics: Univariate Fit

- Univariate fit attempts to determine if the model fits each item marginally
  - Limited information statistic
- Not *that* useful in IRT
  - Scale of fit is usually small
  - Most items "fit"
- H1:Observed Probability
- H0: IRT Model Prediction
- Chi-Square critical values
  - (0.05) = 3.84
  - (0.01) = 6.63

UNIVARIATE MODEL FIT INFORMATION

	Estimated Probabilities		Standardized Residual (z-score)
Variable	H1	H0	
X1			
Category 1	0.468	0.479	-0.511
Category 2	0.532	0.521	0.511
Univariate Pearson Chi-Square			0.261
Univariate Log-Likelihood Chi-Square			0.262
X2			
Category 1	0.424	0.434	-0.487
Category 2	0.576	0.566	0.487
Univariate Pearson Chi-Square			0.237
Univariate Log-Likelihood Chi-Square			0.238
X3			
Category 1	0.485	0.498	-0.611
Category 2	0.515	0.502	0.611
Univariate Pearson Chi-Square			0.373
Univariate Log-Likelihood Chi-Square			0.373

Item Fit Statistics: Bivariate Fit

- Bivariate fit is an index of fit for a pair of items
- Compares observed data with frequency expected under IRT model
  - Produces a 1-df Chi-Squared test for binary items
- Can help identify items that do not fit model
  - Rough approximation

BIVARIATE MODEL FIT INFORMATION

Variable	Variable	Estimated Probabilities H1	H0	Standardized Residual (z-score)
X1	X2			
Category 1	Category 1	0.390	0.338	2.542
Category 1	Category 2	0.078	0.141	-4.185
Category 2	Category 1	0.034	0.096	-4.903
Category 2	Category 2	0.498	0.425	3.438
Bivariate Pearson Chi-Square				47.850
Bivariate Log-Likelihood Chi-Square				57.544
X1	X3			
Category 1	Category 1	0.410	0.363	2.304
Category 1	Category 2	0.058	0.117	-4.245
Category 2	Category 1	0.075	0.136	-4.126
Category 2	Category 2	0.457	0.385	3.429
Bivariate Pearson Chi-Square				41.247
Bivariate Log-Likelihood Chi-Square				47.257
X1	X4			
Category 1	Category 1	0.315	0.325	-0.481
Category 1	Category 2	0.153	0.154	-0.083
Category 2	Category 1	0.157	0.152	0.288
Category 2	Category 2	0.375	0.368	0.315
Bivariate Pearson Chi-Square				0.295
Bivariate Log-Likelihood Chi-Square				0.296

Fraction Subtraction Results: Model Fit

- Univariate model fit
  - Compares model predicted and observed frequencies of responses for all items marginally
  - Of 20 items, none had p-values less than 0.01

- Bivariate model fit
  - Compares model predicted and observed frequencies of responses for all pairs of items
  - Of 190 item pairs 35 had p-values less than 0.01
  - Items most indicated
    - Item 4 (8 pairs) : $3\frac{1}{2} - 2\frac{3}{2}$
    - Item 6 (7 pairs): $\frac{6}{7} - \frac{4}{7}$
    - Item 20 (7 pairs): $4\frac{1}{3} - 1\frac{5}{3}$

- Indicates some items are not fit well by model
  - We will ignore this and continue with analysis as example

