



The University of Georgia

DCMs in Practice

Session 3



Diagnosing Teachers' Multiplicative Reasoning

- *Diagnosing Teachers' Multiplicative Reasoning** (DTMR)
 - NSF funded grant (DRL-0903411)
- Goal was to create a multidimensional test that will assess fine-grained components of teachers' reasoning multiplicatively with fractions
 - Understanding fractions as quantities as in Common Core State Standards
- The test would be used to
 - Tailor professional development to teachers' needs
 - Quantitatively study teachers' fine-grained reasoning abilities
 - ◆ Quantify findings based on extensive qualitative research base
 - ◆ Generalize to larger populations



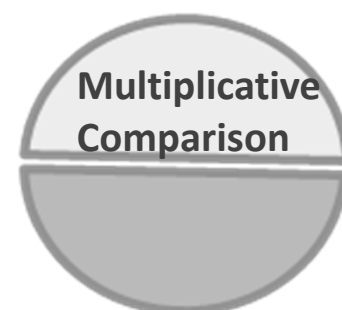
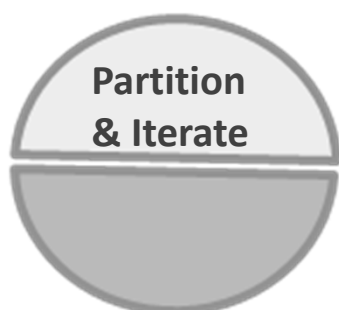
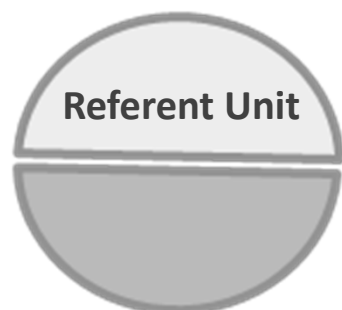
Diagnosis from a Psychometric Model

- Diagnostic classification models would provide the type of diagnostic feedback we wanted to give teachers
 - We were trying to make **decisions** about teachers
- A diagnosis is a decision
 - Does a teacher need professional development on a given concept?



Attributes

- Instead of measuring an overall ability to reason multiplicatively with fractions, we broke down that continuous trait down into more fine-grained cognitive facilities or *attributes*:
 - Ability to identify appropriate **referent units** for numbers
 - Ability to **partition** quantities and **iterate** unit fractions
 - Ability to identify **appropriate** arithmetic operations
 - Ability to make **multiplicative comparisons**
- We were interested in two levels of mastery
 - Each attribute had two categories
 - Mastery of an attribute (= 1) or non-mastery of an attribute (= 0)





The DTMR Fractions Test

Designed to be Multidimensional using a
DCM Framework



Steps in Test Construction

- 3-year test construction process
 - Interdisciplinary team from three (ish) universities
 - ◆ Mathematics Education Researchers & Psychometricians
- Key Steps
 - Operationalizing attributes
 - Writing Items
 - Validating Item/Attribute Alignment
 - Refining Items/Attributes



Defining Attributes

- Complex process
 - Not something that had been explicitly done at this level in mathematics education
- Relied on wealth of qualitative mathematics education research to define a set of workable attributes
 - Did not encompass all components of multiplicative reasoning
 - Focused on a set of components known to be difficult for students and teachers
 - ♦ Much less literature on teacher knowledge than student knowledge for this area
- Continued to refine for first few years of project



Designing Diagnostic Tests

- Items were written so that each item measures one or more of the attributes
 - Mapping is established by content experts
 - ◆ Confirmed by item response interviews
- Q-matrix for first several items on DTMR test:

	RU	PI	APP	MC
Item 1	1	0	0	0
Item 2	0	0	1	0
Item 3	1	0	0	0
Item 4	1	0	0	1



Example Item

- This item is analogous to Item 22 on the DTMR test
 - » Measures Referent Unit (Attribute 1) and Partitioning and Iterating (Attribute 2)

Ms. Roland gave her students the following problem to solve:

Candice has $\frac{4}{5}$ of a meter of cloth. She uses $\frac{1}{8}$ of a meter for a project.

How much cloth does she have left after the project?

She had students use the number line so that they could draw the lengths. Which of the following diagrams shows the solution? Assume all intervals are subdivided equally.

a)



b)



c)



d)



e)





Example Item Solution

- Option A is incorrect
 - » Takes $\frac{1}{8}$ of $\frac{4}{5}$ away from $\frac{4}{5}$ instead of $\frac{1}{8}$ of 1 whole
 - » Evidence of non-mastery of referent unit
- Option B is correct
 - » Correctly partitions the $\frac{1}{5}$ segment so that the whole meter would be segmented into 40 pieces. Then, 5 of the 40 pieces or $\frac{1}{8}$ of the whole are removed from $\frac{4}{5}$ of the meter.

Ms. Roland gave her students the following problem to solve:

*Candice has $\frac{4}{5}$ of a meter of cloth. She uses $\frac{1}{8}$ of a meter for a project.
How much cloth does she have left after the project?*

She had students use the number line so that they could draw the lengths. Which of the following diagrams shows the solution? Assume all intervals are subdivided equally.

a)



b)





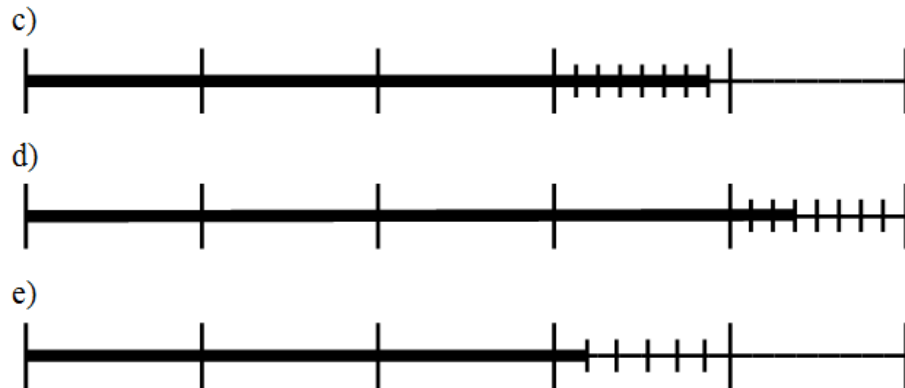
Example Item Solution

- Option C is incorrect
 - » Correctly partitions
 - » Takes $1/8$ of $1/5$ away from $4/5$ instead of $1/8$ of 1 whole
 - Evidence of non-mastery of referent unit
- Option D is incorrect
 - » Correctly partitions
 - » Removes $5/40$ (or $1/8$) from 1 whole instead of $4/5$
 - Evidence of non-mastery of referent unit
- Option E is incorrect

Ms. Roland gave her students the following problem to solve:

*Candice has $4/5$ of a meter of cloth. She uses $1/8$ of a meter for a project.
How much cloth does she have left after the project?*

She had students use the number line so that they could draw the lengths. Which of the following diagrams shows the solution? Assume all intervals are subdivided equally.





Attribute/Item Alignment

- Conducted three rounds of think aloud interviews with a total of 61 in-service teachers
 - These interviews are about an hour and a half long
 - ◆ Praise to our qualitative researchers!
 - This included 3 rounds of interviews (n= 14, 22, 25)
 - ◆ Each time revised test in light of interview data
 - ◆ After 3rd round felt we had a test that would yield valid classifications
 - Time to test it out!



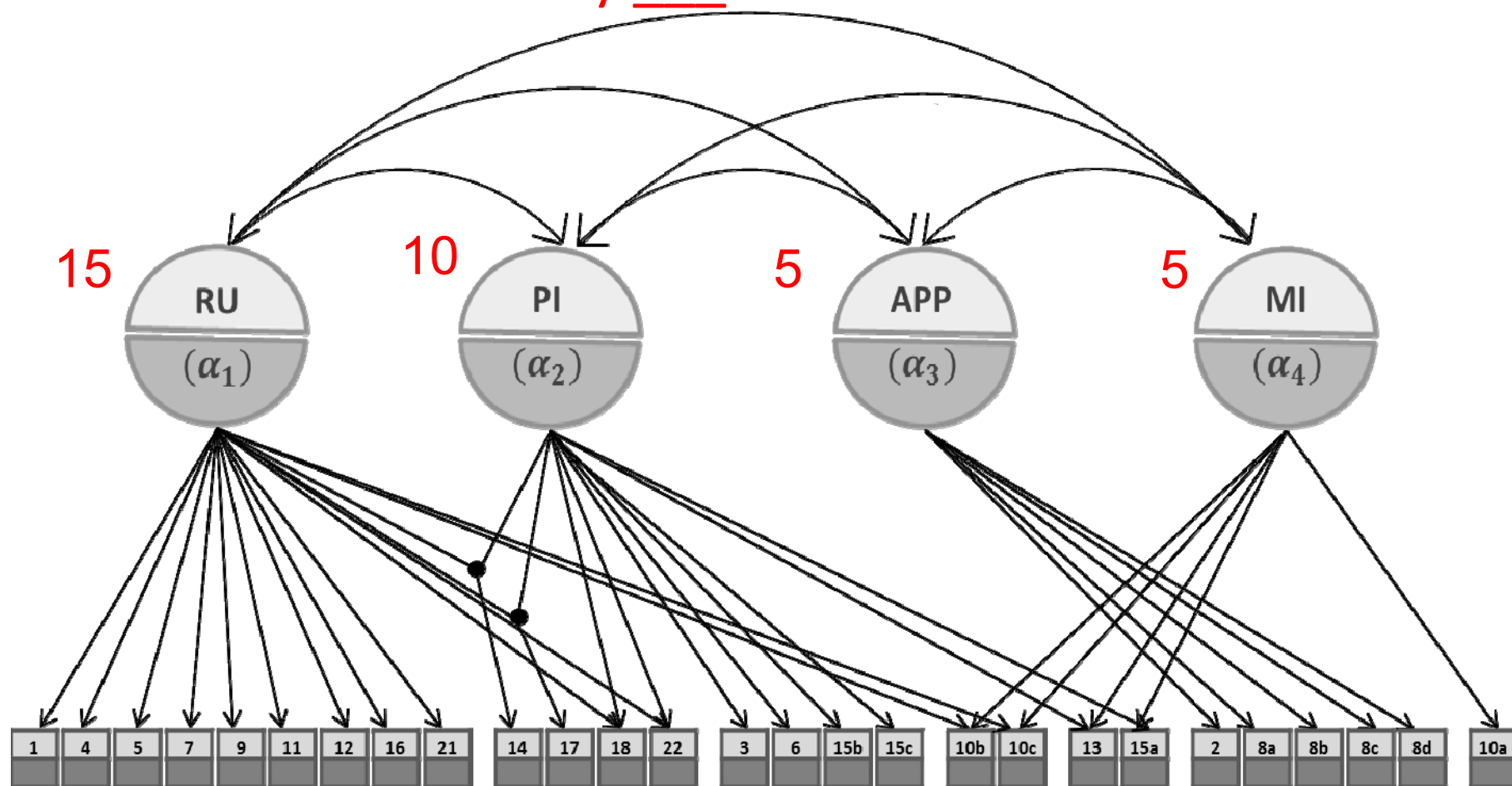
Attribute/Item Alignment

- Analyzed data for
 - True Positive: answered item correctly; used intended attribute
 - False Positive: answered item correctly; did not use intended attribute
 - True Negative: missed item; did not use intended attribute
 - False Negative: missed item; used intended attribute
- Highlights
 - Common issue: Teachers found a way to circumvent attributes
 - ♦ Setting up and solving algebraic equations (instead of reasoning)
 - Open ended items were used in early pilots to more freely elicit reasoning
 - ♦ Helped write distracters



A Path Model of the DTMR Test

- Final Attribute/Item Alignment
- Attribute measured by ___ items





DCM Analysis of DTMR Data using Mplus

And a Little Help from our Friend, SAS



Mplus Files

- Input Files
 - DTMRdemo.inp
 - ◆ Mplus input file
 - DTMRdemo.dat
 - ◆ Data
 - ◆ Why “demo”?
 - Simulated from DTMR final model
- Output Files
 - DTMRdemo.out
 - ◆ Mplus output file
 - Resp_DTMRdemo.dat
 - ◆ Examinee classifications



