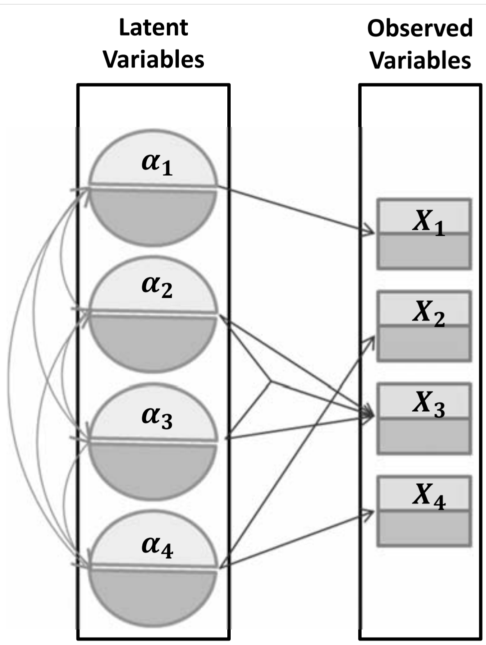
Theoretical Framework for DCMs

Section 2

Diagnostic Classification Model Framework

Structural Component: How latent variables are related to each other (this afternoon)



Measurement Component: How latent variables are related to observed variables (This section)

DCMs: Observable Variables

- DCMs provide a direct link between behavior and a classification status
  - » Observable variables can be:
    - Item responses
    - Observable behaviors
    - Task performance
- Typically the behavior is a test item, but also could represent any observable data used to make a diagnosis
  - » Item is used because most DCMs are used as psychometric models for test behavior

DCMs: Latent Variables

- Different labels have been suggested in the literature for the latent variables:
  - » Latent characteristics
  - » Latent traits
  - » Elements of processes
  - » Attributes
- The term *attribute* will be used today for individual latent variables
  - » Perhaps most frequently used in the measurement literature on diagnostic assessments.
- *Attribute profile* denotes a particular pattern of latent variable values for a particular respondent

Attribute Defined

- Attribute: a *categorical latent variable* representing the diagnostic status of a person
  - » *Categorical*: can be one of a finite number of discrete levels
  - » *Latent*: not directly observable
  - » *Variable*: status changes from person to person
- Synonyms for attributes:
  - » Skills
  - » Factors
  - » Abilities

Types of Attributes

Table 4.1

Exemplary Attribute Definitions from Different Domains

Construct: Number Subtraction Domain: Mathematics Source: de la Torre & Douglas (2004)	
- convert a whole number to a fraction	- borrow from whole number part
- separate a whole number from a fraction	- column borrow to subtract
- simplify before subtracting	- subtract numerators
- find a common denominator	- simplify answer
Construct: Reading Comprehension Domain: English Language Learning Source: Buck, Tatsuoka, & Kostin (1997)	
- synthesize scattered information	- apply relevant background knowledge
- recognize relevant information	- hold relevant information in WM
- know low-frequency vocabulary	- use a word-matching strategy
- identify the gist of a passage	- compare two plausible options and choose
Construct: Figural Analysis Domain: Architecture Source: Katz, Martinez, Sheehan, & Tatsuoka (1998)	
- move or rotate objects	- identify environmental characteristics
- read and translate information	- process a complex diagram
- activate prior knowledge	- understand structural technology
- identify distracting information	- apply a learned procedure

Table from: Rupp, A. A., Templin, J., & Henson, R. (2010). *Diagnostic measurement: Theory, methods, and applications*. New York: Guilford.