“TRADITIONAL” IRT FIT METHODS

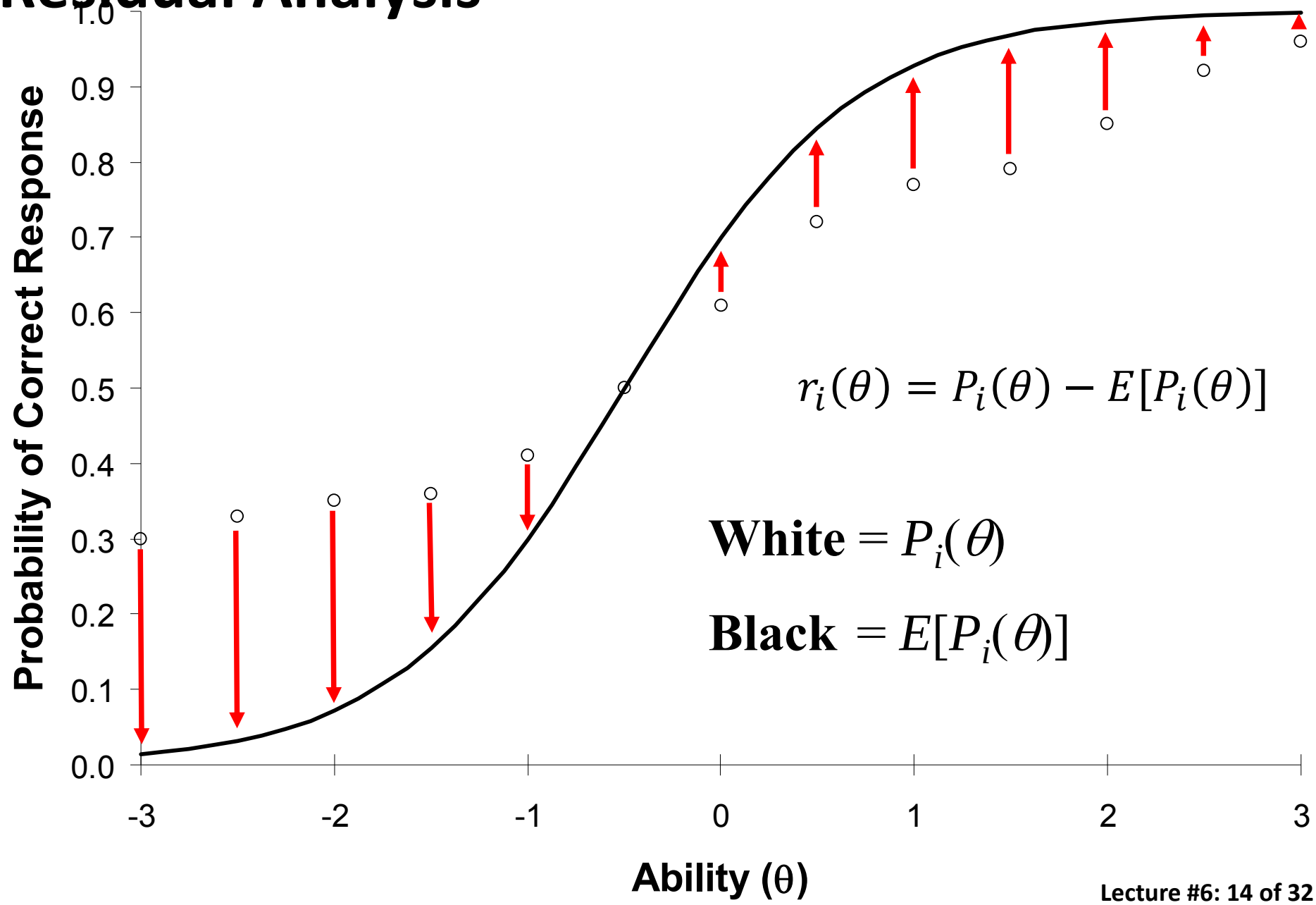
Residual Analysis

- Assess the accuracy of model predictions versus actual data
  - Residual = difference between observed proportion and predicted probability:

$$r_i(\theta) = P_i(\theta) - E[P_i(\theta)]$$

- $P_i(\theta)$ = observed proportion correct for a given **theta** level
- $E[P_i(\theta)]$= expected proportion correct (i.e., probability from the IRT model)

Residual Analysis



Standardized Residuals

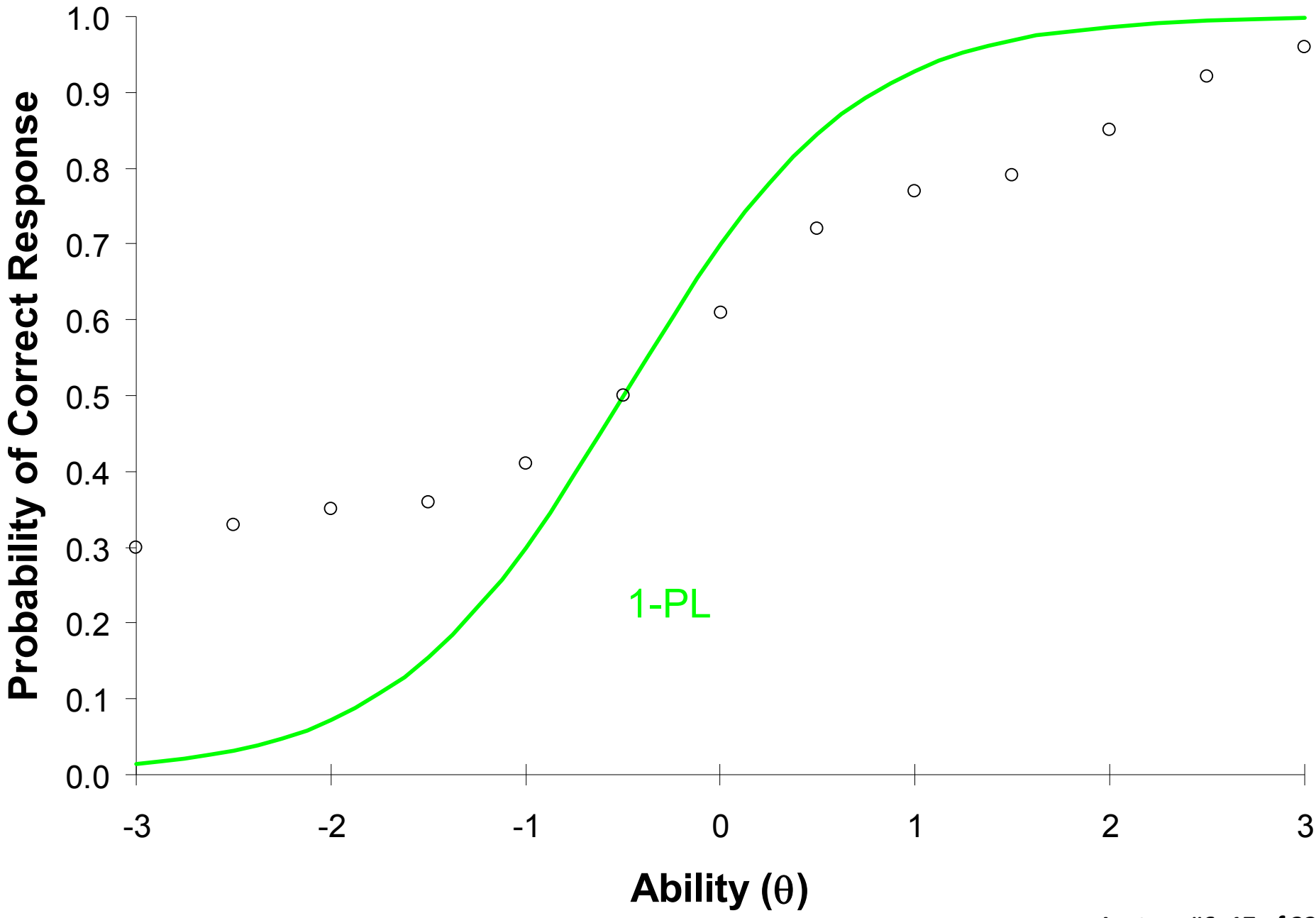
- Raw residuals do not take into account the error associated with the expected proportion correct, so we standardize each by dividing by its standard error:

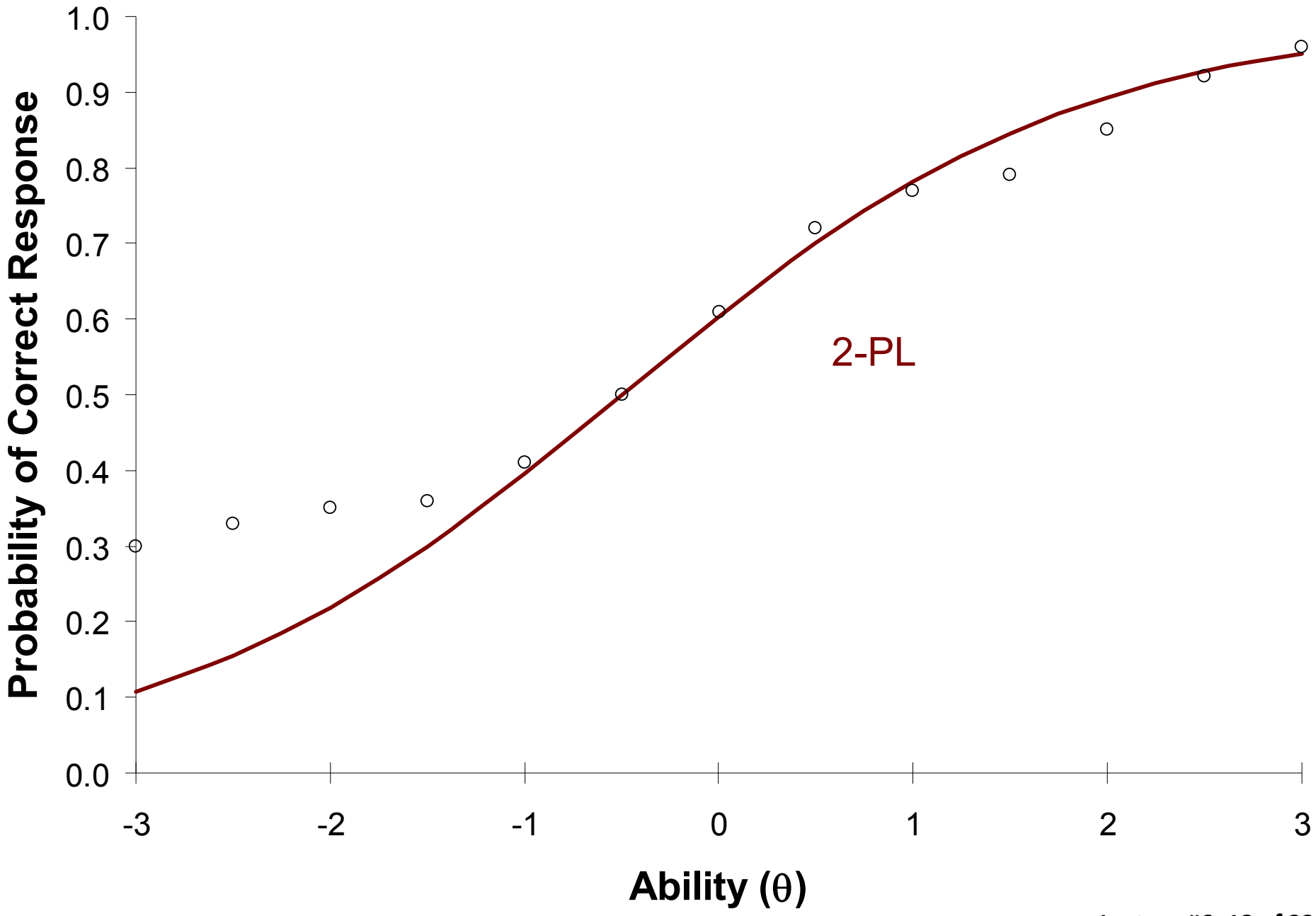
$$SE(E[P_i(\theta)]) = \sqrt{\frac{E[P_i(\theta)]E[1 - P_i(\theta)]}{N(\theta)}}$$

