



# Diagnostic Measurement: Theory, Methods, and Applications

Laine Bradshaw  
University of Georgia

Jonathan Templin  
University of Kansas

Course Website: <http://wp.me/p3nkOf-nu>



# Workshop Overview

**INTRODUCTIONS/COURSE MATERIALS: 8:00 – 8:15am**

- **Section 1: Diagnostic Measurement Introduction (8:15 – 9:45am)**

- Conceptually, how is “diagnostic” measurement different from more traditional measurement?
- Why/when would you use diagnostic classification models (DCMs)?

**BREAK: 9:45-10:00am**

- **Section 2: Theoretical Framework of DCMs (10:00 – 11:45am)**

- Latent variables in diagnostic measurement
- Specification of a general DCM, the log-linear cognitive diagnosis model (LCDM)

- **Questions/Discussion (11:45-12:00 noon)**

**LUNCH: 12:00-1:00pm**

- **Section 3: DCMs in Practice (1:00 – 2:30 pm)**

- Applying DCMs to a test of rational numbers
- How-to for estimating DCMs with Mplus

**BREAK: 2:30-2:45pm**

- **Section 4: Structural Model Specifications(2:45 – 3:45pm)**

- Specifying structural model
- Making alterations in Mplus

- **Section 5: Questions/Discussion (3:45 – 4:45pm)**

- Discussions about our/your practical experiences with DCMs
- Opportunity to ask about more specific content not covered in general introduction

**EVALUATE TRAINING SESSION: 4:45-5:00pm**



The University of Georgia

# Conceptual Foundations of Diagnostic Measurement

## Session 1



# Session Overview

- Key definitions
- Conceptual examples
- Why diagnostic models should be used instead of traditional classification methods
- Concluding remarks



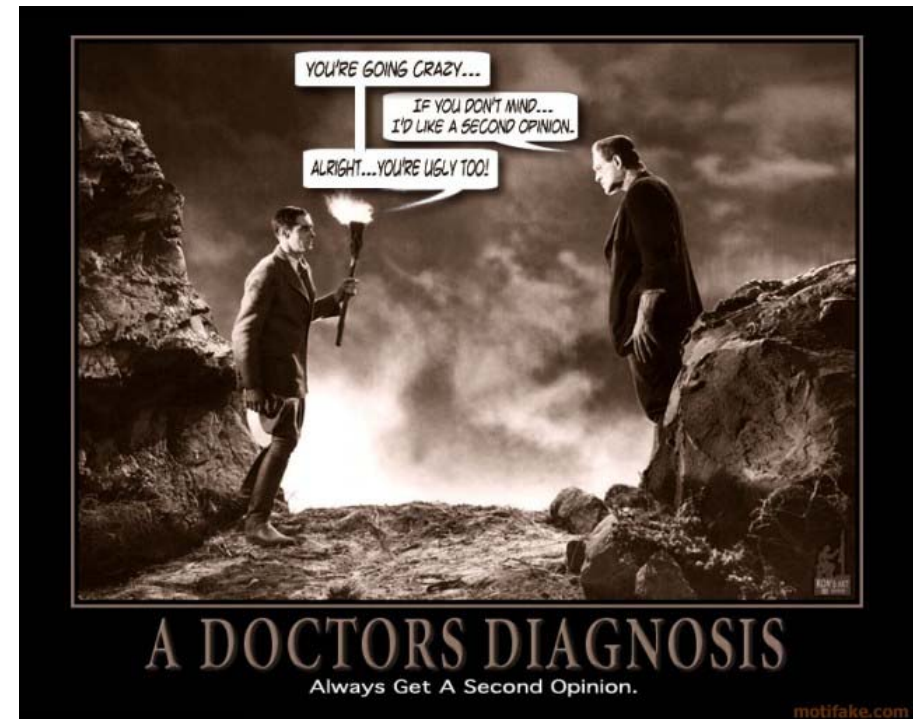
Session 1: Conceptual Foundations of Diagnostic Measurement

# DEFINITIONS



# What are Diagnoses?

- The word and meaning of diagnosis is very commonly used in language
- The roots of the word diagnosis:
  - gnosis: to know
  - dia: from two
- Meaning of diagnoses are deeply ingrained in our society
  - Seldom merits a second thought





# Definitions

- *American Heritage Dictionary* definition of *diagnosis*:
  - Generally
    - ◆ (a) A critical analysis of the nature of something
    - ◆ (b) The conclusion reached by such analysis
  - Medicine
    - ◆ (a) The act or process of identifying or determining the nature and cause of a disease or injury through evaluation of a patient's history, examination, and review of laboratory data
    - ◆ (b) The opinion derived from such an evaluation
  - Biology
    - ◆ (a) A brief description of the distinguishing characteristics of an organism, as for taxonomic classification (p. 500)



# Diagnosis: Defined

- A diagnosis is the decision that is being made based on information
- Within psychological testing, providing a test score gives the information that is used for a diagnosis
  - BUT, the score is not the diagnosis
  - For this workshop, a diagnosis is by its nature *discrete*
    - ♦ Classification





# Day-to-Day Diagnosis

- Decisions happen every day:
  - Decide to wear a coat or bring an umbrella
  - Decide to study
  - Decide what to watch on TV tonight
- In all cases:
  - Information (or data) is collected
  - Inferences are made from data based on what is likely to be the true state of reality



# Diagnosis (Formalized)

- In diagnostic measurement, the procedures of diagnosis are formalized:
  - We make a set of observations
    - ♦ Usually through a set of test questions
  - Based on these questions we make a decision as to the underlying state (or states) of a person
    - ♦ The decision is the diagnosis



# Diagnosis (Formalized)

- Diagnoses featured in this workshop:
  - Educational Measurement
    - ◆ The competencies (skills) that a person has or has not mastered
      - Leads to possible tailored instruction and remediation
  - Psychiatric Assessment
    - ◆ The DSM criteria that a person meets
      - Leads to a broader diagnosis of a disorder



# Workshop Terminology

- **Respondents**: The people from whom behavioral data are collected
  - Behavioral data considered test item responses for workshop
  - Not limited to only item responses
- **Items**: Test items used to classify/diagnose respondents
- **Diagnostic Assessment**: The method used to elicit behavioral data
- **Attributes**: Unobserved categorical characteristics underlying the behaviors (i.e., diagnostic status)
  - Latent variables linked to behaviors diagnostic classification models
- **Psychometric Models**: Models used to analyze item response data
  - Diagnostic Classification Models (DCMs) is the name of the models used to obtain classifications/diagnoses



# Diagnostic Classification Model Names

- Diagnostic classification models (DCMs) have been called many different things
  - Skills assessment models
  - Cognitive diagnosis models
  - Cognitive psychometric models
  - Latent response models
  - Restricted (constrained) latent class models
  - Multiple classification models
  - Structured located latent class models
  - Structured item response theory



# Psychometric Soapbox

- DCMs are but a small set of tools that must be adapted for a common purpose
  - Part of a methodological toolbox that is used to classify respondents
  - Should also include content experts and end-users of the diagnoses
- DCMs link empirical observations and respondents characteristics
  - The models are only as good as underlying theories



