Analysis of Covariance

ERSH 8320

Today's Lecture

- Analysis of Covariance - Chapter 15.
- ANCOVA v. ATI (in our author's mind).
- When ANCOVA is invalid: group equating.
- ANCOVA with multiple covariates.

ANCOVA For Control

- Analysis of covariance (ANCOVA) is used for two different purposes:

  1. Statistical control of relevant variables that are not part of the model.

  2. Adjustment for initial differences among groups being compared.

- ANCOVA for #1 is warrented, and is the focus of today's lecture.

- ANCOVA for #2 is questionable at best.

ANCOVA For Control

- It is often beneficial to control outside factors that may affect the results of an analysis.
- These factors often are unable to be controlled by the experimenter due to design constraints.
  - Random assignment to groups is needed, but often not possible.

Teaching Method Example

- As an example, suppose you want to study the effects of achievement due to teaching method.
- You cannot control the intelligence of the individual you are testing.
- If you control for intelligence, significant results will indicate that one testing method was better than the other, regardless how smart the individual is at the beginning of the class.
- Being able to control for influential factors allows for a much more powerful conclusion.

Teaching Method Example

- There are two ways to control for intelligence in this example:

  1. Experimental design:
     - Group subjects based on ability, then randomly assign the same number of individuals from each group the each treatment.
  2. Statistically:
     - We can partial out the variance associated with the factor and then look at the results. This is called adding a covariate.

- It can be much simpler to use statistical methods to control for intelligence.

The Logic of ANCOVA

- Recall that a residualized variable is a variable which has all information shared with a predictor variable has been removed.

- The correlation between the residualized variable and the predictor is then zero.

- So, back to our example, if we take the predicted achievement of the subject and subtract off the expected achievement, this would be standardizing.

- We can then look at the effects of our treatment based on this standardized achievement.

The Logic of ANCOVA

- The same logic follows for the ANCOVA, which can be expressed by the following equation:

$$Y_{ij} = \bar{Y} + T_j + b(X_{ij} - \bar{X}_{ij}) + e_{ij}$$

- Y_{ij} is the score of subject i under treatment j.
- $\bar{Y}$ is the grand mean of the dependent variable.
- T_j is the effect of treatment j.
- b is a common regression coefficient for Y on X.
- X_{ij} is the score of the covariate for subject i in treatment j.
- $\bar{X}$ is the grand mean of the covariate.
- e_{ij} is the error associated with the score of subject i under treatment j.

The Logic of ANCOVA

- Notice how the previous equation:

$$Y_{ij} = \bar{Y} + T_j + b(X_{ij} - \bar{X}_{ij}) + e_{ij},$$

can be re-written as:

$$Y_{ij} - b(X_{ij} - \bar{X}_{ij}) = \bar{Y} + T_j + e_{ij}.$$

- This helps to better understand what we are doing.
- We are standardizing achievement based on the intelligence of the individual.
- Note the lack of the interaction term here in this model statement.

Homogeneity of Regression Coefficients

- Controlling for the covariate involves applying the common regression coefficient that we discussed last time to the deviation of X from the grand mean of X.
- The common regression coefficient is the coefficient found when estimating the ANCOVA model without the interaction term.
- Not using the interaction term means that the regression line for each group must be equivalent with respect to the slope.
- Hence the term “common regression coefficient.”

Then what is ANCOVA?

- Our author differentiates between ANCOVA (where no interaction term is present) and ATI designs (where interactions are present).
- ANCOVA (according to our author) is a valid procedure once we assume that differences among the b parameters for the regression of Y on X in the different treatment groups are not statistically different.
- In other words, once we determine that no interaction is present, ANCOVA is a valid procedure to use.

Then what is ANCOVA?