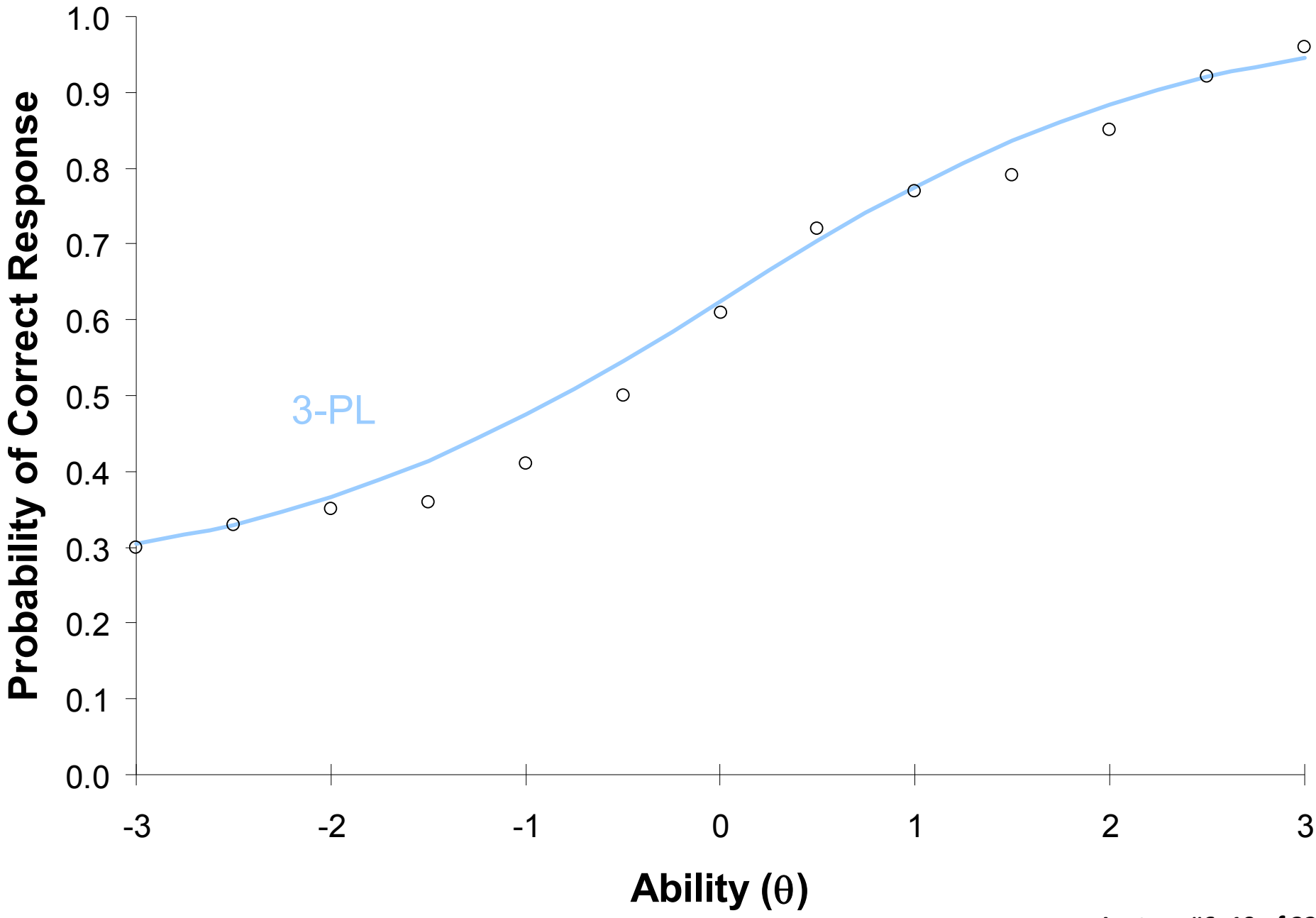
Standardized Residuals

$$SR_i(\theta) = \frac{P_i(\theta) - E[P_i(\theta)]}{\sqrt{\frac{E[P_i(\theta)]E[1 - P_i(\theta)]}{N}}}$$