Session 1: Conceptual Foundations of Diagnostic Measurement

# CONCEPTUAL EXAMPLE



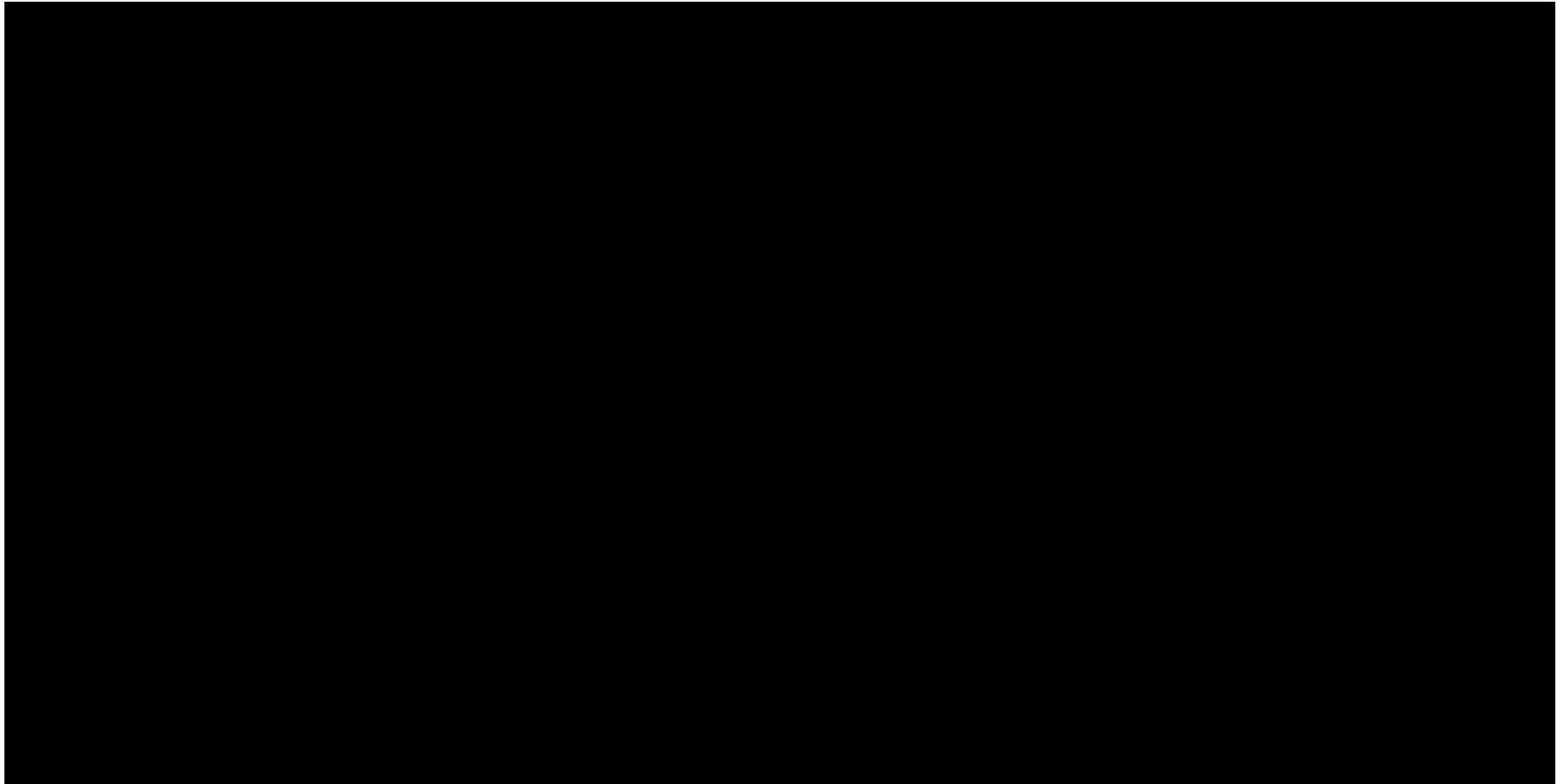
# Motivation for DCMs

- Testing more today than we ever have
  - Accountability movement
- What are we getting out of testing?
  - Often, a single score
  - How useful is this score:
    - ♦ To make decisions about students?
    - ♦ To reflect students' knowledge base or deficiencies?
    - ♦ To inform instruction?
- What if a test didn't give a single score?
  - Instead made decisions about students
    - ♦ With respect to multiple, discrete facets of a content area
- A **diagnostic classification model** is a tool that can be used to make these kinds of decisions





# What do we learn from assessments?





# Diagnostic Need in Education

- In education, there is a need for more specific feedback about what students do and do not understand
- Imagine if we could
  - Diagnose mastery with respect to a set of skills
    - ◆ Complement standards-based curriculum (K-12)
    - ◆ Monitor students' strengths and weaknesses
    - ◆ Place students into appropriate courses
  - Tailor instruction to individual needs
- Concept is not contentious
  - Common Core calls for diagnostic assessments
    - ◆ NCLB required diagnostic score reports
  - Teachers seek more diagnostic score reports



# **TRADITIONAL MEASUREMENT MODELS**



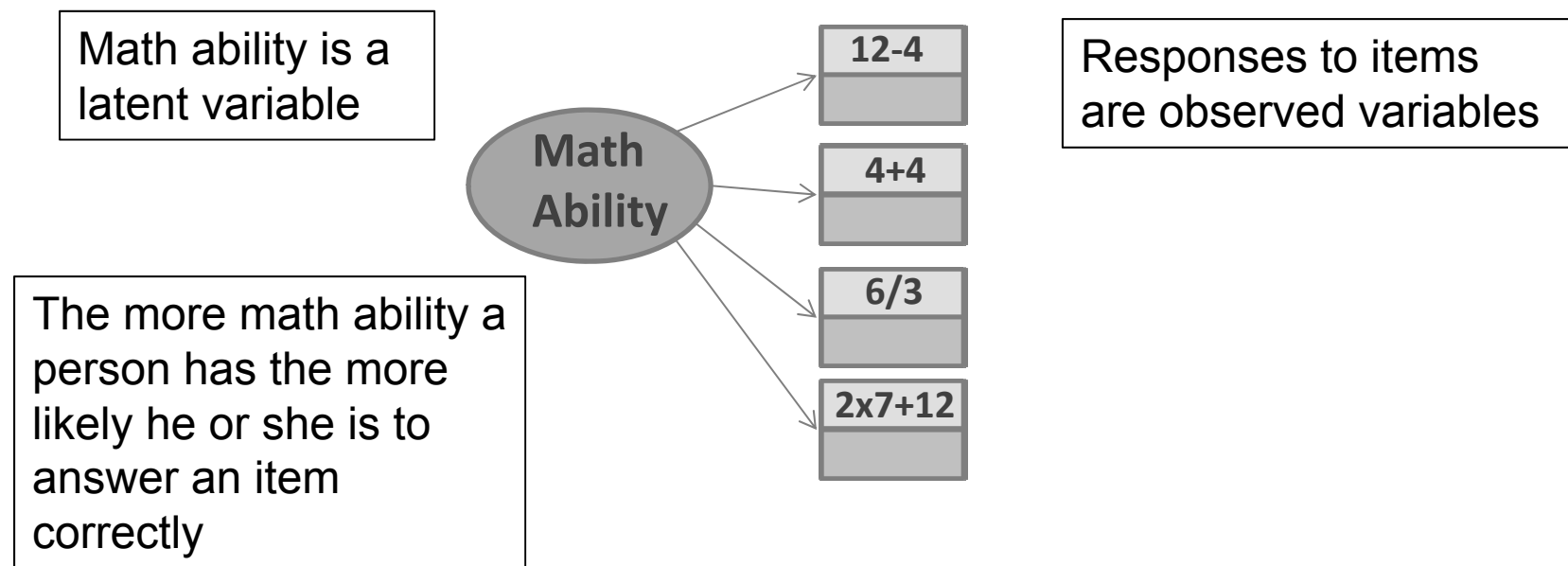
# Traditional Measurement Concepts

- Imagine that an elementary teacher wants to test basic math ability
- Using traditional psychometric approaches, the teacher could estimate an ability or test score for each respondent
  - Classical Test Theory: Assign respondents a test score
  - Item Response Theory: Assign respondents a latent (scaled) score
- By knowing each respondent's score, the students are ordered along a continuum