SAS Macro for Estimating DTMR Data

- You can download this from the course website:
<http://wp.me/p3nkOf-nu>



SAS Macro

- In SAS_MACRO folder:
- LCDM_Mplus2.sas
 - Creates Mplus syntax
 - Parses results into SAS output files
 - ◆ Useful for viewing/reporting results
 - ◆ Today Mplus output is shown, not SAS output
- DTMRdemo.sas
 - You actually run this file
 - Edit lines 23 -82 for your input information
 - You change this file when you want to change specifications for the model being run



Editing DTMRdemo.sas (1)

```
* Defining needed macro variables as global;
%GLOBAL macroloc filesave filename saslibname Qname dataname IDname
      itemstem itemlist numitem ordervar maxitemorder
      attstem attcat numatt numclass structon structorder loosen processors;

* Location of original data files - CHANGE ALL OF THEM;
* Permanent SAS library;          LIBNAME folder      "FILE IS C:\Users\Laine\Dropbox\workshop\NCME 2013\Simulate\DTMRdemo"
* Path to SAS macro file;         %LET macroloc=       C:\Users\Laine\Dropbox\workshop\NCME 2013\Simulate\DTMRdemo;
* Path to import/export files from; %LET filesave=      C:\Users\Laine\Dropbox\workshop\NCME 2013\Simulate\DTMRdemo;

* Name prefix for files to be created; %LET filename =    DTMR_demo1;
* Name of SAS library files are stored in; %LET saslibname=    work;
* Name of SAS dataset for Q matrix; %LET Qname=          DTMR_q;
* Name of SAS dataset with original data; %LET dataname=      DTMRdemo;

* Name of person ID variable (required); %LET IDname=       ID;
* Item stem in Q matrix (cant be "item"); %LET itemstem=     x;
* List of items to be modeled; %LET itemlist=            x1-x27;
* Total number of items; %LET numitem=                   27;
* Variable for order of item model; %LET ordervar=        itemorder;
* Max order of interaction in item model; %LET maxitemorder= 2;

* Attribute stem in Q matrix; %LET attstem=              attribute;
* Number of categories for attributes; %LET attcat =      2; * currently only set to 2;
* Total number of attributes; %LET numatt=               4;
* Number of total classes (2^A); %LET numclass=          16;
* Use structural model (0=N,1=Y); %LET structon=         1;
* Order of interaction in structural model; %LET structorder= 3;

* Loosen convergence criteria (0=N,1=Y)?; %LET loosen=    0;
* Number of processors available for Mplus; %LET processors= 8;
*****;
```



- Code is commented, making many input lines self-explanatory

For others:

- Itemorder
 - You set this in the next section for each item
- Attstem
 - Name for attributes (i.e., Attribute 1 vs. Att 1 vs. A1)
- Attcat = 2 for dichotomous attributes (e.g., mastery vs non-mastery)
- Structon
 - 1 produces code for the log-linear structural model
 - 0 runs the log-linear model in the background as default, but doesn't give you the structural parameters
- Convergence criteria
 - Zero leaves it at Mplus' default
- Processors available
 - Check the computer you're on



Editing DTMRdemo.SAS (2)

- Input Q-matrix
- Fill in the appropriate values in yellow under DATALINES;
 - Copy and paste from Excel
- The last column is the highest interaction term for the item

```
* Import Q-matrix into SAS;  
DATA &saslibname..&Qname.;  
  INPUT &itemstem. attribute1-attribute4 &ordervar.;  
  DATALINES;  
1    1    0    0    0    1  
2    0    0    1    0    1  
3    0    1    0    0    1  
  
...  
  
25   1    1    0    0    1  
26   1    0    0    0    1  
27   1    1    0    0    1  
; RUN;
```



Editing DTMRdemo.SAS (2)

- From SAS macro, can specify LCDM or C-RUM using this input, but not DINA, DINO, or other sub-models
 - Open the input file SAS produces to make these changes
 - Needed for Item “19” and “24”

```
* Import Q-matrix into SAS;  
DATA &saslibname..&Qname.;  
INPUT &itemstem. attribute1-attribute4 &ordervar.;  
DATALINES;  
1 1 0 0 0 1  
2 0 0 1 0 1  
3 0 1 0 0 1  
  
25 1 1 0 0 1  
26 1 0 0 0 1  
27 1 1 0 0 1  
; RUN;
```



Original to Demo Item Mapping

- Macro renames items

Macro Item	DTMR Item
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8a
9	8b
10	8c
11	8d
12	9
13	10a
14	10b

Macro Item	DTMR Item
15	10c
16	11
17	12
18	13
19	14
20	15a
21	15b
22	15c
23	16
24	17
25	18
26	21
27	22



Editing DTMRExample.SAS (3)

- Change this to your data file name

```
* Import original data into SAS dataset;  
DATA &saslibname..&dataname.;  
    INFILE "&filesave.\DTMRdemo.dat" TRUNCOVER;  
    INPUT &IDname. &itemlist.;  
RUN;
```



Did it run?

- It will yell at you if it didn't
- Or not give you standard errors for your parameters

```
THE MODEL ESTIMATION TERMINATED NORMALLY
```

```
THE CHI-SQUARE TEST CANNOT BE COMPUTED BECAUSE THE FREQUENCY TABLE FOR THE  
LATENT CLASS INDICATOR MODEL PART IS TOO LARGE.
```

```
MODEL FIT INFORMATION
```

```
Number of Free Parameters          71
```

```
Loglikelihood
```

```
    H0 Value                      -14257.169
```

```
    H0 Scaling Correction Factor    1.063  
    for MLR
```

```
Information Criteria
```

```
    Akaike (AIC)                   28656.338
```

```
    Bayesian (BIC)                  29004.075
```

```
    Sample-Size Adjusted BIC        28778.576
```



SAS Macro Tutorial

- For step-by-step how-to for using the SAS Macro and interpreting SAS macro output:

[SAS Macro Tutorial](#)

by Daniel Jurich at James Madison University



The University of Georgia

DTMR Test Results



Results Overview

- Data Collection
- Estimation
- Model Fit
- Items
 - How well did they function?
- Attribute Patterns
 - How many teachers are masters of each attribute?
 - What are the attribute mastery probabilities for a single teacher?
- Attribute Correlations
 - How highly correlated are the attributes?
 - Are any attributes dependent on another?



Data Collection

- National sample of 990 in-service middle grades mathematics teachers
- Sample stratified by
 - Region of the country (4 levels)
 - ◆ Northeast, Midwest, South, West
 - Urban-centric locale (12 levels)
 - ◆ City or suburb
 - Small, medium, large
 - ◆ Town or rural
 - Fringe, distant, remote
- Response rate: $\approx 20\%$
 - Received 990 of 5400 teachers
 - Demographics comparable to other math education national samples



Estimation

- LCDM as general modeling framework
 - Estimated with Mplus
 - Top-down approach
 - ♦ Estimate all higher order interactions and then remove non-significant interactions
- LCDM for Testlet Effects (Bifactor LCDM)
 - Estimated first
 - ♦ Three testlets on test
 - Did not converge
 - ♦ Testlet effects were negligible?
 - ♦ More research needed



Note about Model Fit

- Before we look at parameter estimates in the output, we would want to look at the model-data fit
 - See Rupp, Templin, Henson (2010) chapter
- This section will focus on
 - Interpreting output, the estimated parameters
 - ◆ Item parameter estimates
 - ◆ Examinee classifications
 - Evaluating the results
 - ◆ Were items highly “diagnostic”?



INTERPRETING ITEM PARAMETER RESULTS



Was this a “good” item?

- Example item seen previously
 - » Measures Referent Unit (Attribute 1) and Partitioning and Iterating (Attribute 2)

Ms. Roland gave her students the following problem to solve:

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How much cloth does she have left after the project?

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a)



b)



c)



d)



e)






LCDM Function for this Item

- Referent unit (α_1) and partitioning and iterating (α_2) are measured
 - Q-matrix entries:

	RU	PI	APP	MC
Item 22	1	1	0	0

- LCDM item response function:

$$\log \frac{P(X_{ei} = 1 | \alpha_e)}{P(X_{ei} = 0 | \alpha_e)} = \boxed{\lambda_{i,0}} + \boxed{\lambda_{i,1(1)}}(\alpha_{e1}) + \boxed{\lambda_{i,1(2)}}(\alpha_{e2}) + \boxed{\lambda_{i,2(12)}}(\alpha_{e1} \cdot \alpha_{e2})$$

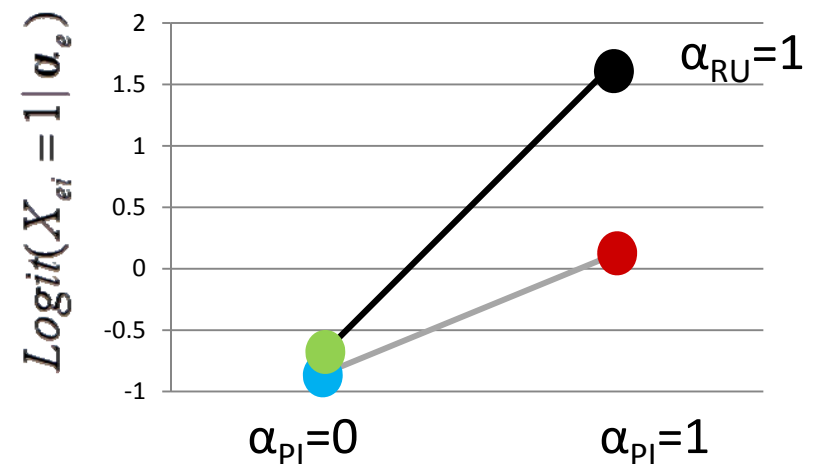
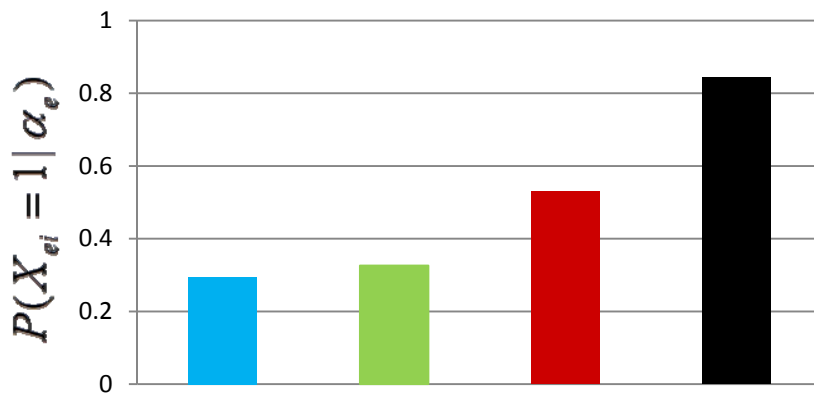

Intercept (Guessing) Main Effect (RU) Main Effect (PI) Interaction (Between RU and PI)



Example Item Response Function

$$\log \frac{P(X_{ei} = 1 | \alpha_e)}{P(X_{ei} = 0 | \alpha_e)} = -.871 + .146(\alpha_{e1}) + .991(\alpha_{e2}) + 1.415(\alpha_{e1} \cdot \alpha_{e2})$$

- On the logit scale, we can see the main effects are positive and the interaction is positive
- Item parameters provide construct validation
 - Is the item actually measuring the attribute?