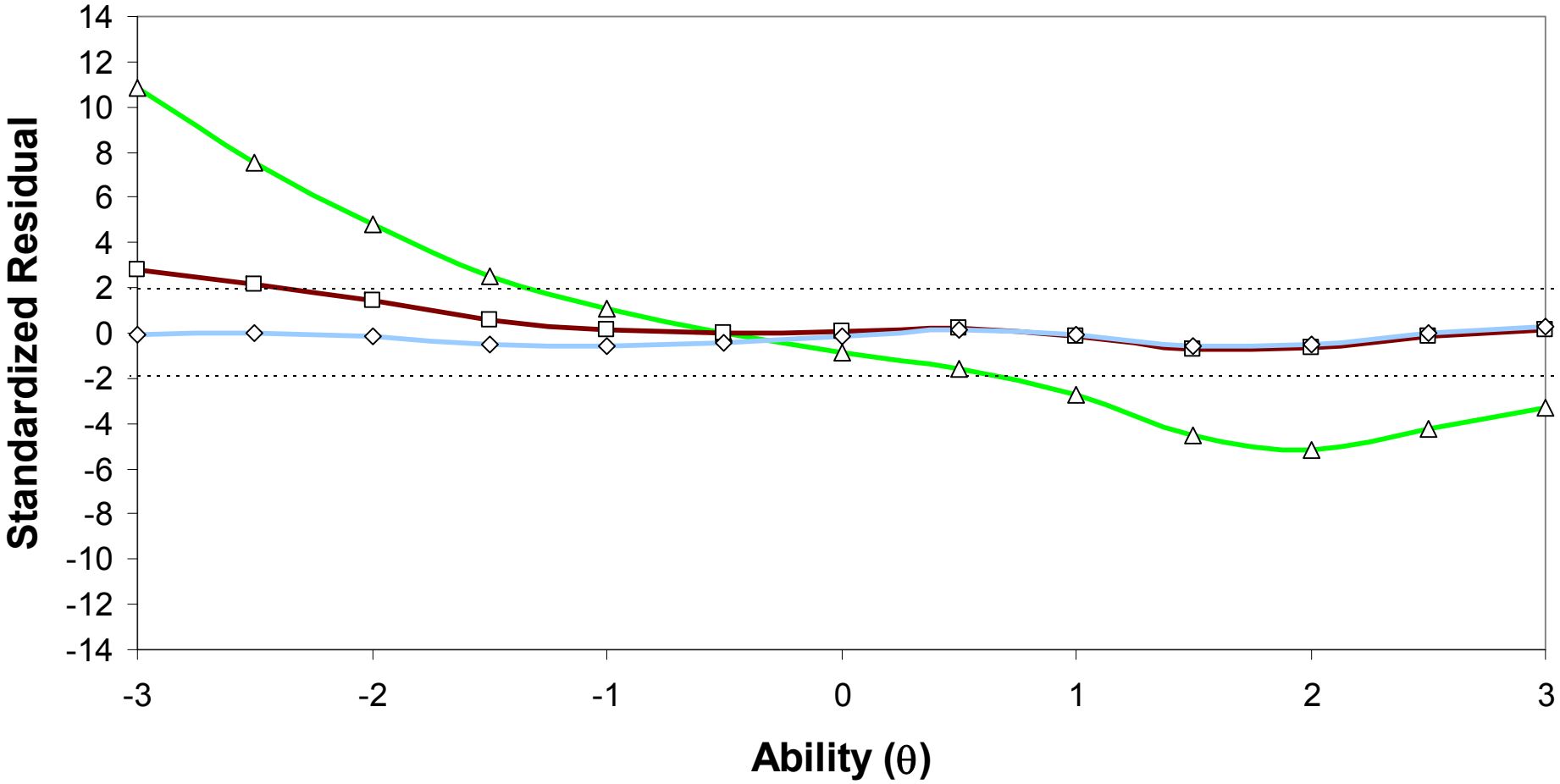
SR values should be homoscedastic for each item and follow an approximately standard normal distribution across all items of the test.