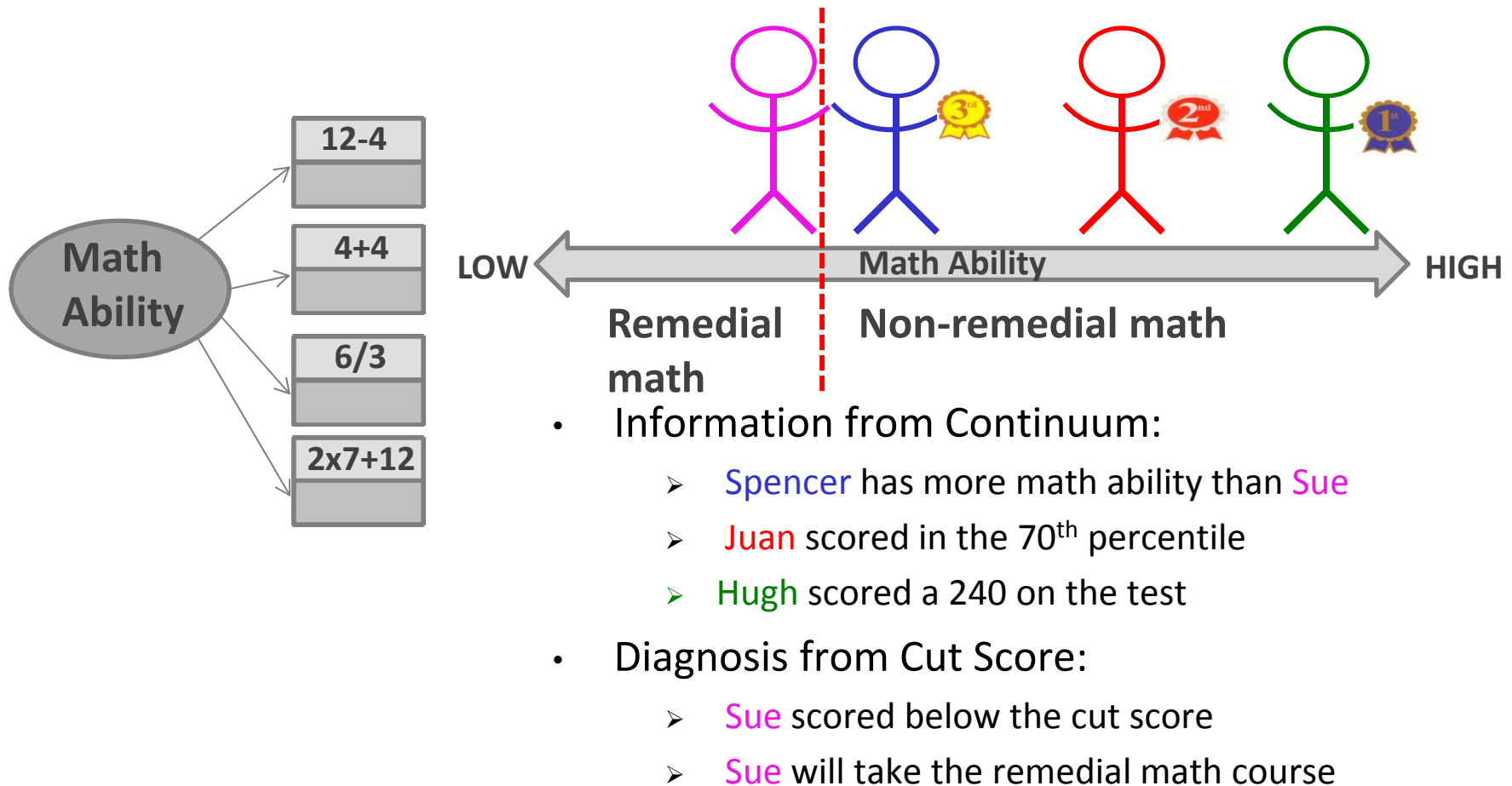
# Traditional Measurement Models

- Traditional testing procedures measure an **overall** ability in an area with a **continuous** latent variable





# Traditional Testing and Classification Methods





# Traditional Psychometrics

- What results is a (weak) ordering of respondents
  - Ordering is called weak because of error in estimates
  - Hugh>Juan>Spencer>Sue
- Questions that traditional psychometrics cannot answer:
  - Why is Sue so low?
    - ♦ How can we get her some help?
  - How much ability is “enough” to pass?
    - ♦ How much is enough to be proficient?
  - What math skills have the students mastered?
    - ♦ Which skills have they yet to master?

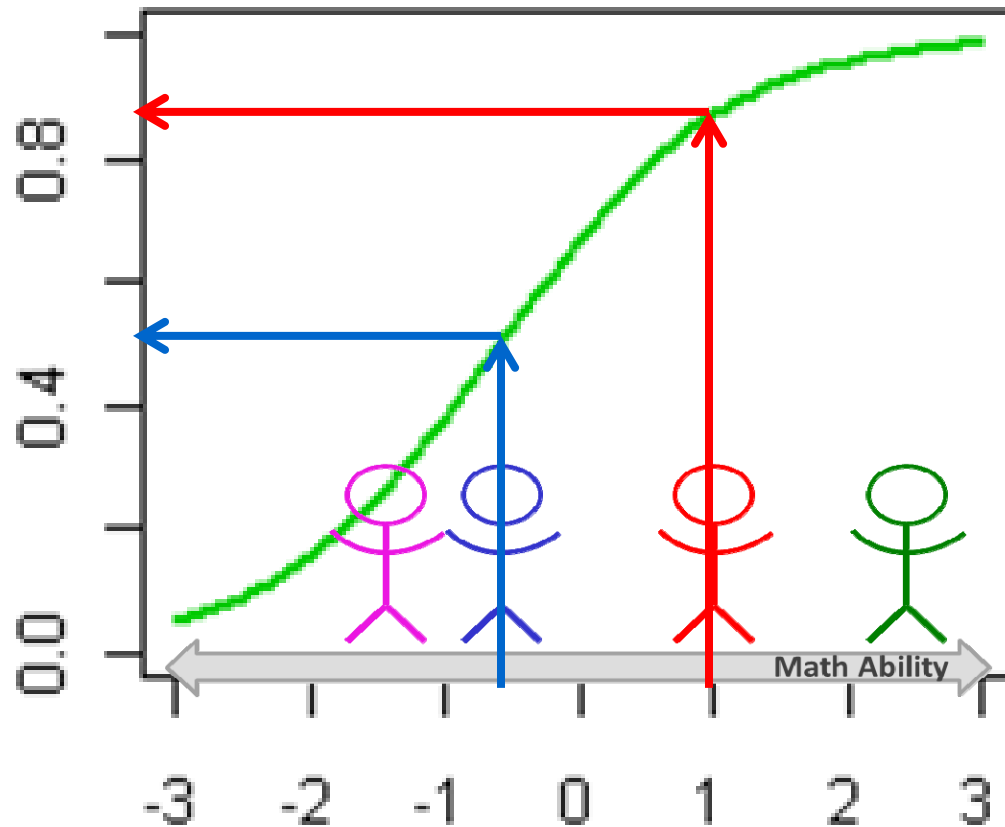


# Item Characteristic Curve

- The item response is a function of a student's ability

The probability that **Juan** will answer this item correctly is .88.

The probability that **Spencer** will answer this item correctly is .52.