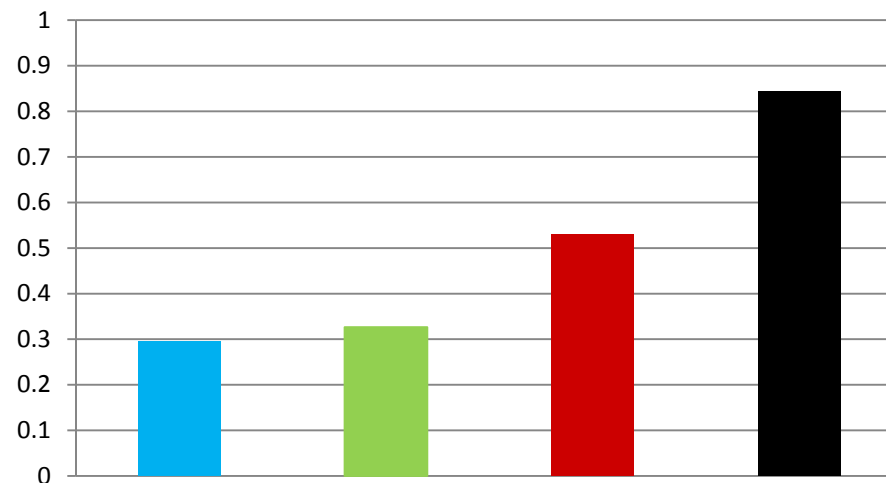


Content Validity

- Results provide some evidence for content validity
 - Is the item actually measuring the attribute?

ICBC

Probability of
Answering Item
Correctly



Key	
✓	Mastery
—	Non-mastery

Referent Unit
Partitioning & Iterating

—	✓	—	✓
—	—	✓	✓

Attribute Mastery Level



Mplus Output: Structural Parameter Estimates

- Structural parameter estimates
- More on these in the next section

Parameter	Estimate	SE	Estimate/SE	p
New/Additional Parameters				
G_0	-0.183	0.142	-1.288	0.198
G_11	-4.542	0.758	-5.989	0.000
G_12	-1.721	0.319	-5.390	0.000
G_13	-1.338	0.358	-3.742	0.000
G_14	-1.112	0.241	-4.618	0.000
G_212	2.119	0.515	4.117	0.000
G_213	1.373	0.557	2.465	0.014
G_214	1.559	0.488	3.196	0.001
G_223	1.603	0.374	4.285	0.000
G_224	0.725	0.362	2.005	0.045
G_234	1.517	0.348	4.354	0.000
L1_0	-1.118	0.123	-9.106	0.000
L1_11	2.239	0.201	11.120	0.000
L2_0	0.585	0.130	4.492	0.000
L2_13	1.271	0.215	5.924	0.000
L3_0	-2.069	0.218	-9.497	0.000
L3_12	1.695	0.239	7.084	0.000
L4_0	-1.191	0.109	-10.906	0.000
L4_11	0.648	0.188	3.448	0.001
L5_0	-1.668	0.138	-12.047	0.000
L5_11	1.517	0.196	7.726	0.000
---	---	---	---	---



Mplus Output: Item Parameter Estimates

- Item parameter estimates
- For item i :
 - $\lambda_{i,0} = \text{Li_0}$
 - $\lambda_{i,1(1)} = \text{Li_11}$
 - $\lambda_{i,1(2)} = \text{Li_12}$
 - $\lambda_{i,2(12)} = \text{Li_212}$

Parameter	Estimate	SE	Estimate/SE	p
New/Additional Parameters				
G_0	-0.183	0.142	-1.288	0.198
G_11	-4.542	0.758	-5.989	0.000
G_12	-1.721	0.319	-5.390	0.000
G_13	-1.338	0.358	-3.742	0.000
G_14	-1.112	0.241	-4.618	0.000
G_212	2.119	0.515	4.117	0.000
G_213	1.373	0.557	2.465	0.014
G_214	1.559	0.488	3.196	0.001
G_223	1.603	0.374	4.285	0.000
G_224	0.725	0.362	2.005	0.045
G_234	1.517	0.348	4.354	0.000
L1_0	-1.118	0.123	-9.106	0.000
L1_11	2.239	0.201	11.120	0.000
L2_0	0.585	0.130	4.492	0.000
L2_13	1.271	0.215	5.924	0.000
L3_0	-2.069	0.218	-9.497	0.000
L3_12	1.695	0.239	7.084	0.000
L4_0	-1.191	0.109	-10.906	0.000
L4_11	0.648	0.188	3.448	0.001
L5_0	-1.668	0.138	-12.047	0.000
L5_11	1.517	0.196	7.726	0.000
---	---	---	---	---



Parameter Interpretation

- To demonstrate parameter interpretation, let's look at Item 18
 - Attributes measured:
 - ♦ Referent Unit (Attribute 1)
 - ♦ Partitioning and Iterating (Attribute 2)
- Parameter estimates:

Parameter	Estimate	SE	p-value
$\lambda_{18,0}$	-0.994	0.135	0.000
$\lambda_{18,1,(1)}$	1.132	0.260	0.000
$\lambda_{18,1,(2)}$	1.100	0.237	0.000



LCDM Intercepts

- Estimated Intercept: -0.994 (0.135)
- Indicates the logit of a correct response for a non-master of all attributes
 - Here, non-masters have an average probability of a correct response: $\exp(-0.994) / (1 + \exp(-0.994)) = 0.27$
- Hypothesis test is not important
 - Tests whether non-masters have a probability of a correct response of .50
- Problematic when very high
 - Difficult to identify other parameters
 - Indicates issues with test, Q-matrix, or attributes



Higher Order Model Parameters

- Interpretation of main effects and interactions proceeds sequentially:
- If interactions are present:
 - Examine highest level of interaction
 - ◆ If significantly different from zero, leave in model
 - ◆ If not, term can be omitted
 - Only 2 interaction terms found to be statistically significant on this test
 - ◆ Item 14 and 17
- When significant interactions are present, main effects cannot be easily interpreted
 - Sometimes called conditional main effects
 - Need to know *combination* of attributes mastered to fully describe item response function
- If interactions are not present:
 - Interpret main effects



Interpreting Main Effects

- Main effects in LCDM cannot be tested for significance in the typical way
 - Lower bound is zero (for monotonicity)
 - p -values are inaccurate as they approach zero
 - Use practical significance
 - ◆ How much of an increase in probability for mastery of attribute is meaningful?
 - Or estimate the model with and without the main effect to compare fit
 - ◆ More in Model Fit section

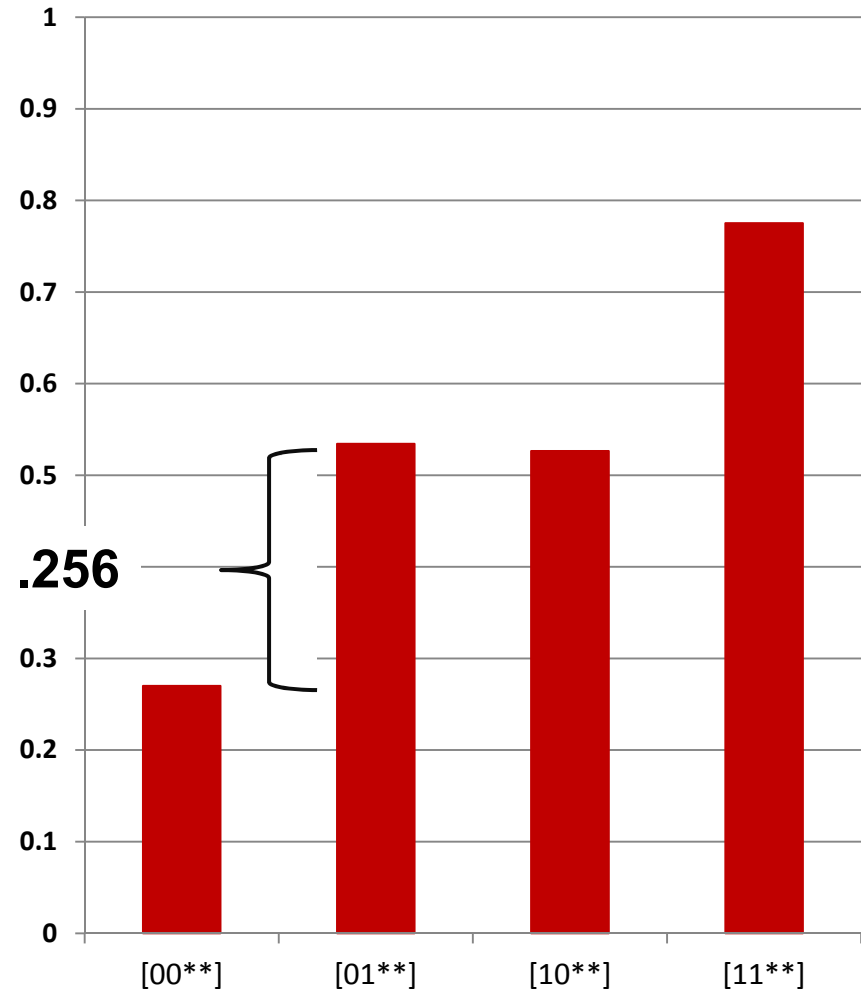


Item 18 Partitioning and Iterating (P&I)

Main Effect (Att 2)

Parameter	Estimate	SE	p-value
$\lambda_{18,0}$	-0.994	0.135	0.000
$\lambda_{18,1,(1)}$	1.132	0.260	0.000
$\lambda_{18,1,(2)}$	1.100	0.237	0.000

- When Referent Unit has not been mastered:
 - P&I main effect : $\lambda_{18,1,(2)} = 1.100$
 - Respondents who have mastered P&I have an increase in logit of 1.1 over respondents who are non-masters
 - Respondents who have mastered P&I have an increase in probability of .256 over respondents who are non-masters



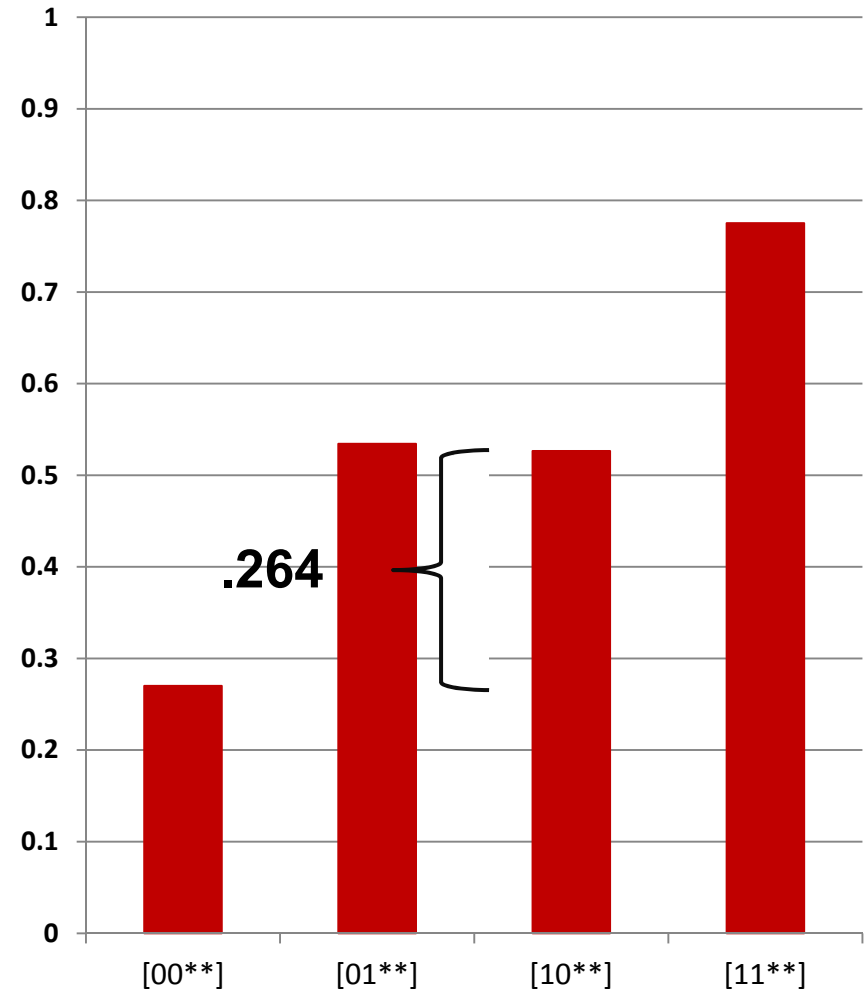
Unique Attribute Profiles for Item 18



Item 18 Referent Unit (RU) Main Effect (Att 1)

Parameter	Estimate	SE	p-value
$\lambda_{18,0}$	-0.994	0.135	0.000
$\lambda_{18,1,(1)}$	1.132	0.260	0.000
$\lambda_{18,1,(2)}$	1.100	0.237	0.000