Probability of Correct Response
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3
-2
-1
0
1
2
3
Ability (θ)
1-PL


Probability of Correct Response
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3
-2
-1
0
1
2
3
Ability (θ)
2-PL


Probability of Correct Response
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
3-PL
-3
-2
-1
0
1
2
3
Ability (θ)

SR values are homoscedastic for this item when fit by a 3-PL model, but systematic errors are present for the 1- and 2-PL models

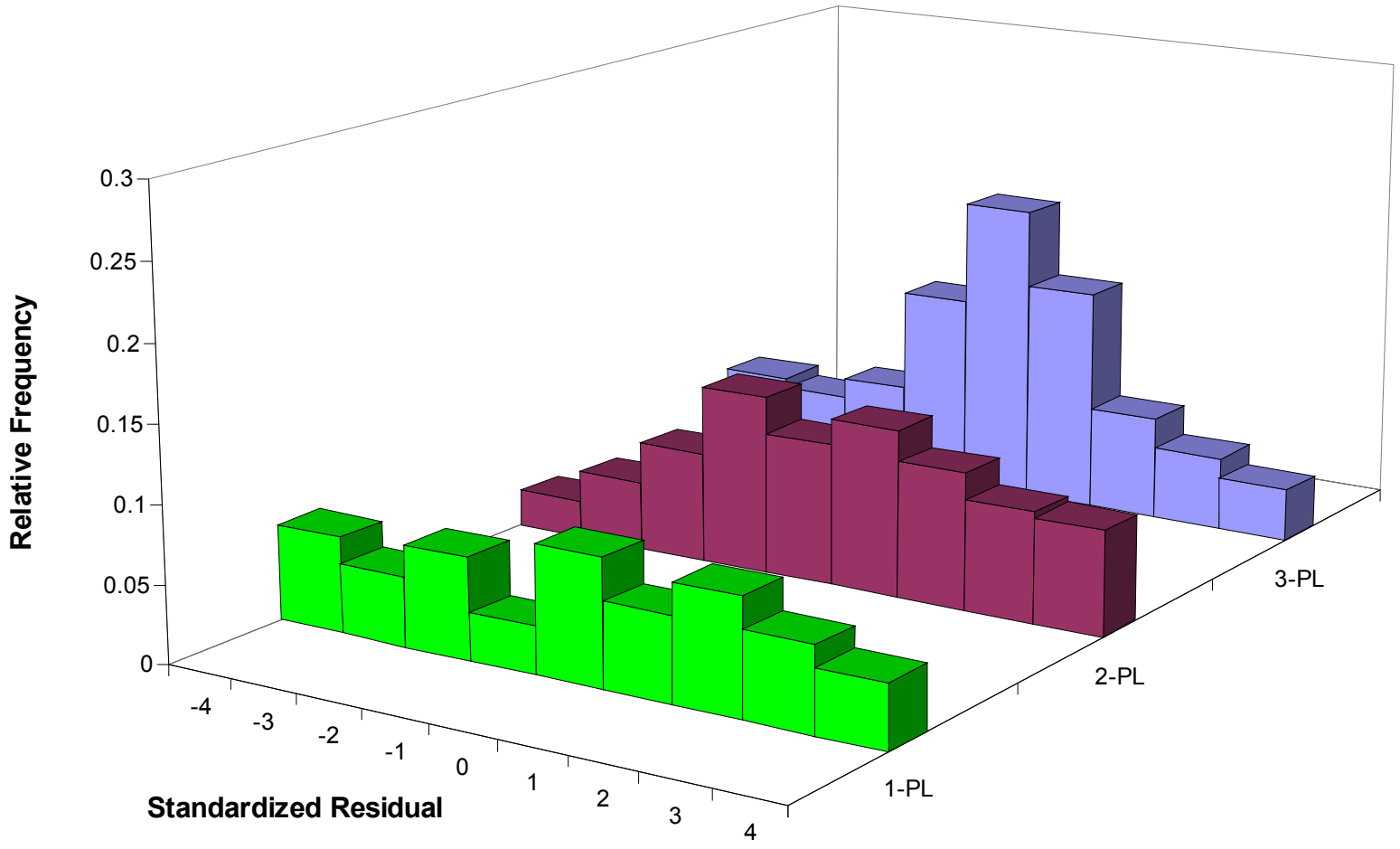


Test-level Fit

- Similar to the comparison done for individual items, but instead we compare Expected Proportion Correct (TCC) to observed proportion correct (raw score/N)

- SRs across the test should be homoscedastic and follow an approximate normal distribution

Across all items, SR values are approximately normally distributed when fit by a 3-PL model, but more uniform for the 1- and 2-PL models



Significance Testing
Q1 chi-square (Yen, 1981)