# Multiple Dimensions of Ability

- As an alternative, we could have expressed math ability as a set of basic skills:
  - Addition
  - Subtraction
  - Multiplication
  - Division



# Multiple Dimensions of Ability

- The set of skills represent the multiple dimensions of elementary mathematics ability
- Other psychometric approaches have been developed for multiple dimensions
  - Classical Test Theory - Scale Subscores
  - Multidimensional Item Response Theory (MIRT)
- Yet, issues in application have remained:
  - Reliability of estimates is often poor for most practical test lengths
  - Dimensions are often very highly correlated
  - Large samples are needed to calibrate item parameters in MIRT

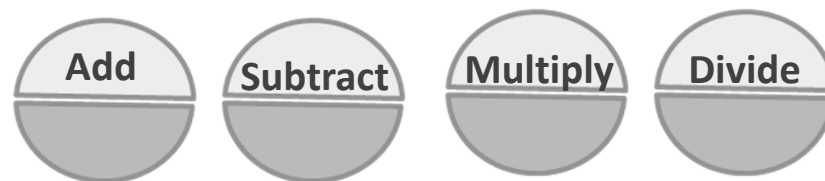


# **DIAGNOSTIC MEASUREMENT MODELS**



# Math Ability Example: A Diagnostic Approach

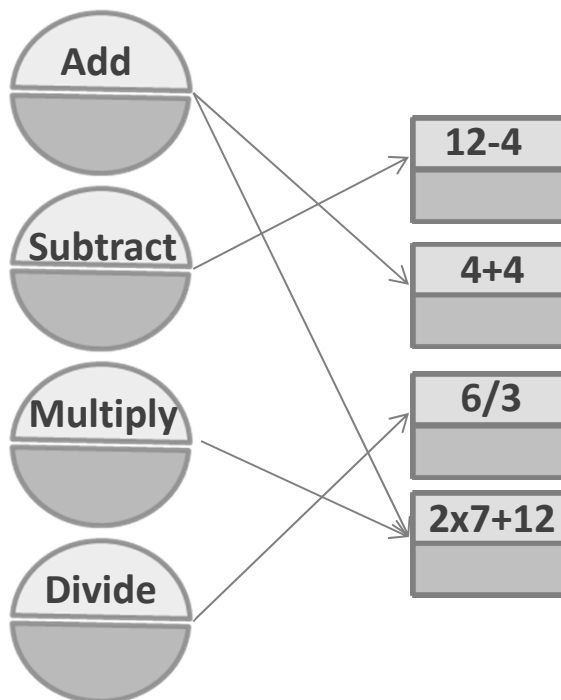
- Instead of measuring an overall math ability, “math” is expressed as a set of skills or **attributes**:
  - Add
  - Subtract
  - Multiply
  - Divide
- These attributes are categorical latent variables
  - Not a continuum
- Attributes are often dichotomous
  - Two levels or groups or categories
  - The two groups may be labeled differently:
    - ◆ Mastery/non-mastery
    - ◆ Proficient/Emerging
    - ◆ Ready/not ready
  - Mastery of an attribute ( $\alpha = 1$ ) or non-mastery of an attribute ( $\alpha = 0$ )





# DCMs: A Conceptual Example

- What would our test look like?
  - Each item measures one or more attribute
- The attributes measured by each item are recorded in a **Q-matrix**
  - Describes whether an item measures an attribute ( $q = 1$ ) or not ( $q = 0$ )
  - Mapping is established by content experts

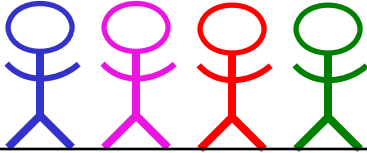
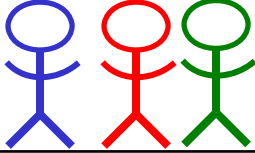

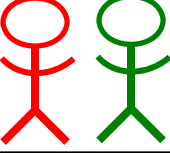
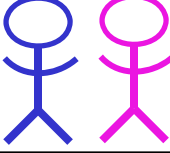

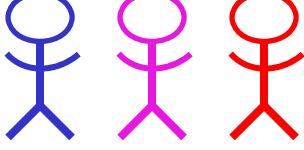


	Add	Subtract	Multiply	Divide
12-4	0	1	0	0
4+4	1	0	0	0
6/3	0	0	0	1
2x7+12	1	0	1	0



# Diagnostic Classification Models





















- DCMs uses responses to items to place students into **groups** according to **multiple** skills
  - No cut score is used to put students into groups; the model is built to do that

	Masters	Non-masters
Add		
Subtract		
Multiply		
Divide		



# Diagnostic Classification Models

- Students receive skill-specific feedback

	Add	Subtract	Multiply	Divide
				
				
				
				

**Spencer** has mastered addition and subtraction, but should improve his ability to multiply and divide.

Or in Bill Gate-ese: Hey, **Spencer** you've got multiplication and division screwed up, but you're fine on addition and subtraction.





















- The model provides a probability each attribute is mastered
- Notice there is no "score" in a traditional or grading sense



# Possible Patterns of Attribute Mastery

The pattern of skills students have is often referred to as a mastery profile or an **attribute profile**

- 4 patterns below are 4 of the possible 16 profiles of mastery that exist if there are 4 attributes:

	Add	Subtract	Multiply	Divide
				
				
				
				

All possible patterns:

Pattern	Add	Sub.	Multiply	Divide
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	1
5	0	1	0	0
6	0	1	0	1
7	0	1	1	0
8	0	1	1	1
9	1	0	0	0
10	1	0	0	1
11	1	0	1	0
12	1	0	1	1
13	1	1	0	0
14	1	1	0	1
15	1	1	1	0
16	1	1	1	1

A classification of each individual skill results in a classification into one of these patterns

























# Expected Examinee Responses

Q-matrix

	+	-	x	/
12-4	0	1	0	0
4+4	1	0	0	0
6/3	0	0	0	1
2x7+12	1	0	1	0

- Given a student's profile of mastery and the set of skills measured by each item, we can determine how we expect a student to respond to each item.

Student Profiles

	Add	Subtract	Multiply	Divide
				
				
				
				

These students will likely answer the following items correctly:

All Items

Items 1, 2, 4

Items 1,2

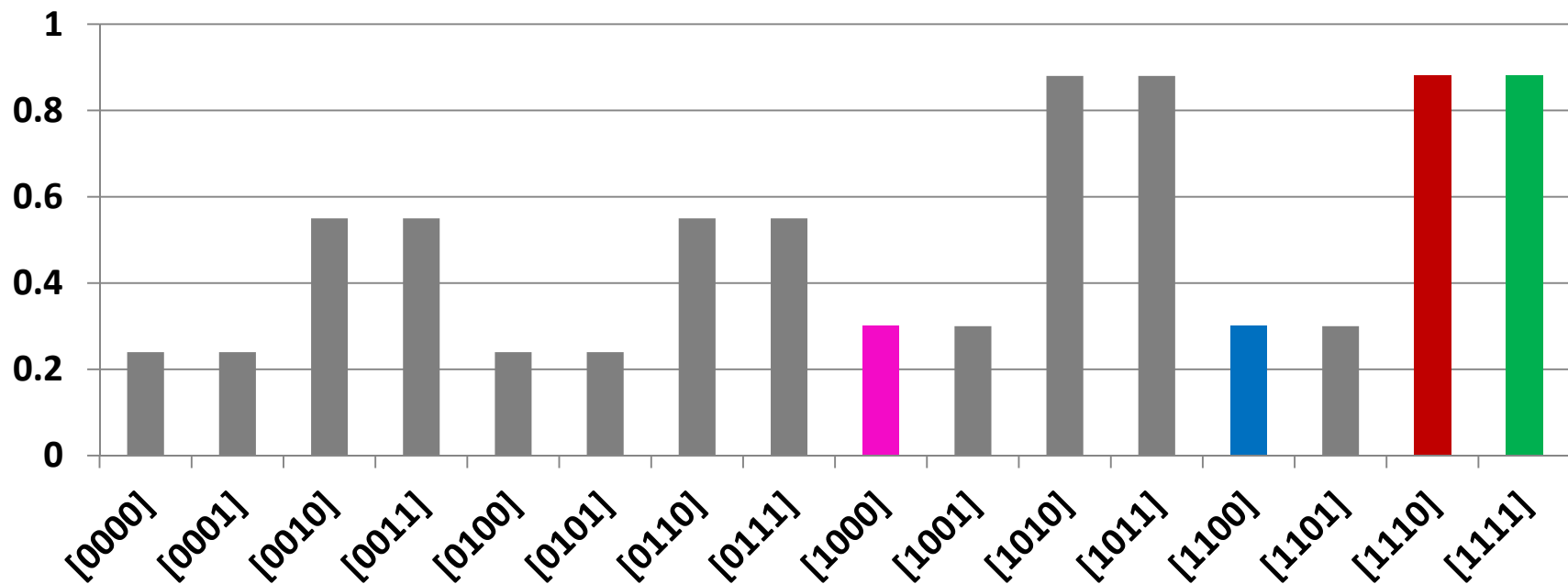
Item 2

*Which items a student answers correctly depends on (1) which attributes he or she has and, (2) which attributes are being measured by the item.*