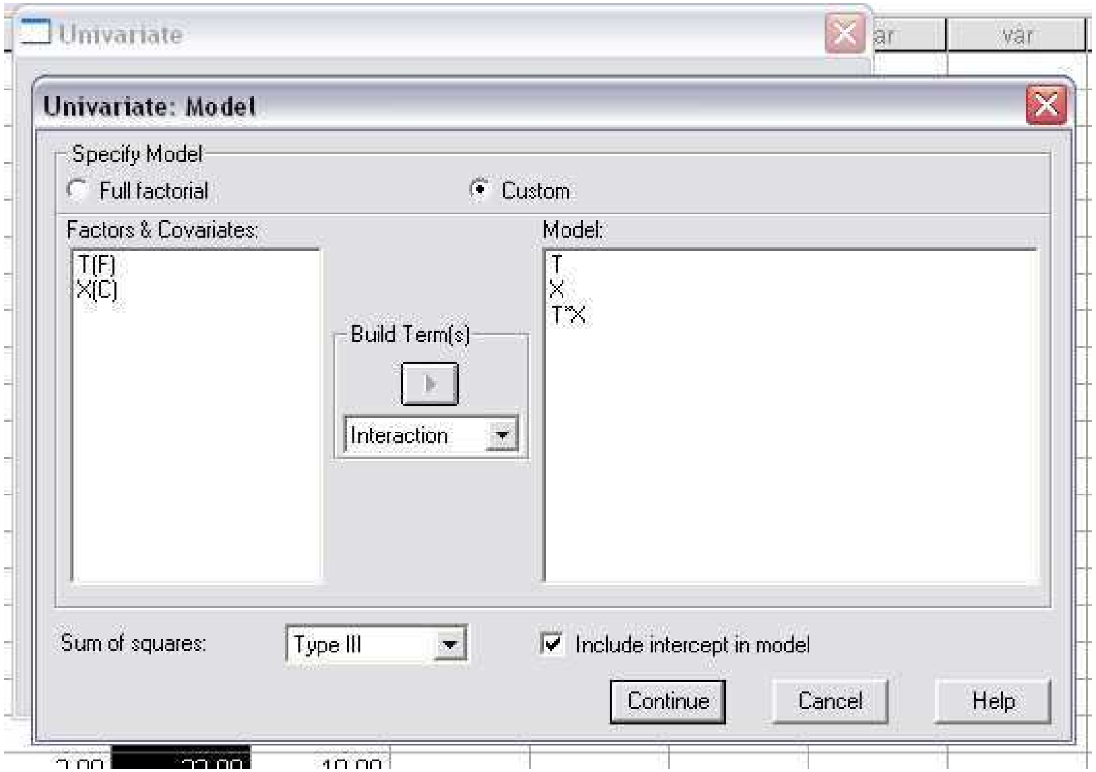
- In other words, we must assume the covariate affects each variable the same (the regression lines would have the same slope or be parallel).
- You can test this assumption by performing the techniques from the previous chapter based on separate regression equations.
- Once we are happy with this assumption being met, the techniques used in ANCOVA are analogous to testing the intercepts of each regression equation to see if they are equal.
- The “ANCOVA” technique is much simpler and more powerful, which is why we it is done this way.

ANCOVA Example

- As an example, consider an experiment with:
  - Four different treatment groups (T).
  - A dependent variable (X).
  - A single covariate (Y).

In SPSS...

- Using Analyze...General Linear Model...Univariate
- First, test for presence of significant interaction.
- This must be done under the Model Box:



In SPSS...

Tests of Between-Subjects Effects

Dependent Variable: Dependent Variable

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	222.234[a]	7	31.748	10.193	.000
Intercept	300.680	1	300.680	96.540	.000
T	1.914	3	.638	.205	.892
X	152.279	1	152.279	48.893	.000
T * X	1.006	3	.335	.108	.955
Error	99.666	32	3.115		
Total	12642.000	40			
Corrected Total	321.900	39			

a. R Squared = .690 (Adjusted R Squared = .623)

Omitting the Interaction

- Using Analyze...General Linear Model...Univariate
- Second, remove interaction term.
- Save the means for each group.