- When Partitioning & Iterating has not been mastered:
 - RU main effect : $\lambda_{18,1,(1)} = 1.132$
 - Respondents who have mastered Referent Unit have an increase in logit of 1.132 over respondents who are non-masters
 - Respondents who have mastered Referent Unit have an increase in probability of .264 over respondents who are non-masters of RU



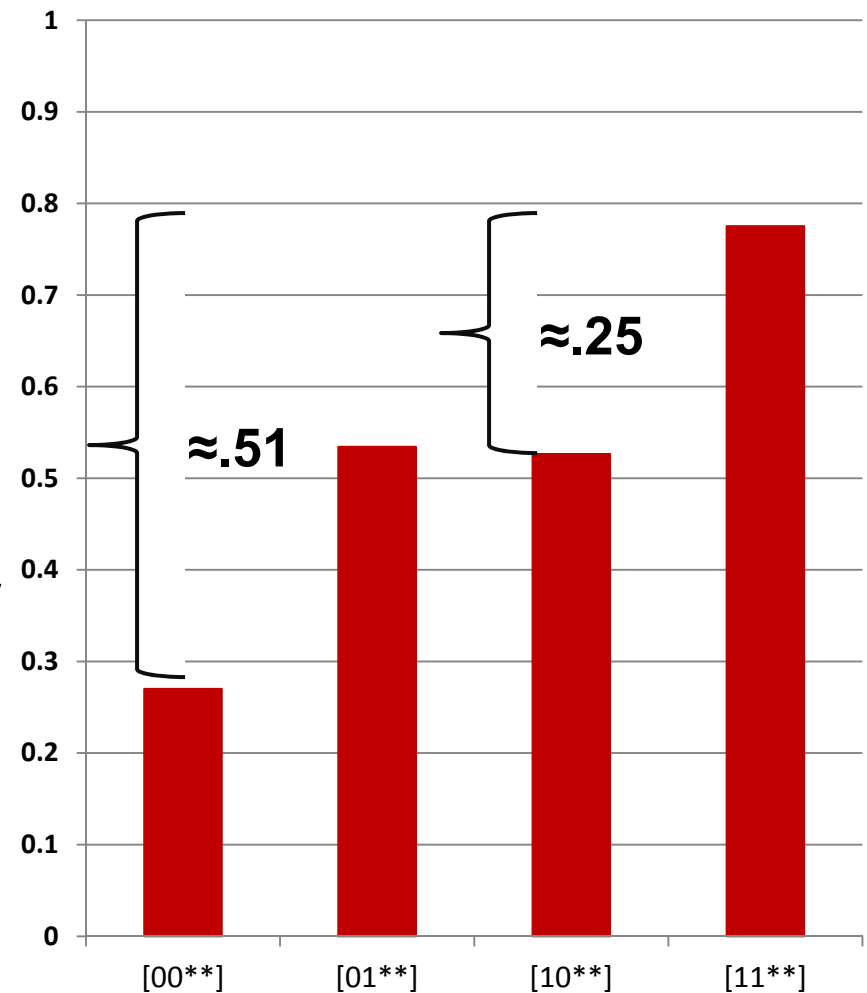
Unique Attribute Profiles for Item 18



Item 18 Referent Unit (RU) Main Effect (Att 1)

Parameter	Estimate	SE	p-value
$\lambda_{18,0}$	-0.994	0.135	0.000
$\lambda_{18,1,(1)}$	1.132	0.260	0.000
$\lambda_{18,1,(2)}$	1.100	0.237	0.000

- When both Referent Unit and Partitioning & Iterating has been mastered:
 - Respondents have a .775 probability of answering the item correctly
 - 51% increase of non-masters of both
 - About 25% increase over masters of only 1



Unique Attribute Profiles for Item 18



Preview to Mplus Coding

- Mplus sets class-specific item response probabilities using the appropriate combinations of item parameters for each class
- Output looks like this:

```

      I21$1      1.502      0.128      11.733      0.000
      I22$1      1.247      0.161       7.742      0.000

Latent Class 5

Thresholds
I1$1      1.118      0.123       9.106      0.000
I2$1     -0.585      0.130     -4.492      0.000
I3$1      0.373      0.106       3.507      0.000
I4$1      1.191      0.109      10.906      0.000
I5$1      1.668      0.138      12.047      0.000
I6$1      1.732      0.136      12.726      0.000
I7$1      0.726      0.089       8.143      0.000
I8A$1      0.615      0.249       2.468      0.014
I8B$1      0.091      0.172       0.529      0.596
I8C$1     -0.283      0.128     -2.215      0.027
I8D$1      1.032      0.166       6.214      0.000
I9$1      1.224      0.100      12.246      0.000
I10A$1     0.503      0.184       2.738      0.006
I10B$1     4.014      0.737       5.444      0.000
I10C$1     4.888      0.868       5.629      0.000
I11$1      0.875      0.097       9.031      0.000
I12$1      1.288      0.111      11.646      0.000
I13$1      0.287      0.206       1.389      0.165
I14$1      2.143      0.140      15.354      0.000
I15A$1    -0.243      0.292     -0.834      0.404
I15B$1    -2.376      0.263     -9.023      0.000
I15C$1    -2.605      0.302     -8.631      0.000
I16$1      0.857      0.096       8.943      0.000
I17$1      0.804      0.204       3.944      0.000
I18$1     -0.106      0.194     -0.545      0.585
I21$1      1.502      0.128      11.733      0.000
I22$1     -0.183      0.207     -0.885      0.376

Latent Class 6

Thresholds
I1$1      1.118      0.123       9.106      0.000
```



Note about Thresholds

- Item parameters sum to give us (negative) thresholds
 - Negative because Mplus models $P(X=0)/P(X=1)$ odds
- Example: Item 18 parameters

	-----	-----	-----	-----
L18_0	-0.994	0.135	-7.366	0.000
L18_11	1.132	0.260	4.357	0.000
L18_12	1.100	0.237	4.632	0.000

- For classes 1- 4: $[00^{**}]$, threshold = .994
- For classes 5-8: $[01^{**}]$, threshold = $-.994 + 1.1 = 1.06$
- For class 9-12: $[10^{**}]$, threshold = $-.994 + 1.132 = 1.38$
- For class 13-16: $[11^{**}]$, threshold = $-.994 + 1.1 + 1.32 = 1.238$



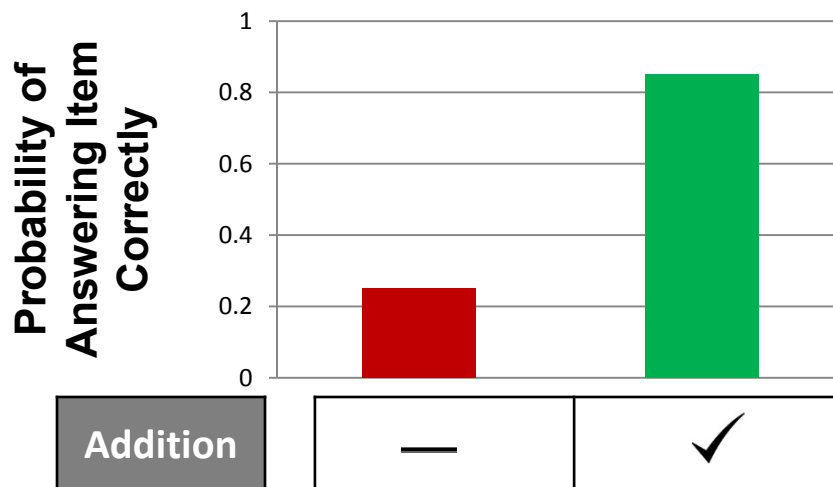
DIAGNOSTIC QUALITY OF ITEMS



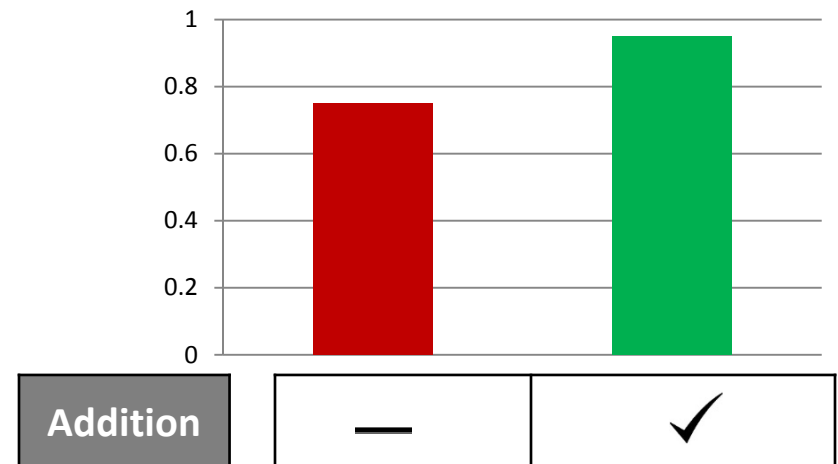
What makes a “good” diagnostic item?

- For “good” items, masters of the attribute(s) answer the item correctly and non-masters answer the item incorrectly
- For example, consider two hypothetical items:

Item A: $4+12=$ ____



Item B: $1+12=$ ____

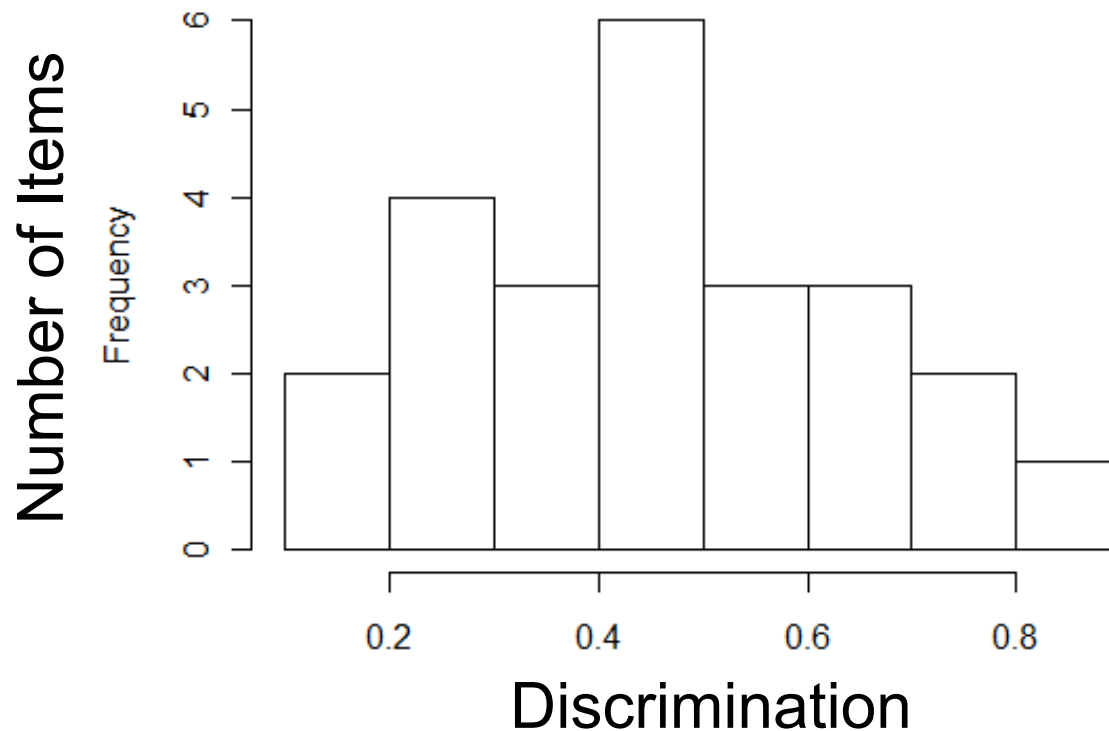


- Masters and non-masters should respond differently to the items
 - Evidence for the Q-matrix alignment
 - Is attribute required to answer the item correctly?