# Item Characteristic Bar Chart

- Example item:  $2 \times 7 + 12 = ?$ 
  - Measures Attribute 1 (addition) and Attribute 3 (multiplication)



- **Juan's** probability of correct response is .88
- **Spencer's** probability of correct response is .3



# Traditional Score Report

- Example from Georgia's federally mandated end-of-course assessments

## Criterion-Referenced Competency Tests (CRCT) • Spring 2009

Name: **JOHNSON, JENNY A**  
GTID: **5678123499**  
Gender: **F**  
Grade: **5**

Class: **HOIT**  
School: **NORTH SCHOOL**  
System: **NORTH DISTRICT**

**Lexile: 650L**

CONTENT AREA	SCALE SCORE	PERFORMANCE LEVEL
Reading-GPS	<b>800</b>	<b>Meets</b>
English/Language Arts-GPS	<b>790</b>	<b>Does Not Meet</b>
Mathematics-GPS	<b>870</b>	<b>Exceeds</b>
Science-GPS	<b>870</b>	<b>Exceeds</b>
Social Studies-GPS	<b>796</b>	<b>Does Not Meet</b>

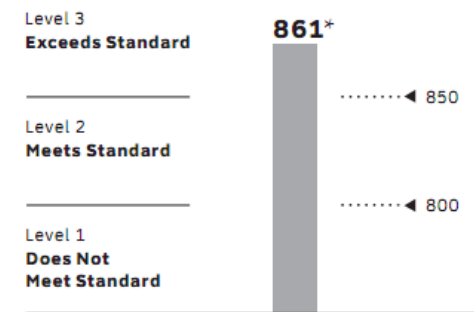
Subject

Score

Classification

**Diagnostic Information**

Get to classification from score by using cut scores.



This student's score is **861**, which is in performance Level 3 and **exceeds the standard** for Reading.

### Reading Domains

	Number Correct	Number Possible
Literary Comprehension	19	24
Reading for Information	6	8
Reading Skills and Vocabulary Acquisition	7	8

Breakdown content area into sub-domains

Sub-scores



# What can DCMs do differently?

- Example from Georgia's federally mandated end-of-course assessments

## Criterion-Referenced Competency Tests (CRCT) • Spring 2009

Name: **JOHNSON, JENNY A**  
GTID: **5678123499**  
Gender: **F**  
Grade: **5**

Class: **HOIT**  
School: **NORTH SCHOOL**  
System: **NORTH DISTRICT**

**Lexile: 650L**

CONTENT AREA	SCALE SCORE	PERFORMANCE LEVEL
Reading-GPS	<b>800</b>	<b>Meets</b>
English/Language Arts-GPS	<b>790</b>	<b>Does Not Meet</b>
Mathematics-GPS	<b>870</b>	<b>Exceeds</b>
Science-GPS	<b>870</b>	<b>Exceeds</b>
Social Studies-GPS	<b>796</b>	<b>Does Not Meet</b>

Subject

Score

Classification

**DCMs can provide statistical probabilities an individual has mastered each sub-domain.**

Get to classification from score by using cut scores.



This student's score is **861**, which is in performance Level 3 and **exceeds the standard** for Reading.

### Reading Domains

	Number Correct	Number Possible
Literary Comprehension	19	24
Reading for Information	6	8
Reading Skills and Vocabulary Acquisition	7	8

Breakdown content area into sub-domains

Sub-scores



# Diagnostic Score Report

## Diagnostic Scoring Report

Student Name: Daphne

### Review Your Answers

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Your Answer	✓	✓	✓	✓	a	c	✓	c	d	✓	✓	✓	c	✓	d	a	✓	b	a	a	d	c	b	a	c
Correct Answer	d	a	b	d	d	a	b	d	a	c	a	b	d	c	a	d	a	c	b	d	a	a	a	d	b
Difficulty	e	e	m	m	m	m	h	h	h	m	e	e	m	m	m	h	m	m	h	h	h	h	h	h	h

### Score

You correctly answered 10 out of 25 questions.

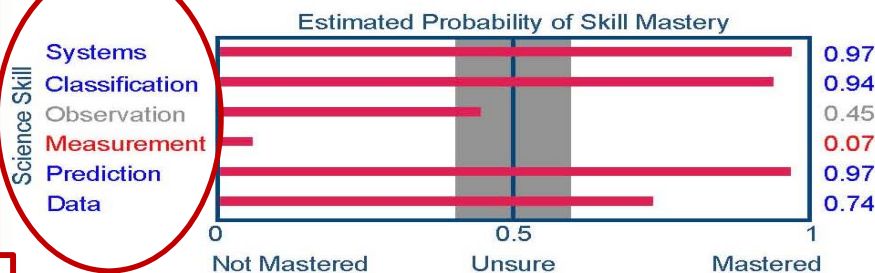
Easy: 4/4; Medium: 5/10; Hard: 1/11

### Guide

✓ - Correct answer; o - Omitted answer

e - Easy; m - Medium; h - Hard

### Improve Your Skills



### Example Questions

3, 14, 2, 17, 19, 23, 9
3, 12, 13, 5, 2, 17, 18, 16, 24, 7
11, 15, 1, 8, 18
22, 20, 10, 11, 5, 6, 18, 25
4, 14, 20, 12, 5, 19, 9
22, 1, 19, 21

Skills or  
Sub-domains



# DCM Conceptual Summary

- Diagnostic classification models (DCMs) are a set of statistical tools that provide diagnostic feedback
- Feedback is diagnostic in the sense that the **decision** itself—not just information to inform the decision—is given from the model
- No need for a two-stage approach of lining up and then cutting



# DCM Conceptual Summary

- DCMs focus on **WHY** a respondent is not performing well as compared to only focusing on **WHO**
- The models define the chances of a correct response based on the respondent's attribute profile
- Many models have been created ranging in complexity
  - In Session #2 we discuss a general DCM
  - The general model subsumes all other latent-variable DCMs
- The model predicts how respondents will answer each item
  - Also allows for classification/diagnoses based on item responses



# How do DCMs Produce Diagnoses?

- Diagnostic decisions come from comparing observed behaviors to two parts of the psychometric model:

1. Item/variable information (item parameters)

Measurement Model

- ♦ How respondents with different diagnostic profiles perform on a set of test items
- ♦ Helps determine which items are better at discriminating between respondents with differing diagnostic profiles

2. Respondent information pertaining to the base-rate or proportion of respondents with diagnoses in the population

Structural Model

- ♦ Provides frequency of diagnosis (or diagnostic profile)
- ♦ Helps validate the plausibility of the observed diagnostic profiles