Grainsizes of Attributes

- The degree of definitional specificity of an attribute is often referred to as the ***definitional grainsize***
- The grainsize is driven by the level of specificity with which one would like to make statements about respondents
- The grainsize of an attribute is the resolution with which an investigator dissects a cognitive response process and describes its constituent components

Attribute Examples

Table 4.2

Attribute Definitions within the Same Domain at Different Levels of Definitional Grain size

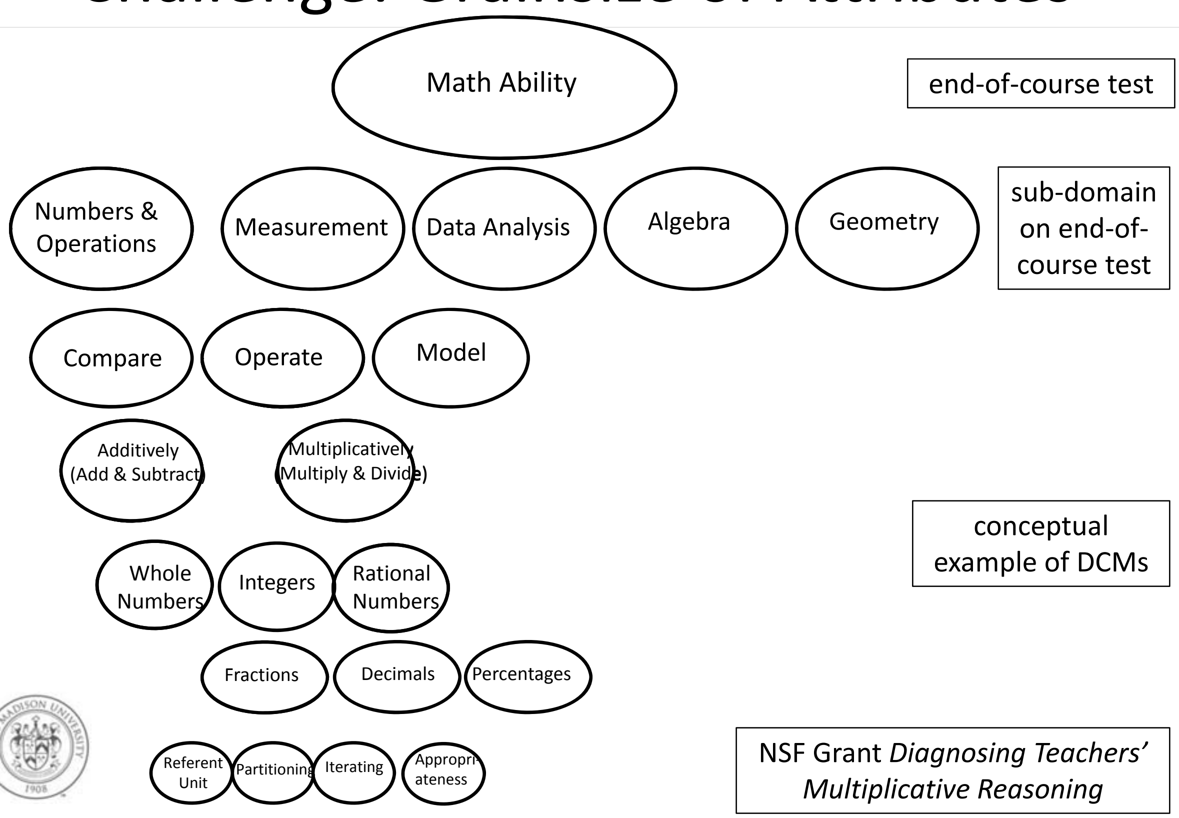
Construct: Number Subtraction Source: de la Torre & Douglas (2004)	
- convert a whole number to a fraction	- borrow from whole number part
- separate a whole number from a fraction	- column borrow to subtract
- simplify before subtracting	- subtract numerators
- find a common denominator	- simplify answer
Construct: Basic Artihmetic Skills Source: Kunina, Rupp, & Wilhelm (2008)	
- form a basic number representation	- multiply and divide (100 to 1000)
- add and subtract (0 to 100)	- perform inverse operations
- add and subtract (100 to 1000)	- solve problems with two operations
- multiply and divide (0 to 100)	- construct a mathematical model
Construct: Solving Linear Equations Source: Gierl, Leighton, & Hunka (2007)	
- understand meaning of symbols and conventions	- solve quadratic equations
- comprehend textual description of problem	- solve multiple equations simultaneously
- perform algebraic manipulations	- construct a tabular representation
- solve linear equations	- construct a graphical representation

Table from: Rupp, A. A., Templin, J., & Henson, R. (2010). *Diagnostic measurement: Theory, methods, and applications*. New York: Guilford.