$$Q1_j = \sum_{i=1}^{m} SR_{ij}^2 \qquad Q1 \sim \chi^2 \qquad df = m - p$$

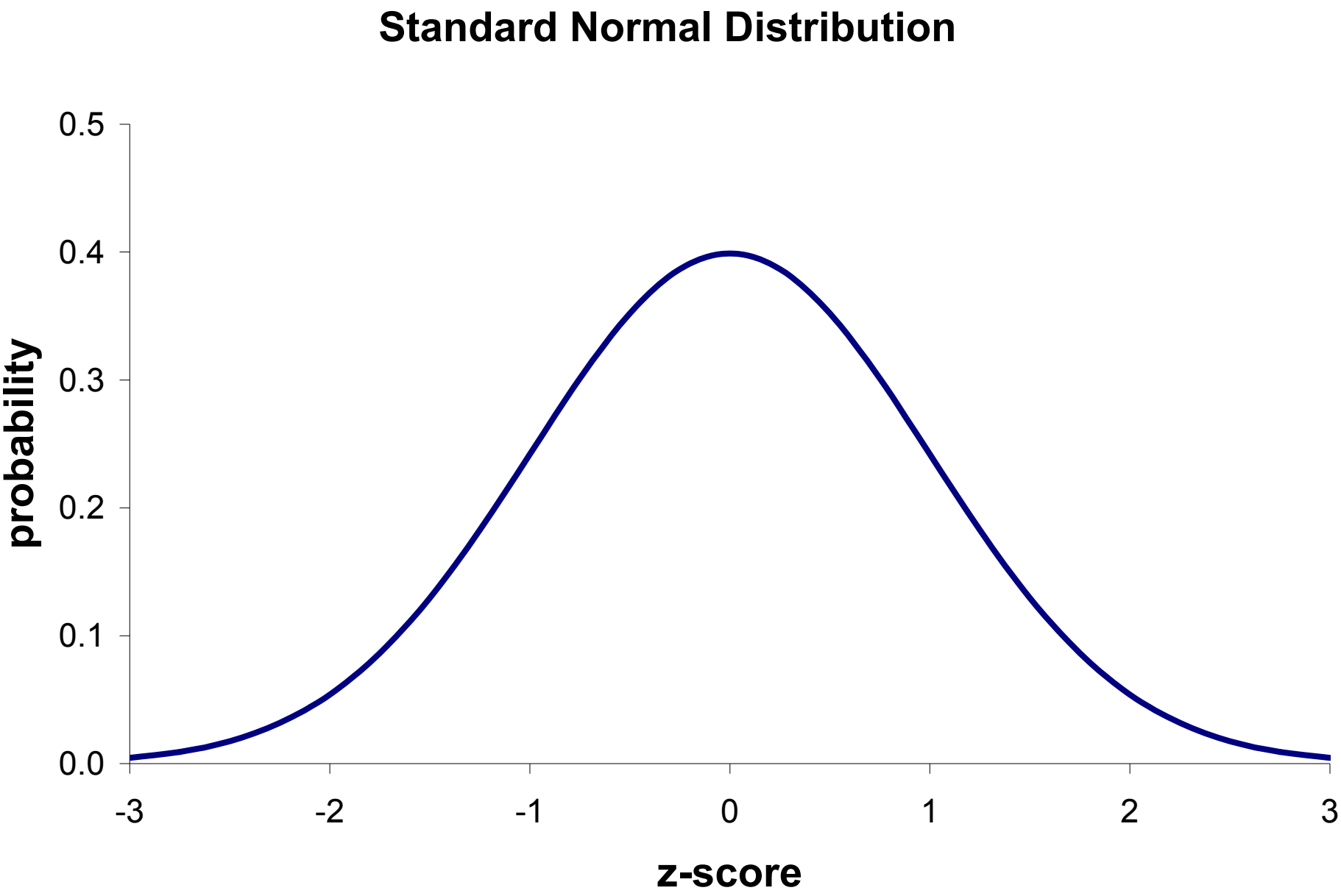
$m =$ # of quadrature points

$p =$ # of item parameters

Chi-square test

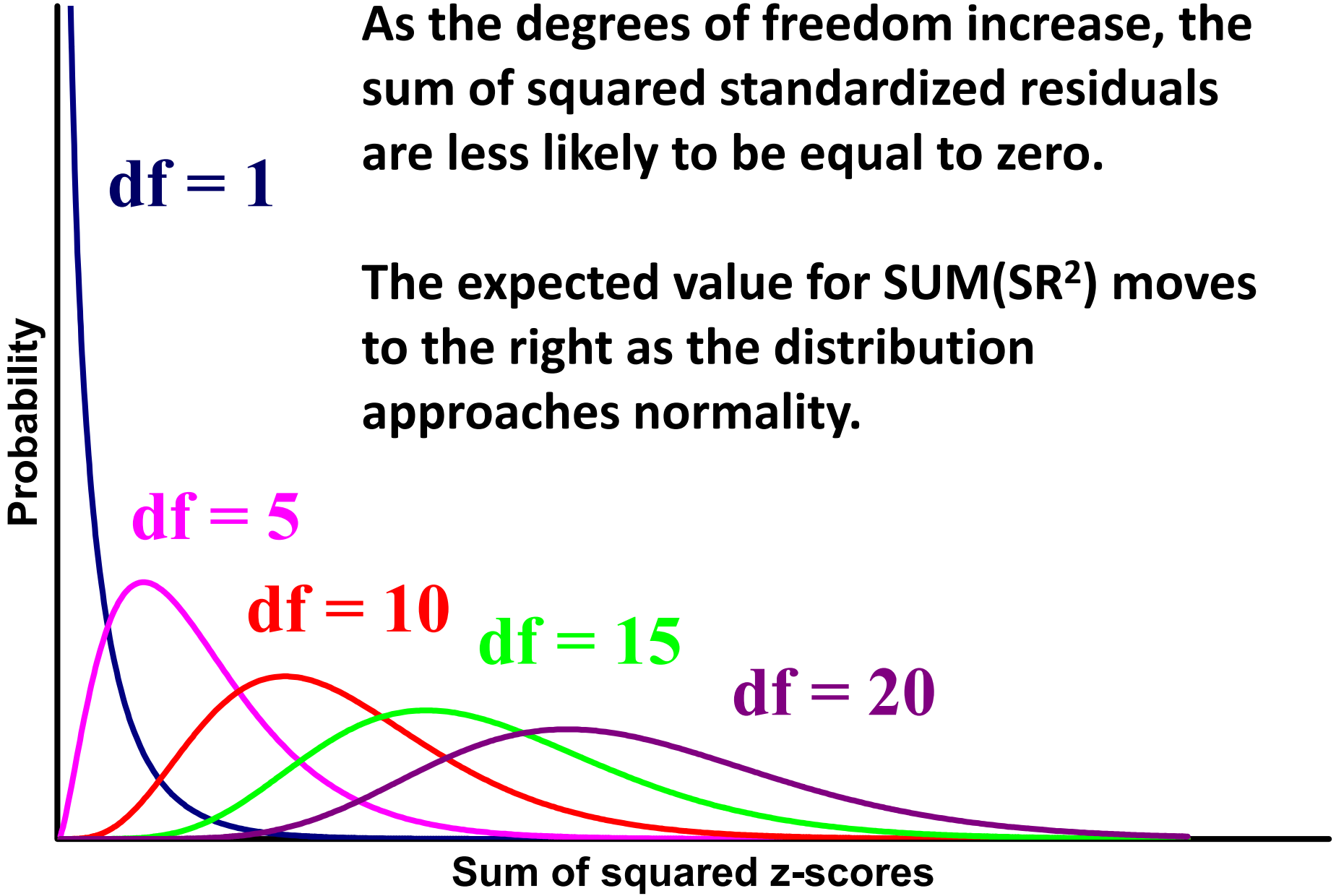
- Standardized residuals are essentially prediction errors that have been turned into z-scores

- The sum of squared z-scores follow a Chi-square distribution
  - Much like Sums of Squares and variances follow a Chi-square distribution in ANOVA


Standard Normal Distribution
probability
0.5
0.4
0.3
0.2
0.1
0.0
-3
-2
-1
0
1
2
3
z-score

As the degrees of freedom increase, the sum of squared standardized residuals are less likely to be equal to zero.

The expected value for SUM(SR^2) moves to the right as the distribution approaches normality.



Goodness-of-Fit

- This is what we call “goodness-of-fit”:

- We hope that the chi-square test will NOT be significant
  - This indicates that the differences between observed and expected is small
  - Significant differences would mean that observed proportions are far from what the model predicted

Significance Testing in BILOG and PARSCALE

- The goodness of fit information contained in BILOG and PARSCALE use the Chi-square test described in the previous slides