# **WHY USE DCMS OVER TRADITIONAL APPROACH?**

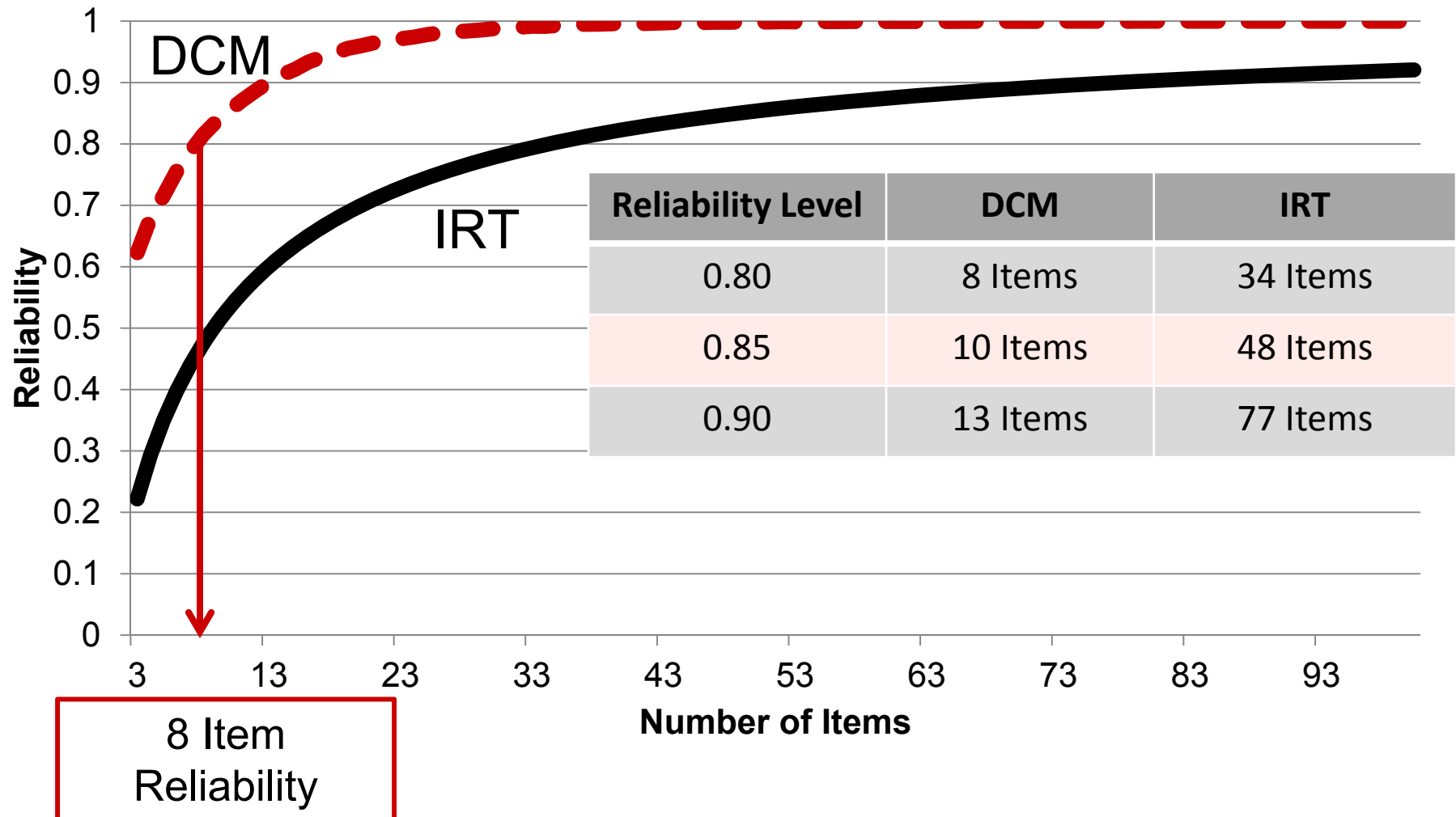


# DCMs as an Alternative

- DCMs do not assign a single score
- Instead, a ***profile*** of ***mastered*** attributes is given to respondents
  - Multidimensional models
- DCMs provide respondents valuable information with fewer data demands
  - Higher reliability than comparable IRT/MIRT models
  - Complex item structures possible



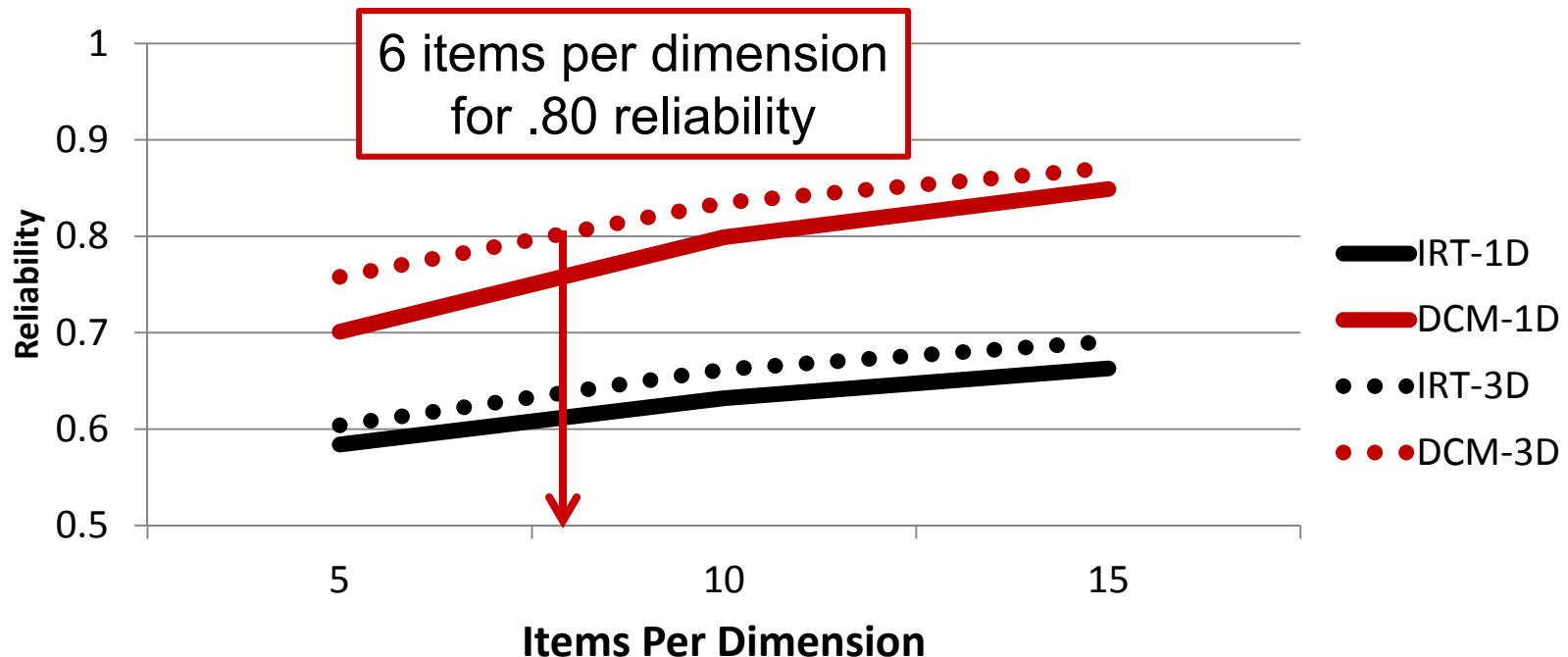
# Data Demands for Reliability (Unidimensional)





## Data Demands for Reliability (Multidimensional)

- Can we measure multiple attributes reliably, too?