Practical Issues with Grainsizes

- It is possible to decompose individual attributes for more complex tasks further
  - » That would increase the number of attributes
- As the number of attributes increases, the number of latent variables in a DCM increases
  - » Attribute profiles and item parameters may become impossible to estimate statistically
- It is important to fix the number of attributes to a statistically manageable number for a given diagnostic assessment length and respondent sample size

Challenge: Grainsize of Attributes



Attributes Exist Because of Theory

- Current applications of DCMs typically involve attributes that are defined via a theory of response processing supported by research in applied cognitive psychology and educational measurement
  - » DCMs help model these multidimensional theories
- In order to use DCMs to represent and, ideally, **validate** cognitive processing models one first needs to have developed a plausible model from theory and empirical investigations

DCM MEASUREMENT MODELS

Overview

- General DCMs
- Background information
  - ANOVA models
  - Logits explained
- The Log-linear Cognitive Diagnosis Model (LCDM; Henson, Templin, & Willse, 2009)
  - Parameter structure with example item demonstration
  - A look at model interaction
  - General form of the LCDM

Development of Psychometric Models

- Over the past several years, numerous DCMs have been developed
- Each DCM makes assumptions about how mastered attributes combine/interact to produce an item response
- With so many models, analysts have been unsure which model would best fit their purpose
  - » Difficult to imagine all items following same assumptions

General Models for Diagnosis

- Recent developments have produced very general diagnostic models
  - » General Diagnostic Model (**GDM**; von Davier, 2005)
  - » Loglinear Cognitive Diagnosis Model (**LCDM**; Henson, Templin, & Willse, 2009)
    - Focus of this workshop
- These general DCMs provide great flexibility
  - » Subsume all other latent variable DCMs
  - » Allow for both additive and non-additive relationships between attributes and items
  - » Sync with other psychometric models allowing for greater understanding of modeling process

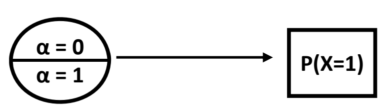
Notation

- **Attributes**: $a = 1,\ldots, A$
- **Respondents**: $r = 1,\ldots,R$
- **Attribute Profiles**: $\boldsymbol{\alpha}_r = [\alpha_{r1}, \alpha_{r2},\ldots, \alpha_{rA}]$
  - » α_{ra} is 0 or 1
- **Latent Classes**: $c = 1,\ldots,C$
  - » We have $C = 2^A$ latent classes – one for each possible attribute profile
- **Items**: $i = 1,\ldots,I$
  - » Restricted to dichotomous item responses (X_{ri} is 0 or 1)
- **Q-matrix**: Elements q_{ia} for an item i and attribute a
  - » q_{ia} is 0 or 1

BACKGROUND INFORMATION: ANOVA MODELS

Background Information – ANOVA

- The LCDM models the probability of a correct response to an item as a function of the latent attributes of a respondent



- The latent attributes are categorical, meaning a respondent can have only a few possible statuses
  - » Each status corresponds to a predicted probability of a correct response

- As such, the LCDM is very similar to an ANOVA model
  - » Predicting the dependent variable as a function of the experimental group of a respondent

ANOVA Refresher

- As a refresher on ANOVA, lets imagine that we are interested in the factors that have an effect on a college student's STAT 101 final *exam score* (denoted by Y)

- We are interested in a two-factor study where the *exam score* may be affected by:
  - » Prior Coursework
    - Took AP Stats in HS or did not
  - » Major type
    - STEM major vs non-STEM major

- This design is known as a 2-Way ANOVA

ANOVA Model

- Here is the 2 x 2 Factorial design:

	No AP Stats	AP Stats
Non-STEM Major	$\bar{Y}_{nonSTEM,noAP}$	$\bar{Y}_{nonSTEM,AP}$
STEM Major	$\bar{Y}_{STEM,noAP}$	$\bar{Y}_{STEM,AP}$

- The ANOVA model for a respondent's exam score is

$$Y_{rmc} = \mu + A_m + B_c + (AB)_{mc} + \varepsilon_{rmc}$$

ANOVA Model

- The ANOVA model allows us to test for the presence of:
  - » A main effect associated with *major type* (A_m)
  - » A main effect associated with *prior coursework* (B_c)
  - » An interaction effect associated with *major type* and *prior coursework* $(AB)_{mc}$

$$Y_{rmc} = \mu + A_m + B_c + \left(AB\right)_{mc} + \varepsilon_{rmc}$$

ANOVA with Dummy Coded Variables

- The ANOVA model can also be re-written using two dummy-coded variables D_{rm} and D_{rc}
  - » Becomes a linear model (i.e., regression model)
- D_{rmajor}
  - » $D_{rm}=0$, if respondent *r* is a ***non-STEM*** major
  - » $D_{rm}=1$, if respondent *r* is a ***STEM*** major
- $D_{rcoursework}$
  - » $D_{rc}=0$ if respondent *r* **did not take *AP Stats***
  - » $D_{rc}=1$ if respondent *r* **took *AP Stats***

ANOVA with Dummy Coded Variables

- The ANOVA model then becomes:

	$D_{rc}=0$ No AP Stats	$D_{rc}=1$ AP Stats
$D_{rm}=0$ Non-STEM Major	$\bar{Y}_{nonSTEM,noAP}$	$\bar{Y}_{nonSTEM,AP}$
$D_{rm}=1$ STEM Major	$\bar{Y}_{STEM,noAP}$	$\bar{Y}_{STEM,AP}$

$$Y_{rmc} = \beta_0 + \beta_m D_{rm} + \beta_c D_{rc} + \beta_{m*c} D_{rm} D_{rc} + e_{rmc}$$

ANOVA Effects Explained

$$Y_{rtl} = \beta_0 + \beta_m D_{rm} + \beta_c D_{rc} + \beta_{m*c} D_{rm} D_{rc} + e_{rmc}$$

- β_0 is the predicted mean for the non-STEM major and no AP Stats coursework condition (reference group)
  - The intercept
- β_m is the predicted change of the mean when comparing non-STEM to STEM majors for students who did not take AP Stats (Simple Main Effect)
- β_c is the predicted change of the mean when comparing AP Stat students to non-AP Stat students for students are not STEM majors (Simple Main Effect)
- β_{m*c} is the predicted additional mean change that is not explained by the shift in major type and shift in prior coursework, when both occur (2-Way Interaction)
- Respondents in the same condition have the same predicted value
  - » Four different groups yield four predicted values

ANOVA and the LCDM

- The ANOVA model and the Log-linear cognitive diagnosis model (LCDM) take the same modeling approach
  - » Predict a response using dummy coded variables
    - In LCDM dummy coded variables are latent attributes
  - » Using a set of main effects and interactions
    - Links attributes to item response
  - » Where possible, we may look for ways to reduce the model
    - Removing non-significant interactions and/or main effects

Differences Between LCDM and ANOVA

- The LCDM and the ANOVA model differ in two ways:
  - » Instead of a continuous outcome such as exam score, the LCDM models a function of the probability of a correct response
    - The logit of a correct response (defined next)
  - » Instead of observed "factors" as predictors the LCDM uses discrete *latent* variables
    - The attributes being measured
- Attributes are given dummy codes (act as latent factors)
  - » α_{ra} = 1 if respondent *r* has **mastered** attribute *a*
  - » α_{ra} = 0 if respondent *r* has **not mastered** attribute *a*