Item Discrimination

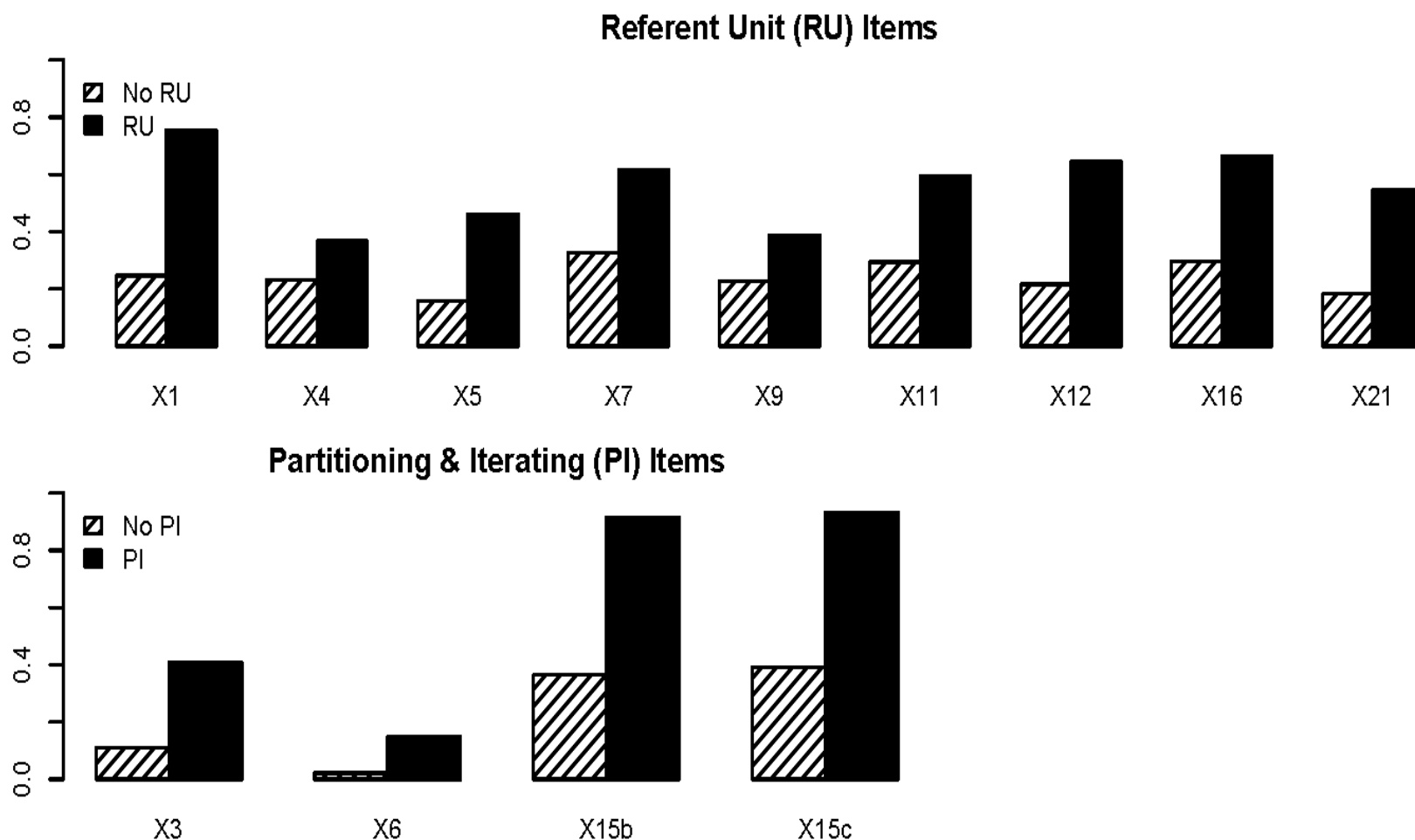
- This “goodness” quality is discrimination
 - Differences in the probability masters and non-masters answer the item correctly





Simple Structure Items

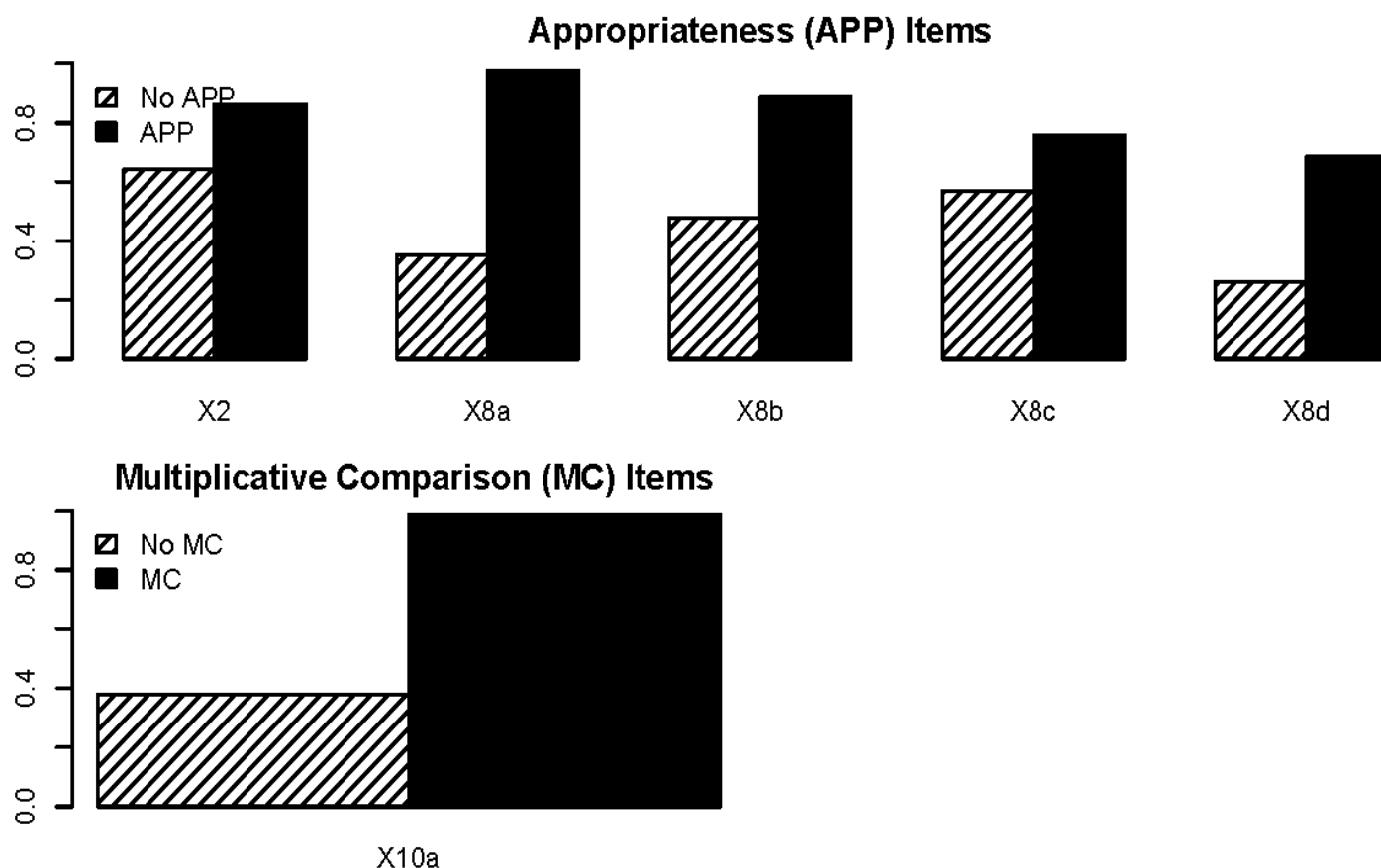
- Small intercepts and large main effects increase item discrimination





Simple Structure Items

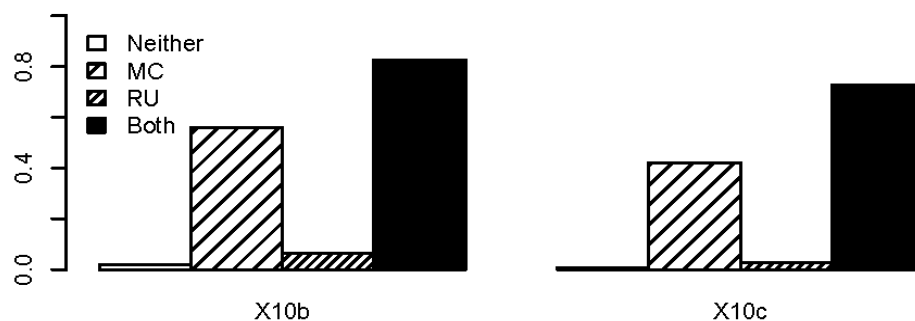
- Odds ratios were calculated as an effect size
 - Almost all items had medium to large effects



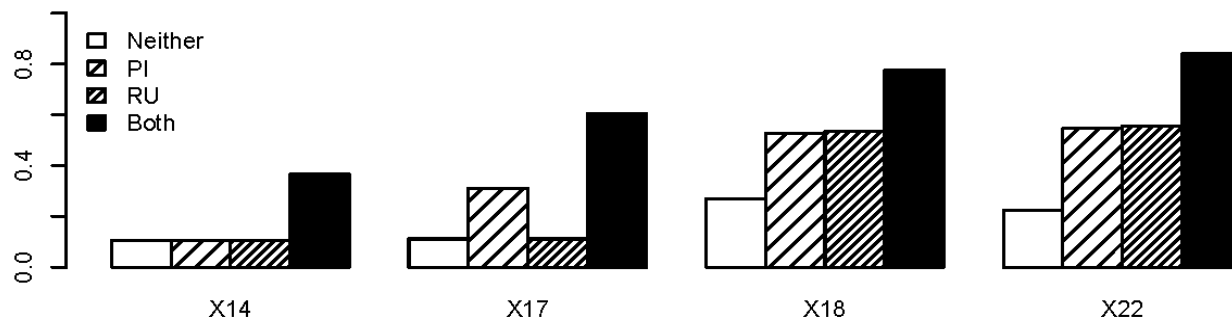


Complex Structure Items

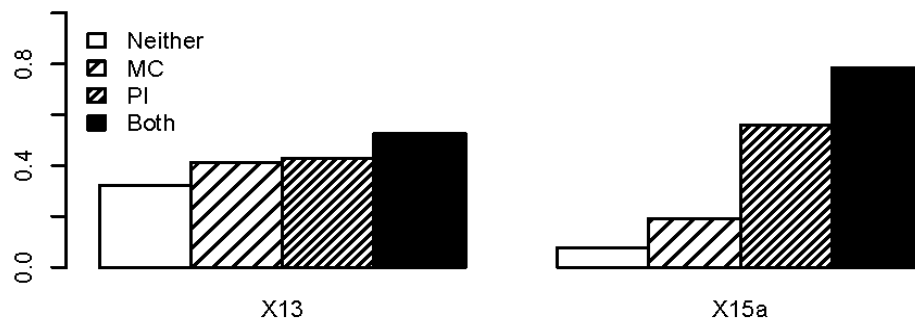
ICBC RU and MC Items



ICBC RU and PI Items



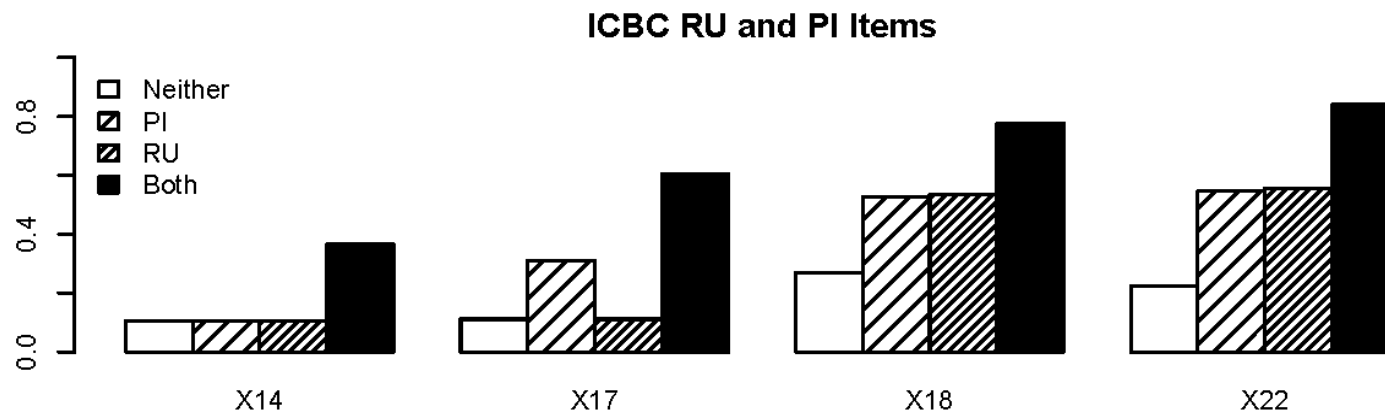
ICBC PI and MC Items





Flexibility of LCDM Framework

- Notice the compensatory, non-compensatory nature of the items vary across the test
 - DINA model would force every item to look like Item 14
 - Empirically, we found only one item functioned like Item 14



- Important not to assume these strict assumptions
 - ♦ Even if they are very popular in the literature
 - ♦ Don't do it!