  - These values can be found for all items

SUBTEST PRETEST ; ITEM PARAMETERS AFTER CYCLE 30

ITEM	INTERCEPT S.E.	SLOPE S.E.	THRESHOLD S.E.	LOADING S.E.	ASYMPTOTE S.E.	CHISQ (PROB)	DF
MATH01	1.041	0.651	-1.599	0.545	0.186	29.0	9.0
	0.107*	0.082*	0.242*	0.069*	0.084*	(0.0007)	
MATH02	2.230	0.600	-3.717	0.514	0.199	9.5	5.0
	0.165*	0.114*	0.610*	0.098*	0.089*	(0.0920)	
MATH03	0.428	0.693	-0.618	0.569	0.159	63.2	7.0
	0.106*	0.084*	0.190*	0.069*	0.071*	(0.0000)	
MATH04	-0.601	1.391	0.432	0.812	0.217	10.7	8.0
	0.216*	0.268*	0.095*	0.156*	0.040*	(0.2204)	

The problem of sample size

- Statistical tests of model-data fit present an interesting duality:

- Due to sensitivity to sample size, almost any departure of data from the model results in rejecting H0

- For small samples, model-data misfit can be overlooked, and SEs for item parameters are large

CONCLUDING REMARKS

Concluding Remarks: Model Fit

- Assessment of model fit in IRT Models is currently a difficult task
  - Easily accessible options are limited
  - Can quickly find options that take longer to assess fit than to estimate model
  - Mplus options are adequate for initial screening

- IRT models share this problem general categorical data analysis techniques

- Other model fit options are available and forthcoming