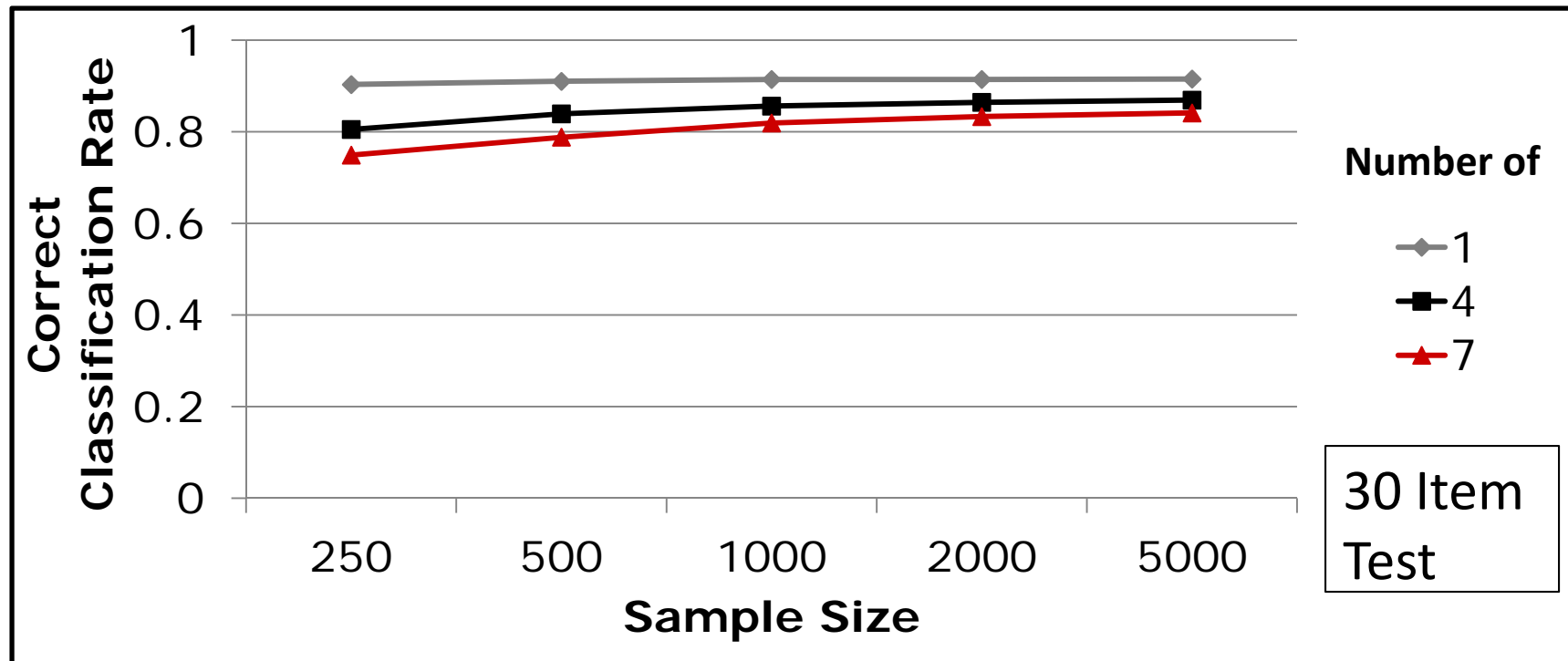


- 18 item diagnostic test → diagnose mastery of 3 attributes
- 34 item traditional test → scale unidimensional ability



# DCM Classification Accuracy

- What about the accuracy of the classification:
  - As sample size decreases?
  - As dimensionality increases?



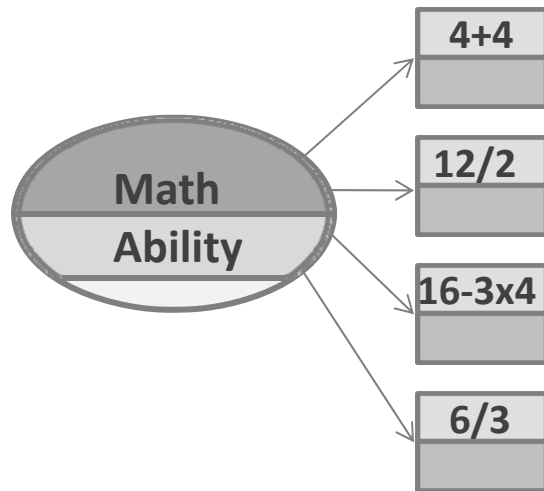


Classifying into Multiple Groups

# **OTHER CONCEPTUAL EXAMPLES OF DCMS**

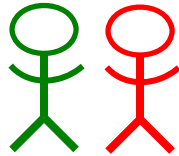




# Unidimensional, Multicategory DCMs



- Math ability is unidimensional
- Math ability is categorical, not continuous

- DCMs can be unidimensional and used for AYP-like classification purposes

	Advanced	Proficient	Basic
Math Ability			

- The classification is statistical (directly from the model)



# What can DCMs do differently?

- Example from Georgia's federally mandated end-of-course assessments

## Criterion-Referenced Competency Tests (CRCT) • Spring 2009

Name: **JOHNSON, JENNY A**  
GTID: **5678123499**  
Gender: **F**  
Grade: **5**

Class: **HOIT**  
School: **NORTH SCHOOL**  
System: **NORTH DISTRICT**

**Lexile: 650L**

CONTENT AREA	SCALE SCORE	PERFORMANCE LEVEL
Reading-GPS	<b>800</b>	<b>Meets</b>
English/Language Arts-GPS	<b>790</b>	<b>Does Not Meet</b>
Mathematics-GPS	<b>870</b>	<b>Exceeds</b>
Science-GPS	<b>870</b>	<b>Exceeds</b>
Social Studies-GPS	<b>796</b>	<b>Does Not Meet</b>

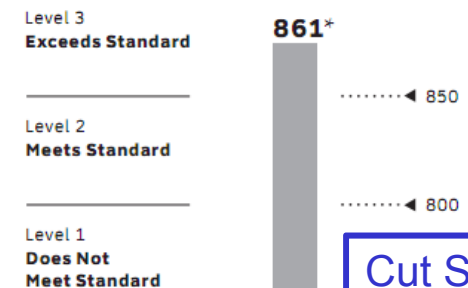
Subject

Score

Classification

DCMs can directly classify students into proficiency categories by assuming ability is unidimensional, but categorical.

## Reading GPS



This student's score is **861**, which is in performance Level 3 and **exceeds the standard** for Reading.

## Reading Domains

	Number Correct	Number Possible
Literary Comprehension	19	24
Reading for Information	6	8
Reading Skills and Vocabulary Acquisition	7	8