INTERPRETING EXAMINEE CLASSIFICATIONS



Groups According to Attribute Mastery

Possible patterns or classes:

Pattern	RU	PI	APP	MC
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	1
5	0	1	0	0
6	0	1	0	1
7	0	1	1	0
8	0	1	1	1
9	1	0	0	0
10	1	0	0	1
11	1	0	1	0
12	1	0	1	1
13	1	1	0	0
14	1	1	0	1
15	1	1	1	0
16	1	1	1	1



Mplus Classification Output

- Proportion of examinees in each class

FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
BASED ON THE ESTIMATED MODEL

Latent
Classes

1	210.34540	0.21247
2	69.20994	0.06991
3	55.19197	0.05575
4	82.74175	0.08358
5	37.63398	0.03801
6	25.56661	0.02582
7	49.04489	0.04954
8	151.80987	0.15334
9	2.24094	0.00226
10	3.50645	0.00354
11	2.32019	0.00234
12	16.54147	0.01671
13	3.33667	0.00337
14	10.77973	0.01089
15	17.15841	0.01733
16	252.57174	0.25512



Examinee-level Classifications

- Resp_DTMRdemo.dat
 - Output file for examinee classifications
- First columns are any variables read into Mplus
 - scored responses to items, ID, etc.
- Last 17 Columns:

File	Edit	Format	View	Help													
.00727	0.00005	0.00108	0.00000	0.00004	0.00074	0.01243	0.00001	0.00039	0.00000	0.00008	0.00000	0.00001	2.00000				
.06529	0.01325	0.01052	0.17482	0.63210	0.00002	0.00002	0.00024	0.00073	0.00030	0.00023	0.01576	0.05369	8.00000				
.00017	0.00751	0.00405	0.23626	0.58083	0.00000	0.00000	0.00001	0.00002	0.00041	0.00021	0.05140	0.11902	8.00000				
.00006	0.00062	0.00005	0.00000	0.00000	0.00033	0.00002	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000				
.00002	0.17696	0.00026	0.00368	0.00003	0.00815	0.00001	0.00013	0.00000	0.00280	0.00000	0.00023	0.00000	1.00000				
.05191	0.00000	0.00009	0.00000	0.01260	0.00001	0.00590	0.00013	0.67245	0.00000	0.00045	0.00005	0.25459	12.00000				
.01711	0.00039	0.00518	0.00509	0.31108	0.00001	0.00014	0.00013	0.00664	0.00021	0.00269	0.01110	0.63839	16.00000				
.00132	0.46098	0.12812	0.00959	0.01214	0.00106	0.00126	0.00002	0.00009	0.01351	0.01785	0.00111	0.00668	5.00000				
.27511	0.00001	0.00069	0.00000	0.00134	0.00002	0.00248	0.00001	0.00383	0.00000	0.00000	0.00000	0.00003	2.00000				
.02990	0.00007	0.26870	0.00003	0.55216	0.00000	0.00139	0.00000	0.00226	0.00000	0.00803	0.00000	0.06515	8.00000				
.00000	0.00000	0.00027	0.00000	0.00182	0.00000	0.00000	0.00000	0.00002	0.00001	0.03683	0.00004	0.96101	16.00000				
.00864	0.01249	0.00005	0.39310	0.00762	0.00006	0.00000	0.00161	0.00003	0.00002	0.00000	0.00199	0.00004	3.00000				
.00061	0.09389	0.01000	0.00080	0.00039	0.00166	0.00015	0.00001	0.00000	0.00030	0.00003	0.00001	0.00000	1.00000				
.00985	0.00055	0.03081	0.00203	0.51944	0.00001	0.00038	0.00002	0.00511	0.00012	0.00632	0.00175	0.42060	8.00000				
.0													0000				
.2													0028				
.0													6718				
.0													0000				
.0													2750				
.0													0000				
.00016	0.01796	0.00011	0.00056	0.00002	0.00179	0.00001	0.00004	0.00000	0.00005	0.00000	0.00001	0.00000	1.00000				
.00185	0.00046	0.00000	0.00100	0.00001	0.00003	0.00000	0.00005	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000				
.32047	0.00329	0.00062	0.04345	0.03755	0.00002	0.00000	0.00023	0.00017	0.00000	0.00000	0.00004	0.00003	3.00000				

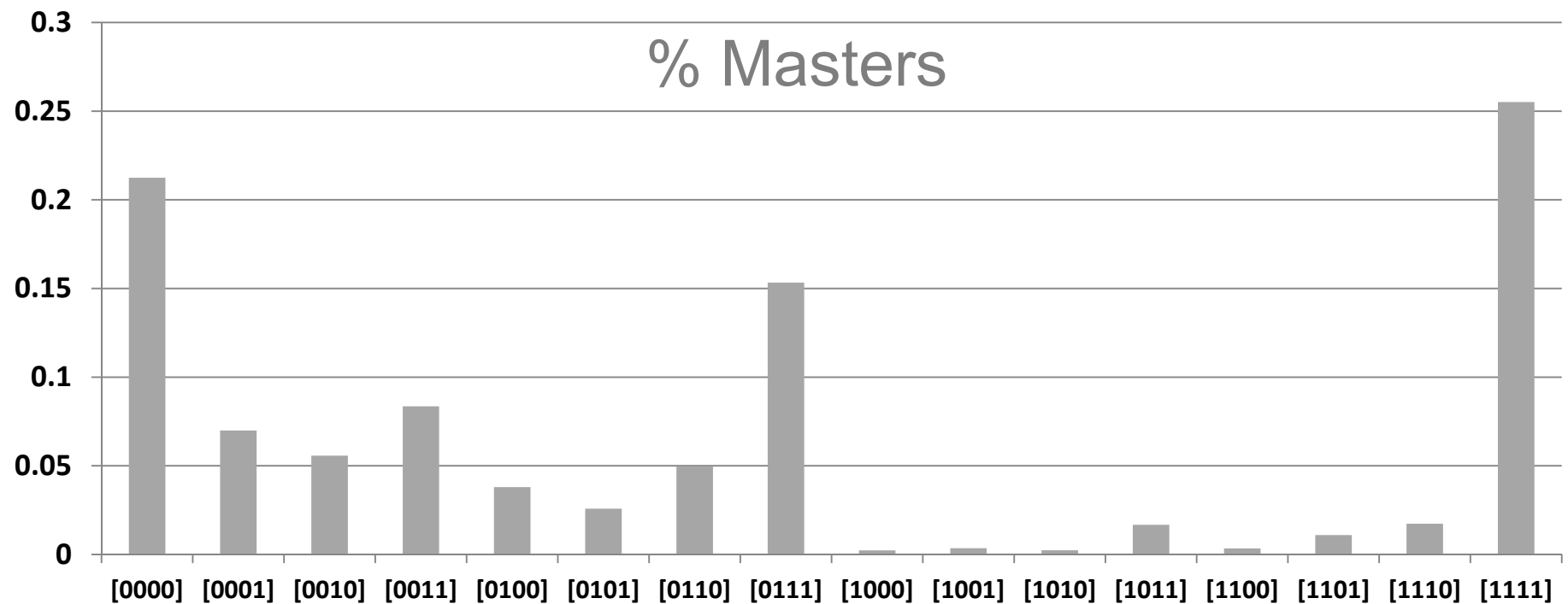
Probability of Class Membership
for Each of the 16 Classes