Tests of Between-Subjects Effects

Dependent Variable: Dependent Variable

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	221.227[a]	4	55.307	19.228	.000
Intercept	313.501	1	313.501	108.992	.000
X	165.127	1	165.127	57.408	.000
T	65.042	3	21.681	7.538	.001
Error	100.673	35	2.876		
Total	12642.000	40			
Corrected Total	321.900	39			

a. R Squared = .687 (Adjusted R Squared = .652)

Estimated Marginal Means

Treatment Condition

Dependent Variable: Dependent Variable

Treatment Condition	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	15.521[a]	.538	14.430	16.613
2.00	18.027[a]	.537	16.937	19.116
3.00	19.024[a]	.536	17.935	20.113
4.00	17.628[a]	.537	16.537	18.718

a. Covariates appearing in the model are evaluated at the following values: Covariate = 7.1250.

Omitting the Covariate

- Using Analyze...General Linear Model...Univariate
- Remove the covariate altogether.
- Save the means for each group.

Tests of Between-Subjects Effects

Dependent Variable: Dependent Variable

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	56.100[a]	3	18.700	2.533	.072
Intercept	12320.100	1	12320.100	1668.637	.000
T	56.100	3	18.700	2.533	.072
Error	265.800	36	7.383		
Total	12642.000	40			
Corrected Total	321.900	39			

a. R Squared = .174 (Adjusted R Squared = .105)

Estimated Marginal Means

Treatment Condition

Dependent Variable: Dependent Variable

Treatment Condition	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1.00	15.800	.859	14.057	17.543
2.00	17.900	.859	16.157	19.643
3.00	19.100	.859	17.357	20.843
4.00	17.400	.859	15.657	19.143

Adjusted Means

- ANCOVA adjusts the mean for each treatment group by that of the mean deviation for the covariate:

$$\bar{Y}_{j\text{(adj)}} = \bar{Y}_j - b(\bar{X}_j - \bar{X})$$

- Although not easily found in SPSS, consider the following (with $b = 1.013$):

Group	$\bar{Y}$	$\bar{X}$
1	15.8	7.4
2	17.9	7.0
3	19.1	7.2
4	17.4	6.9
Overall	17.55	7.125

Tests for Differences of Adjusted Means

- With the ANCOVA adjusted means, one can now test for differences between the means (using, for example, multiple planned comparisons).
- Using Analyze...General Linear Model...Univariate
- Use the covariate.
- Go under Options...

Pairwise Comparisons

Dependent Variable: Dependent Variable

(I) Treatment Condition	(J) Treatment Condition	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	Upper Bound
1.00	2.00	-2.505*	.760	.014	-4.632	-.379
	3.00	-3.503*	.759	.000	-5.625	-1.380
	4.00	-2.107	.761	.054	-4.236	.023
2.00	1.00	2.505*	.760	.014	.379	4.632
	3.00	-.997	.759	1.000	-3.120	1.125
	4.00	.399	.759	1.000	-1.723	2.520
3.00	1.00	3.503*	.759	.000	1.380	5.625
	2.00	.997	.759	1.000	-1.125	3.120
	4.00	1.396	.760	.447	-.728	3.520
4.00	1.00	2.107	.761	.054	-.023	4.236
	2.00	-.399	.759	1.000	-2.520	1.723
	3.00	-1.396	.760	.447	-3.520	.728

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

ANCOVA Uses

- ANCOVA was initially developed in the context of experimental research to increase the precision of statistical analysis by controlling for sources of systematic variations.
- It is misused when applied is cases of quasi-experimental or nonexperimental research in order to equate groups.
- Outline of steps of ANCOVA with one covariate.
  1. Create a vector of responses for the dependent variable.
  2. Create a vector of responses for the covariate.
  3. Create coded vector of group membership (treatment group).

ANCOVA Uses

- Outline continued:

  4. Multiply covariate by coded variable.
  5. Test whether the proportion of variance accounted for by adding the vector is step #4 to the model is significant. If significant, do not use ANCOVA (interpret groups separately).
  6. Test whether the proportion of variance accounted for by adding the vector is step #3 to the model is significant as compared to vector #2 alone. If yes, there are group differences in treatment groups.
  7. Look at differences between adjusted means of the groups to find the differences present.