LOGITS EXPLAINED

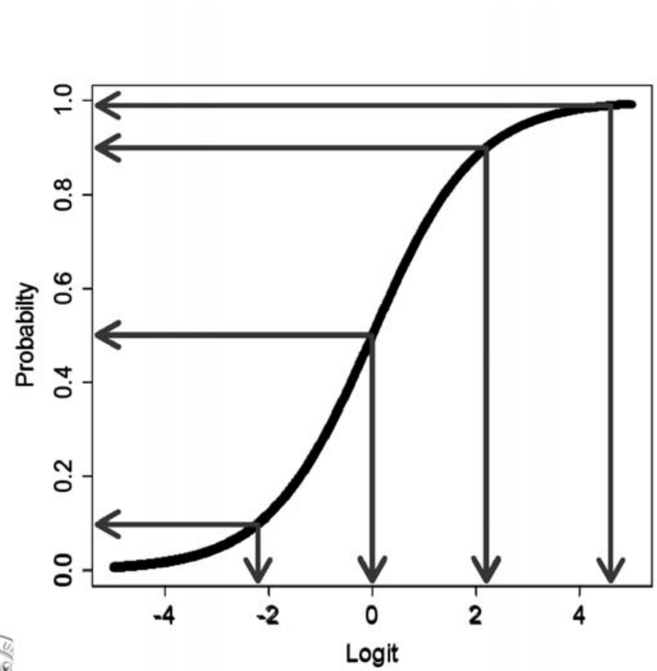
Model Background

- Just as in IRT models, the LCDM models the ***log-odds*** of a correct response conditional on a respondent's attribute pattern $\boldsymbol{\alpha}_r$
  - The log-odds is called a ***logit***

$$\text{Logit}(X_{ri} = 1 \mid \boldsymbol{\alpha}_r) = \ln\left(\frac{P(X_{ri} = 1 \mid \boldsymbol{\alpha}_r)}{1 - P(X_{ri} = 1 \mid \boldsymbol{\alpha}_r)}\right)$$

- The logit is used because the responses are binary
  - Items are either answered correctly (1) or incorrectly (0)
- The linear model is inappropriate for categorical data
  - Can lead to impossible predictions (i.e., probabilities greater than 1 or less than 0)

More on Logits



Probability	Logit
0.5	0.0
0.9	2.2
0.1	-2.2
0.99	4.6

From Logits to Probabilities

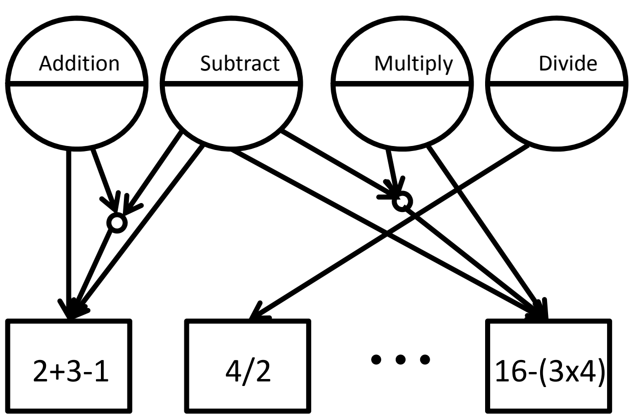
- Whereas logits are useful as the are unbounded continuous variables, categorical data analyses rely on estimated probabilities
- The inverse logit function coverts the unbounded logit to a probability
  - » This is also the form of an IRT model (and logistic regression)

$$P(X_{ri} = 1 \mid \boldsymbol{\alpha}_r) = \frac{\exp(\text{Logit}(X_{ri} = 1 \mid \boldsymbol{\alpha}_r))}{1 + \exp(\text{Logit}(X_{ri} = 1 \mid \boldsymbol{\alpha}_r))}$$