Most
Likely
Class



Attribute Patterns of Mastery

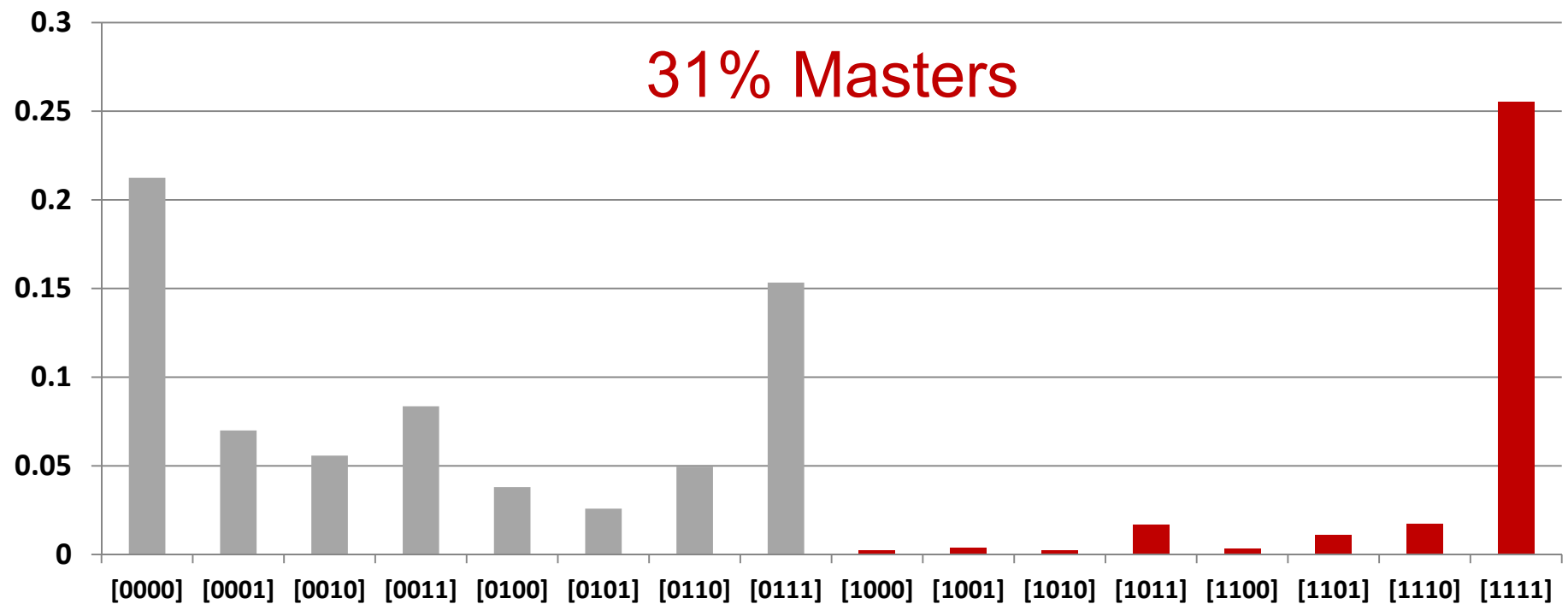
16 Attribute Patterns or Latent Classes





Attribute Patterns of Mastery

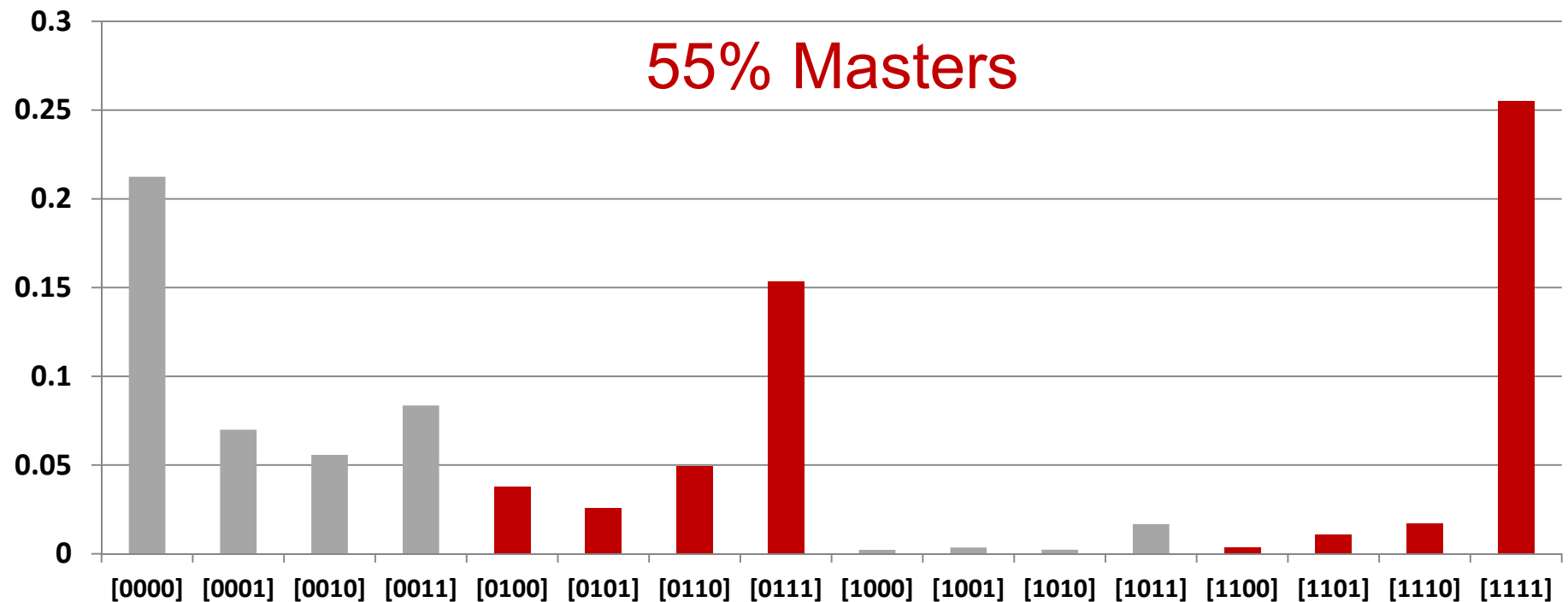
Attribute 1: Referent Unit





Attribute Patterns of Mastery

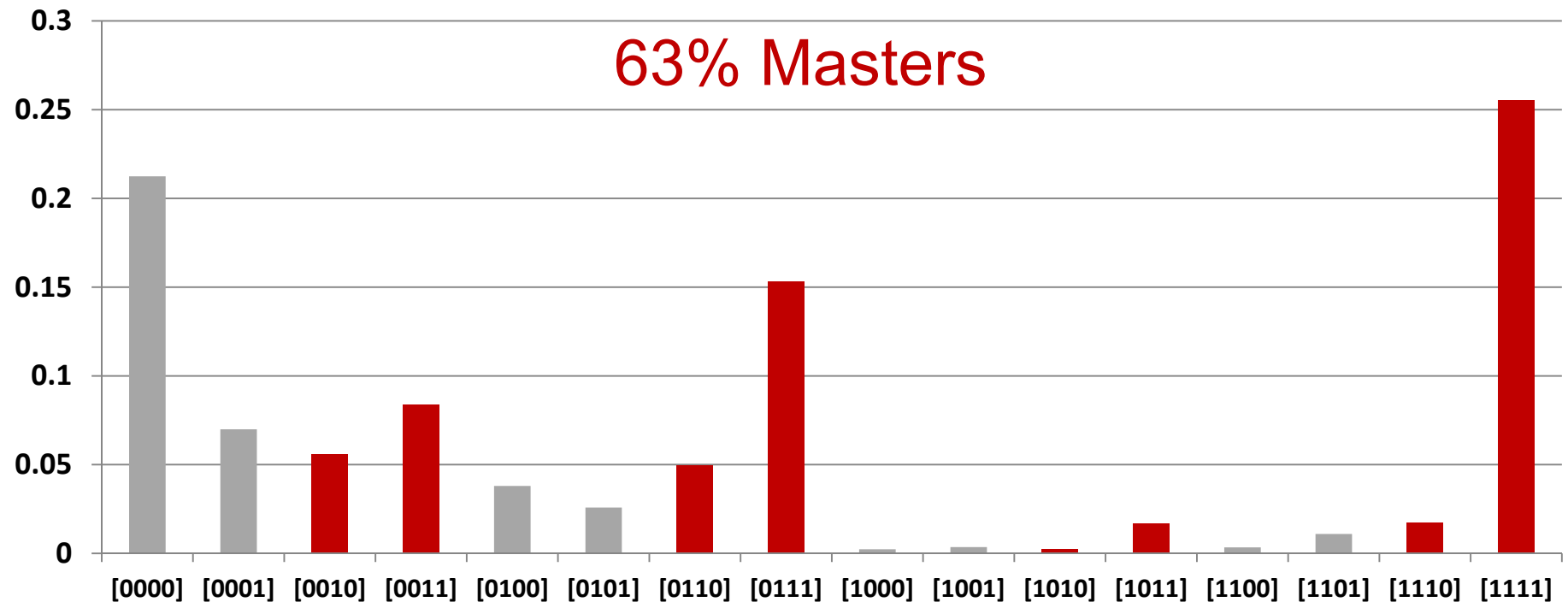
Attribute 2: Partitioning and Iterating





Attribute Patterns of Mastery

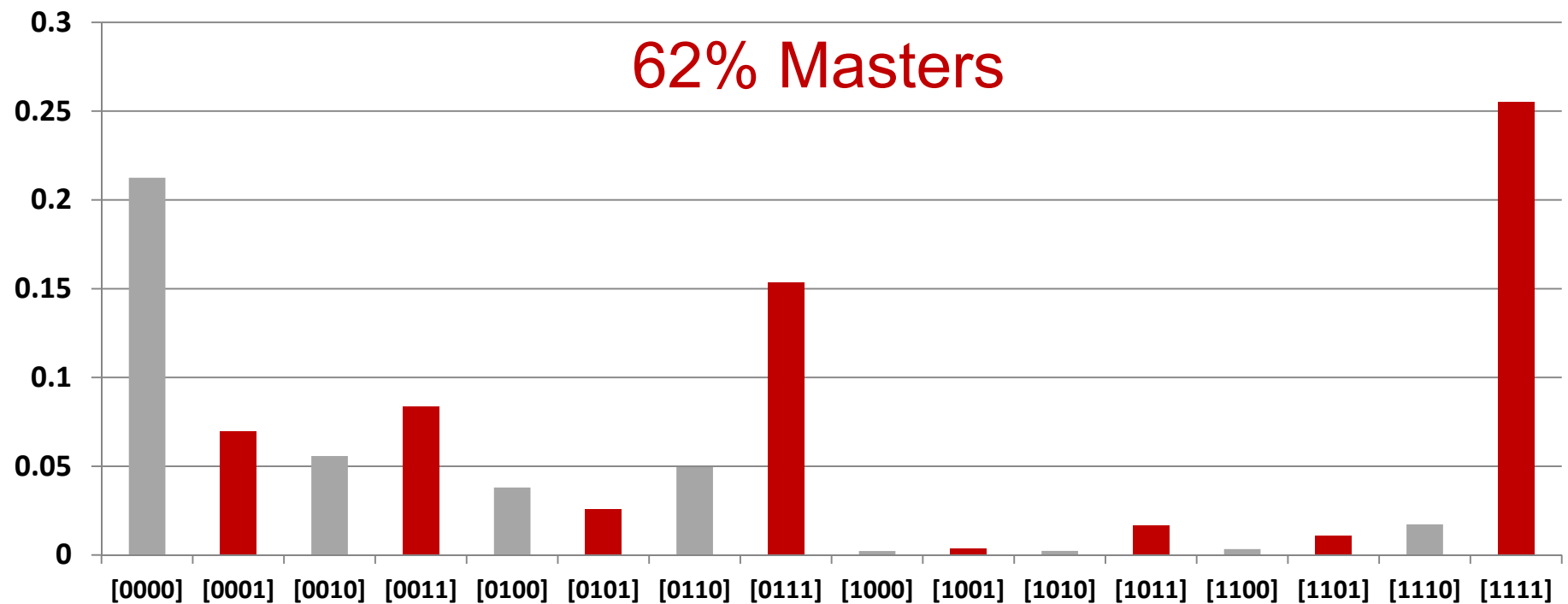
Attribute 3: Appropriateness





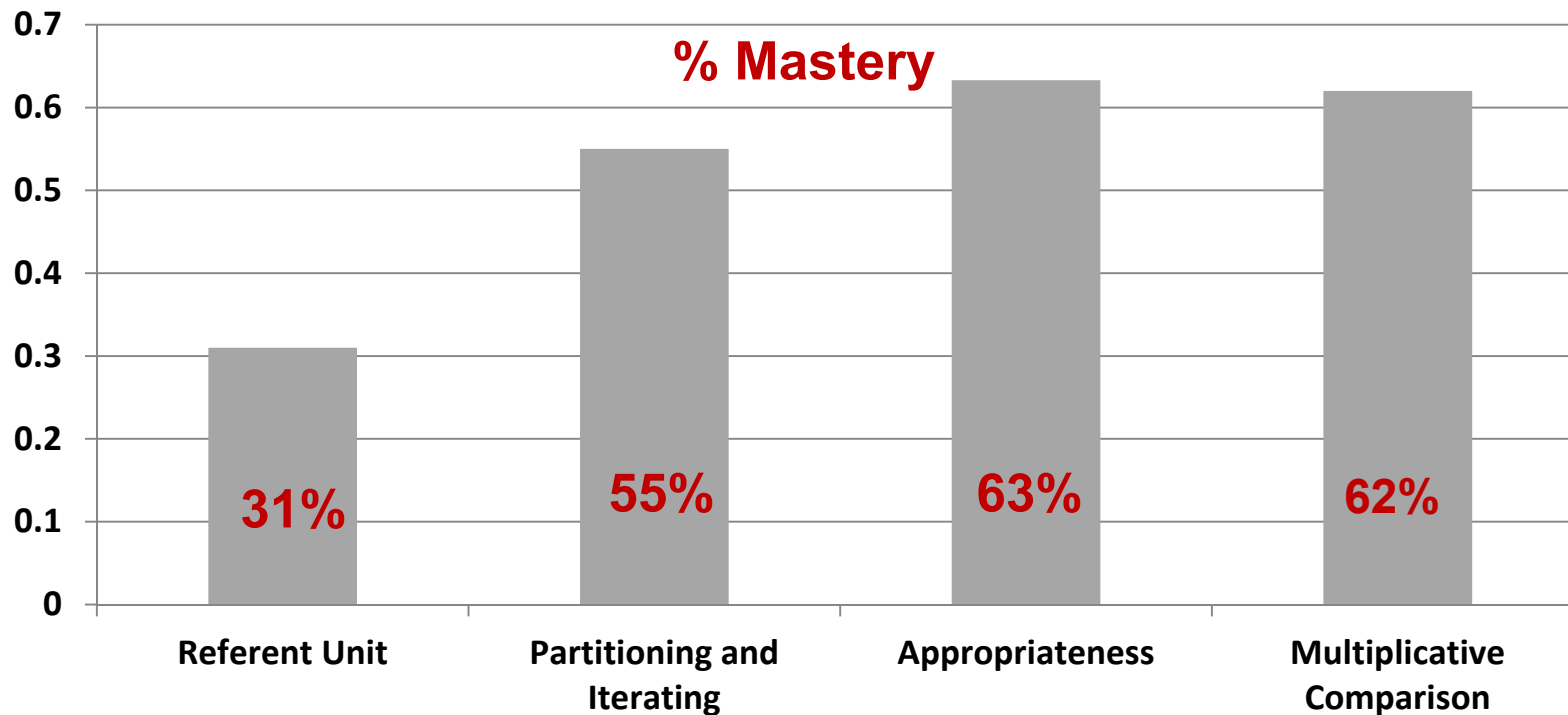
Attribute Patterns of Mastery

Attribute 4: Multiplicative Comparisons





Individual Attribute Mastery

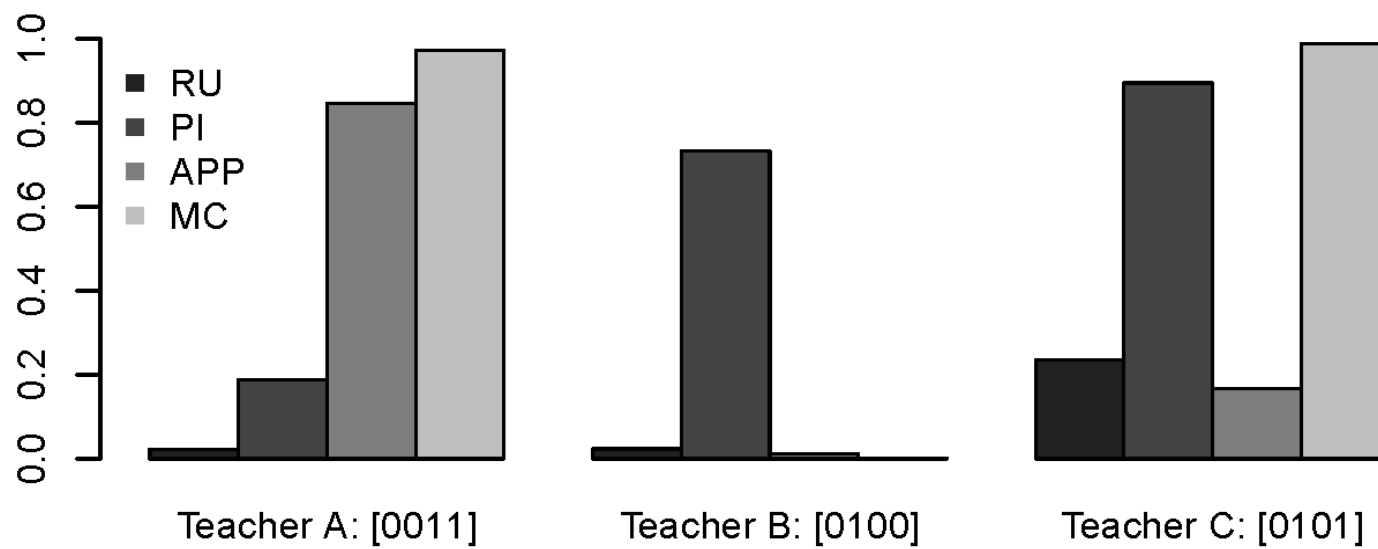


- Information useful for
 - Tailoring professional development
 - Many teachers may benefit from professional development on referent unit
 - Understanding base-rates of attribute mastery in the population of in-service teachers
 - Quantitative Research



Teacher-level Individual Attribute Feedback

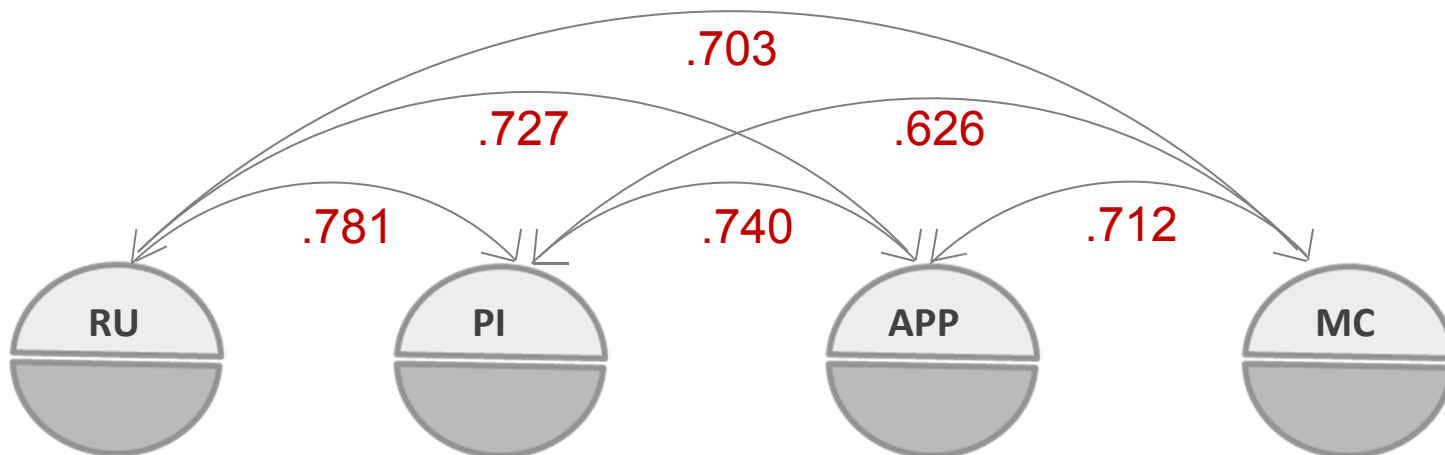
- Comparison of total scores and DCM diagnosis:
 - Teacher A, B, and C both answered 11 out of 27 items correctly
 - Teacher A has attribute pattern [0011]
 - Teacher B has attribute pattern [0100]
 - Teacher C has attribute pattern [0101]
 - ◆ Need different types of professional development





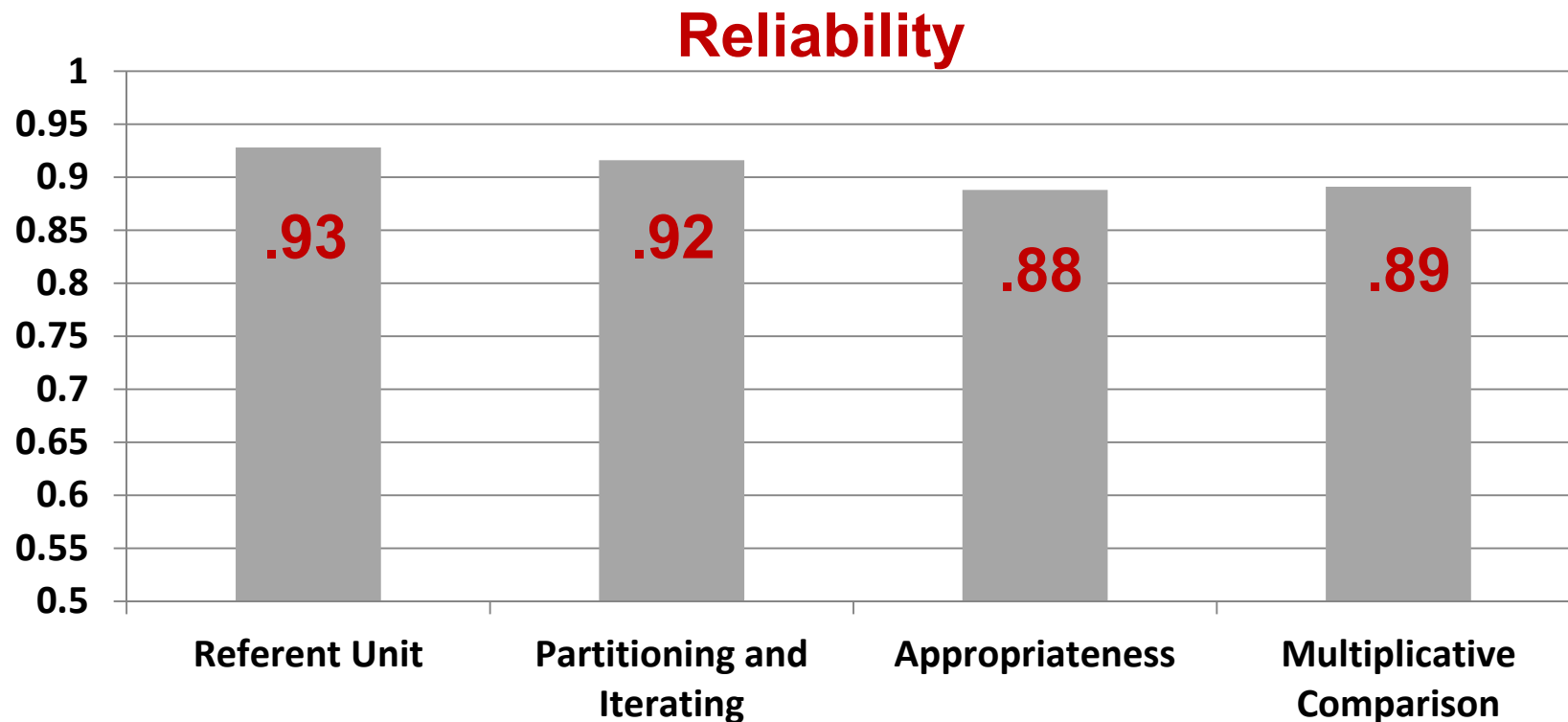
Attribute Correlations

- The attribute patterns are reflections of the correlations among the latent variables
 - **Tetrachoric correlations** (between categorical variables)
 - The relationships among the attributes are parameterized through a log-linear structural model
- Shows we have related but distinct dimensions
- More on attribute relationships in Section 4 about structural models





Individual Attribute Reliability



- Attributes measured by 15, 10, 5 and 5 items, respectively
- Gaining estimation precision by sacrificing latent trait precision



Concluding Remarks



Designing a Multidimensional Test

- Measuring multiple dimensions practically has been the argument for DCMs
 - However, many tests show traits too highly correlated
 - ◆ Typically retrofitting data
- This project is one of the first efforts to prospectively diagnose attributes
 - Further unique in that the attributes are cognitive in nature and very fine-grained
 - One possible model of how to do this in practice