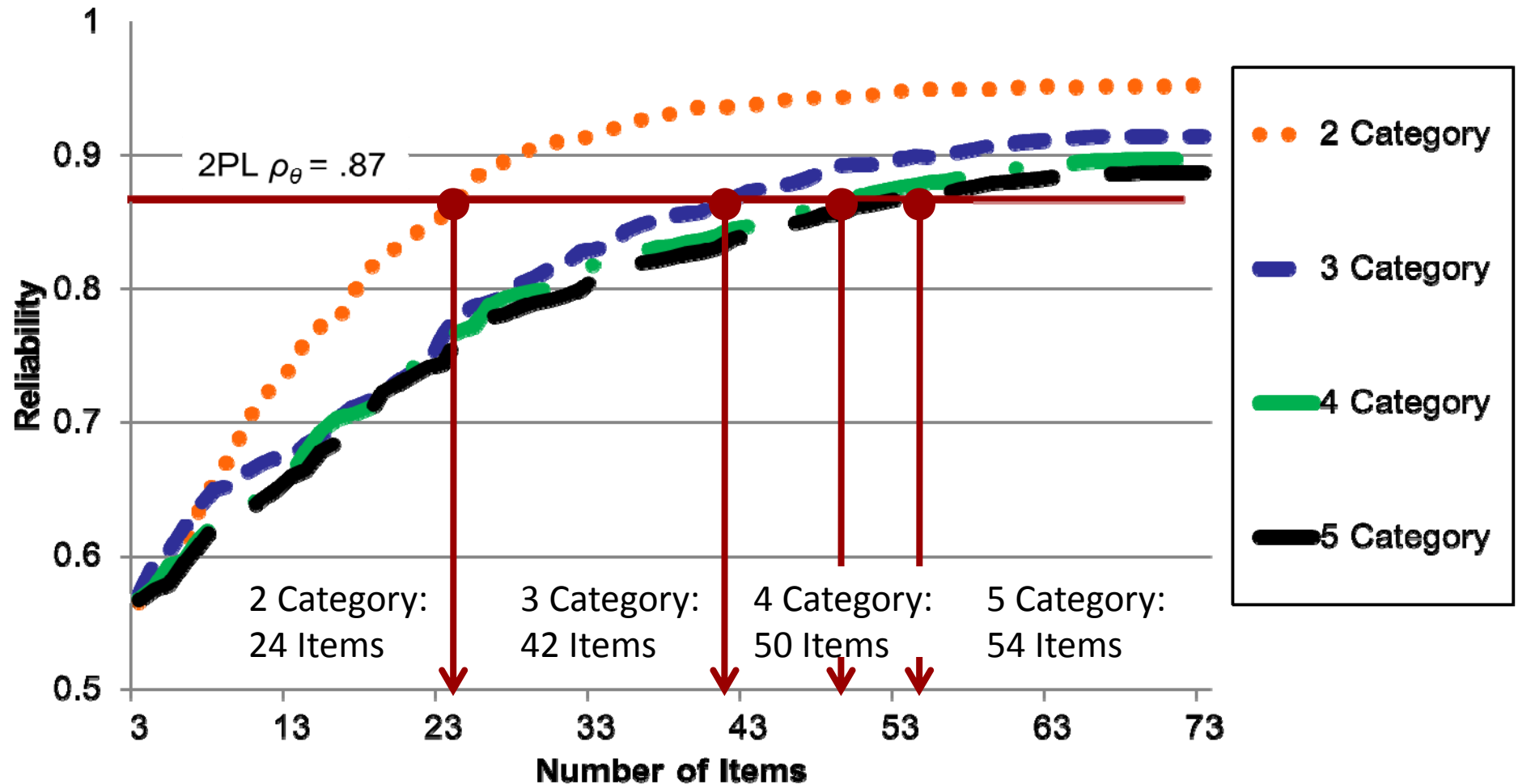
Breakdown content area into sub-domains

Sub-scores





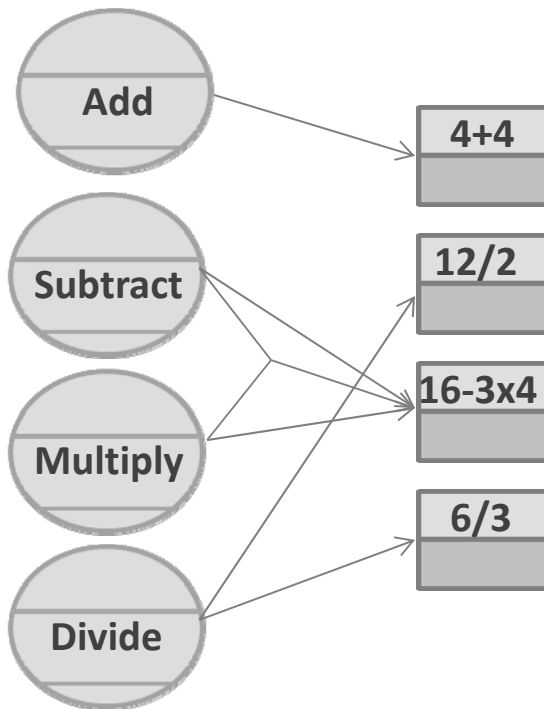
# IRT vs DCM Reliability



- DCMs yield equal reliability with fewer items (which means shorter tests)



















# Multicategory, Multidimensional DCMs



Attributes are still categorical, but now have 3 categories instead of 2

- DCMs can place students into more than 2 groups

	Advanced	Proficient	Basic
Add	   		
Subtract	  		
Multiply	 		
Divide		 	



Session 1: Conceptual Foundations of Diagnostic Measurement

# **IMPLICATIONS FOR LARGE SCALE TESTING PROGRAMS**



# DCM Characteristics

- As mentioned previously, DCMs provide a higher level of reliability for their estimates than comparable IRT or CTT models (Templin & Bradshaw, 2013)
  - It is easier to place a respondent into one of two groups (mastery or non-mastery) than to locate them on a scale
- Such characteristics allow DCMs to potentially change how large scale testing is conducted
  - Most EOC-type tests are for classification
    - ◆ Proficiency standards
  - DCMs provide direct link to classification
    - ◆ And direct access to standards



# Ramifications for Use of DCMs

- Reliable measurement of multiple dimensions is possible
  - Multidimensional proficiency standards
    - ◆ Respondents must demonstrate proficiency on multiple areas to be considered proficient for an overall content domain
  - “Teaching to the test” would therefore represent covering more curricular content to best prepare respondents
- Shorter unidimensional tests
  - Unidimensional DCM application to empirical data:
    - ◆ Test needed only 24 items to have same reliability as IRT with 73 items



## Ramifications for Use of DCMs: Formative Assessment

- Classroom appropriate test lengths
  - Teaching and testing time is limited
- Multivariate feedback
  - Strengths and weaknesses profiles
- No scores
  - Argued as a key element of effective formative testing in research



# The Paradox of DCMs

- DCMs are often pitched as models that allow for measurement of “fine-grained” skills (e.g., Rupp & Templin, 2008)
- Paradox of DCMs:
  - Sacrifice fine-grained measurement of a latent trait for only several categories
  - Increased capacity to measure ability multidimensionally



# When Are DCMs Appropriate?

- Which situations lend themselves more naturally to such diagnosis?
  - The *purpose* of the diagnostic assessment matters most
  - DCMs provide classifications directly
    - ♦ Optimally used when tests are used for classification
      - EOC Tests
      - Licensure/certification
      - Clinical screening
      - College entrance
      - Placement tests
  - DCMs *can* be used as coarse approximations to continuous latent variable models
    - ♦ i.e., EOC example (2-5 category levels shown)





Session 1: Conceptual Foundations of Diagnostic Measurement

# **BENEFITS OF DCMS OVER TRADITIONAL CLASSIFICATION METHODS**



# Previous Methods for Classification

- Making diagnoses on the basis of test responses is not a new concept
  - Classical test theory
  - Item response theory
  - Factor analysis
- Process is a two-stage procedure
  1. Scale respondents
  2. Find appropriate cut-scores
- Classify respondents based on cut-scores



# Problems with the Two-Stage Approach

- The two-stage procedure allows for multiple sources of error to affect the results
- 1. The latent variable scores themselves: estimation error
  - Uncertainty is typically not accounted for in the subsequent classification of respondents (i.e., standard errors)
  - The classification of respondents at different locations on the score continuum with multiple cut-scores is differentially precise
    - ◆ Uncertainty of the latent variable scores varies as a function of the location of the score



# Problems with the Two-Stage Approach

2. Latent variable assumptions: that latent variable scores follow a continuous, typically normal, distribution
  - Estimates reflect the assumed distribution
  - Can introduce errors if the assumption is incorrect
3. Cut-score determination
  - Standard setting is imprecise when used with general abilities
    - ◆ Standard setting methods can be directed to item performance
  - Some theoretical justification needs to be provided for such a cut-off



# Why are DCMs Better for Classification?

- The need for a two-stage procedure to set cut-scores for classification is eliminated when DCMs are used
  - Reduces classification error
- Quantifies and models the measurement error of the observable variables
  - Controlling for measurement error when producing the diagnosis
- DCMs have a natural and direct mechanism for incorporating base-rate information into the analysis
  - No direct way to do so objectively in two-stage procedures
- Item parameters provide information as to the diagnostic quality of each item
  - Not directly estimable in two-stage approaches
  - Can be used to build tests that optimally separate respondents



Session 1: Conceptual Foundations of Diagnostic Measurement

# **CONCLUDING REMARKS**



# Session 1 – Take-home Points

- DCMs provide direct link between diagnosis and behavior
  - Provide diagnostic classifications directly
  - Diagnoses set by psychometric model parameters
- DCMs are effective if classification is the ultimate purpose
  - Reduce error by removing judgments necessary in two-stage approach
- DCMs can be used in many contexts
  - Can be used to create highly informative tests
  - Can be used to measure multiple dimensions