- Note that steps 1-4 are accomplished automatically using SPSS.

ANCOVA with Multiple Covariates

- Often, there is more than one variable that you would like to control for.
- This approach generalizes to any number of covariates that you would like to add.
- All other generalizations are valid, except now to adjust the means you use:

$$\bar{Y}_{j\text{(adj)}} = \bar{Y}_j - b_X(\bar{X}_j - \bar{X}) - b_Z(\bar{Z}_j - \bar{Z})$$

Factorial ANCOVA

- The examples given so far all deal with a single independent variable.
- ANCOVA is not limited in this respect.
- You are able to use multiple factors (categorical independent variables) along with multiple covariates.
- The extension of ANCOVA is similar to that of the factorial design of the ANOVA.
- The variables are coded in the same manner as in the factorial design in ANOVA, then the covariates are added to the model.

ANCOVA for Adjustment

- ANCOVA is also used in some contexts in an attempt to equate groups that are essentially nonequivalent, or to adjust for differences among preexisting groups on a covariate.
- These uses are not always valid applications of this procedure.
- Here are some examples of these uses:

  1. When one or more manipulated variables are used with nonequivalent groups, ANCOVA is used in an attempt to equate these groups (referred to as quasi-experimental design).
  2. In nonexperimental research, ANCOVA is often used to compare the performance of two or more groups on a given variable while controlling for one or more relevant variables.
  3. Sometimes one wants to compare regression equations obtained in intact groups while controlling for relevant variables.

Problems with Interpretation

- Recall that the use of ANCOVA in experimental research is aimed at identifying and removing extraneous variance, thereby increasing the precision of the analysis.
- In addition to the assumptions present in ANCOVA, it is also necessary that the treatments do not affect the covariate either directly or indirectly.
- Any effect will cause the adjustment due to the covariate to remove not only extraneous variance but also variance due to the treatments.

Problems with Interpretation

- The situation on the previous slide, however, is different when ANCOVA is used in quasi-experimental or nonexperimental research in an attempt to equate groups.
- Many statistical and theoretical problems arise when these methods are used in this manner.
- The following are some errors that arise when ANCOVA is used this way:
  - Specification Errors
  - Extrapolation Errors
  - Differential Growth
  - Nonlinearity
  - Measurement Errors

Specification Errors

- Specification errors refer to misspecified models.
- These errors lead to biased estimates of the common regression coefficient for the covariate and an overadjustment or underadjustment of treatment means.

Extrapolation Errors

- When there are considerable differences in the covariate between two groups (i.e. there is little overlap of the distributions), the process of arriving at the adjusted means involves two extrapolations, one for each distribution.
- There are often adjustment errors that arise.

Differential Growth

- ANCOVA is often used to equate two groups when attempting to test group differences in growth due to an independent variable.
- Using this technique assumes the covariate affects each group equally, which may not be the case in terms of growth rate.

Nonlinearity

- One assumption of ANCOVA is that the regression of the dependent variable on the covariate is linear.
- If this relationship is nonlinear, ANCOVA is not a valid approach.
- Plot the relationship before running the analysis to check this assumption.

Measurement Errors

- When measurement errors are present in the covariate, this will lead to underadjustments for the initial differences among groups.
- This may greatly mislead you in terms of your conclusions.
- This problem is omnipresent in research, giving a total of three certainties in life:
  1. Death.
  2. Taxes.
  3. Measurement errors deteriorating any treatment effects.

Final Thought

- ANCOVA is referred to by Pedhazur as regression with categorical and continuous variables when no interaction is present.

- ANCOVA is typically used in experimental designs.

- Care must be taken to ensure valid uses of ANCOVA in quasi-experiment or nonexperimental research protocols.

Next Time

- Final exam discussion.