Key Features of Successful DCM-based Test

1. Test purpose aligned with DCM purpose
2. Clearly-defined attributes
 - Theory delineating construct is strong
3. Strong items
 - High discrimination
 - High statistical information
4. Accurate Q-matrix
 - Strong understanding of how items relate to attributes
5. Accurate model parameterization
 - Model-data fit is critical to making valid inferences
 - Flexible model parameterization helps align theory and model
6. Sufficient data to estimate model
 - Examinees
 - Items
 - Items per attribute
 - ◆ Back to statistical information



Iterative Test Development

- Some grey area in confirmatory analyses
- Conjecture based approach
 - Begin with a hypothesis about the attributes, Q-matrix
 - Refine hypothesis with statistical feedback
 - ◆ LCDM parameters
- Both steps are important
 - Strong theory to inform model
 - Flexible model to inform theory
 - Cycles of test refinement yield stronger theory and more accurate inferences from test results



General Modeling Tips

- High-level interactions are difficult to estimate in most samples
 - More than 2-way interactions may not be possible
- Modeling strategy:
 - Try all item-level interactions
 - ◆ If model does not converge, limit to only 2-way interactions
 - Remove non-significant interactions from model
 - If all interactions and main effects for an attribute are close to zero:
 - ◆ Try removing entry for attribute in Q-matrix
 - ◆ More on this in Section 5 about model-data fit



Wrap-Up and Take-Home Points

- Session 3 demonstrated a potential use of DCMs
- Prospective applications of DCMs are rare
 - Tests aren't designed to measure categorical attributes
 - ◆ Item information is different in DCMs
 - Users haven't had access to software
 - ◆ Previously, most applications use software built by researchers
 - MCMC in Fortran or WinBugs
 - MML in Fortran
 - ◆ Now, researchers can use Mplus and more recently, FlexMIRT



Notes on Usefulness of DCMs

- Full utility of DCMs cannot be understood until applications become more frequent
 - Many papers to this point have used sub-optimal data and problems
- Funding opportunities exist and seem to review well
 - Educational Measurement: NSF (DR-K12); IES (Goals 2 and 5)
 - Psychological Measurement: NIH (NIMH; NIDA; NIA;...)
- Industry seems interested
 - ACT/College Board/ETS/Measurement Inc./Pearson