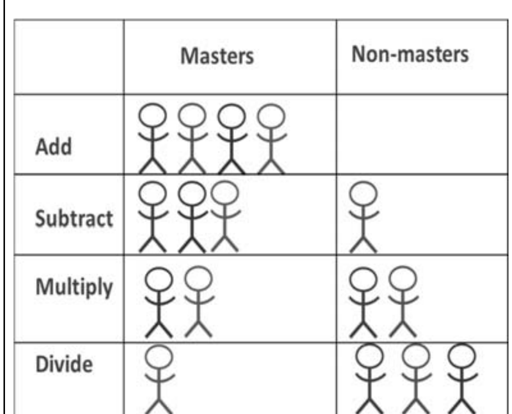
THE LCDM: AN EXAMPLE ITEM

Conceptual Example Revisited

- Our previous example:
  - Items measure combinations of the attributes:



- End goal: Statistically classify students according to these attributes:



LCDM Item Response Function

- The item response probability in the LCDM is a function of the attributes that are measured by an item:

	Add	Subtract	Multiply	Divide
16-3x4	0	1	1	0

$$\log \frac{P(X_{ri}=1\mid\alpha_r)}{P(X_{ri}=0\mid\alpha_r)} = \underbrace{\boxed{\lambda_{i,0}}}_{\text{Intercept}} + \underbrace{\boxed{\lambda_{i,1(2)}}}_{\substack{\text{Main Effect}\\ \text{(Attribute 2)}}}(\alpha_{r2}) + \underbrace{\boxed{\lambda_{i,1(3)}}}_{\substack{\text{Main Effect}\\ \text{(Attribute 3)}}}(\alpha_{r3}) + \underbrace{\boxed{\lambda_{i,2(23)}}}_{\substack{\text{Interaction}\\ \text{(Between Attributes 2 \& 3)}}}(\alpha_{r2}\cdot\alpha_{r3})$$

- Extending the ANOVA analogy, the reference group are examinees who have mastered no attributes.

LCDM Explained

$$\boxed{\mathrm{logit}(X_{ri}=1\mid\boldsymbol{\alpha}_r)} = \boxed{\lambda_{i,0}} + \lambda_{i,1,(2)}\alpha_{r2} + \lambda_{i,1,(3)}\alpha_{r3} + \lambda_{i,2,(2,3)}\alpha_{r2}\alpha_{r3}$$

- logit(X_{ri} = 1) is the logit of a correct response to item *i* by respondent *r*

- $\lambda_{i,0}$ is the intercept
  - » The logit for non-masters of subtraction and multiplication
  - » The reference group is respondents who have not mastered ***either*** attribute (α_{r2} = 0 and α_{r3} = 0)

LCDM Explained

$$\text{logit}(X_{ri}=1\mid\boldsymbol{\alpha}_r)=\lambda_{i,0}+\boxed{\lambda_{i,1,(2)}}\alpha_{r2}+\boxed{\lambda_{i,1,(3)}}\alpha_{r3}+\boxed{\lambda_{i,2,(2,3)}}\alpha_{r2}\alpha_{r3}$$

- $\lambda_{i,1,(2)}$ = **main effect** for subtraction (Attribute 2)
  - » The increase in the logit for mastering subtraction (for someone who has not also mastered multiplication)
- $\lambda_{i,1,(3)}$ = **main effect** for multiplication (Attribute 3)
  - » The increase in the logit for mastering multiplication (for someone who has not also mastered subtraction)
- $\lambda_{i,2,(2,3)}$ is the **interaction** between addition and subtraction (Attributes 2 and 3)
  - » Change in the logit for mastering ***both*** subtraction & multiplication

Understanding LCDM Notation

- The LCDM item parameters have several subscripts:

$$\lambda_{\boxed{i},\boxed{e},\boxed{(a_1,\ldots)}}$$

- Subscript #1 – *i*: the item to which parameters belong
- Subscript #2 – *e*: the level of the effect
  - » 0 is the intercept
  - » 1 is the main effect
  - » 2 is the two-way interaction
  - » 3 is the three-way interaction
  - » A is the A-way effect
- Subscript #3 – (a_1,…): the attributes to which the effect applies
  - » Same number of attributes listed as number in Subscript #2

Examinee Item Response Probabilities

- The item response probability is also a function of which attributes a student has mastered:

	+	-	x	/	$\log \frac{P(X_{ri}=1 \mid \alpha_r)}{P(X_{ri}=0 \mid \alpha_r)} =$
16-3x4	0	1	1	0	$\lambda_{i,0} + \lambda_{i,1(2)}(\alpha_{r2}) + \lambda_{i,1(3)}(\alpha_{r3}) + \lambda_{i,2(23)}(\alpha_{r2} \cdot \alpha_{r3})$

	Add	Subtract	Multiply	Divide	$\log \frac{P(X_{ri}=1 \mid \alpha_r)}{P(X_{ri}=0 \mid \alpha_r)} =$
	✓	✓	✓	✓	$\lambda_{i,0} + \lambda_{i,1(2)} + \lambda_{i,1(3)} + \lambda_{i,2(23)}$
	✓	✓	✓	✗	$\lambda_{i,0} + \lambda_{i,1(2)} + \lambda_{i,1(3)} + \lambda_{i,2(23)}$
	✓	✓	✗	✗	$\lambda_{i,0} + \lambda_{i,1(2)}$
	✓	✗	✗	✗	$\lambda_{i,0}$

Item Response Probabilities by Attribute Pattern

	+	-	x	/	$\log \frac{P(X_{ri}=1 \mid \alpha_r)}{P(X_{ri}=0 \mid \alpha_r)} =$
16-3x4	0	1	1	0	$\lambda_{i,0} + \lambda_{i,1(2)}(\alpha_{r2}) + \lambda_{i,1(3)}(\alpha_{r3}) + \lambda_{i,2(23)}(\alpha_{r2} \cdot \alpha_{r3})$

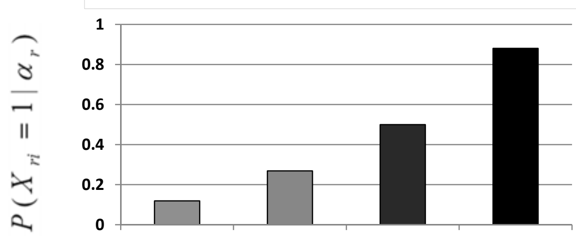
Pattern	Add	Sub.	Multiply	Divide
1	0	**0**	**0**	0
2	0	**0**	**0**	1
3	0	**0**	**1**	0
4	0	**0**	**1**	1
5	0	**1**	**0**	0
6	0	**1**	**0**	1
7	0	**1**	**1**	0
8	0	**1**	**1**	1
9	1	**0**	**0**	0
10	1	**0**	**0**	1
11	1	**0**	**1**	0
12	1	**0**	**1**	1
13	1	**1**	**0**	0
14	1	**1**	**0**	1
15	1	**1**	**1**	0
16	1	**1**	**1**	1

- For an item that measures 2 attributes, there are 4 distinct response functions for the 16 patterns:

$\boxed{\lambda_{i,0}}$ $\boxed{\lambda_{i,0} + \lambda_{i,1(2)}}$ $\boxed{\lambda_{i,0} + \lambda_{i,1(3)}}$

$\boxed{\lambda_{i,0} + \lambda_{i,1(2)} + \lambda_{i,1(3)} + \lambda_{i,2(13)}}$

- Different response functions yield different response probabilities:



LCDM with Example Numbers

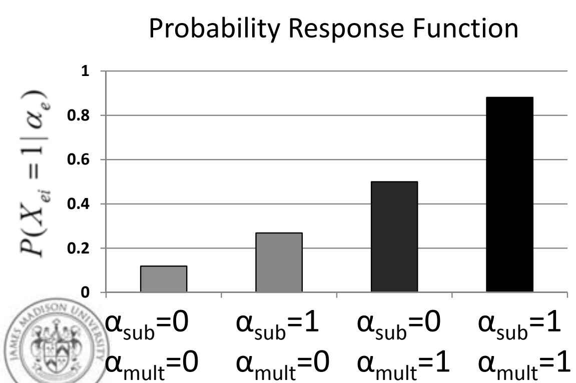
- The value of the probabilities are calculated from the item parameters
- Imagine the previous item had these item parameters:

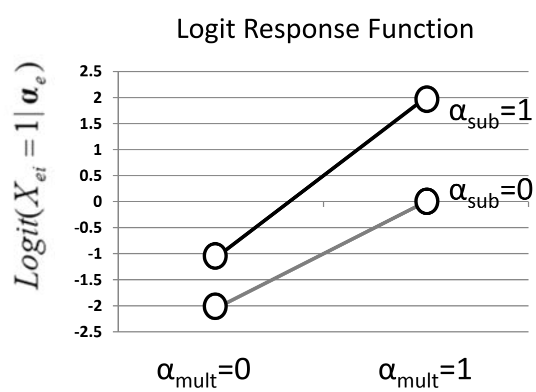
Parameter	Estimate	Effect Name
$\lambda_{i,0}$	-2	Intercept
$\lambda_{i,1,(2)}$	1	Subtraction Simple Main Effect
$\lambda_{i,1,(3)}$	2	Multiplication Simple Main Effect
$\lambda_{i,2,(2,3)}$	1	Subtraction/Multiplication Interaction

LCDM Predicted Logits and Probabilities

$$Logit(X_{ri}=1\mid\boldsymbol{\alpha}_r) = -2 + 1(\alpha_{r2}) + 2(\alpha_{r3}) + 1(\alpha_{r2}\alpha_{r3})$$

α_2	α_3	LCDM Logit Function	Logit	Probability
0	0	$\lambda_{i,0}+\lambda_{i,1,(2)}*(0)+\lambda_{i,1,(3)}*(0)+\lambda_{i,2,(2,3)}*(0)*(0)$	-2	0.12
1	0	$\lambda_{i,0}+\lambda_{i,1,(2)}*(1)+\lambda_{i,1,(3)}*(0)+\lambda_{i,2,(2,3)}*(0)*(1)$	-1	0.27
0	1	$\lambda_{i,0}+\lambda_{i,1,(2)}*(0)+\lambda_{i,1,(3)}*(1)+\lambda_{i,2,(2,3)}*(1)*(0)$	0	0.50
1	1	$\lambda_{i,0}+\lambda_{i,1,(2)}*(1)+\lambda_{i,1,(3)}*(1)+\lambda_{i,2,(2,3)}*(1)*(1)$	2	0.88





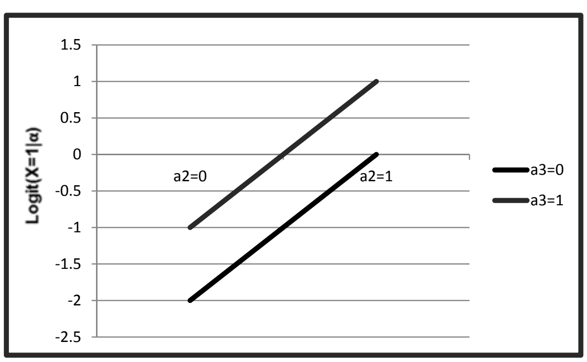
LCDM: INTERACTIONS

LCDM Interaction Plots

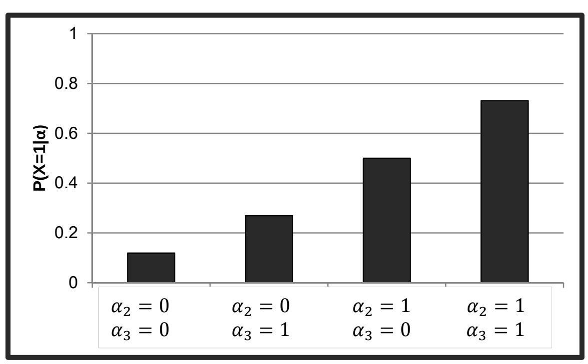
- The LCDM interaction term can be investigated via plots
- Imagine this item:
- **No interaction**
  - » parallel lines for the logit

Parameter	Estimate	Effect Name
$\lambda_{i,0}$	-2	Intercept
$\lambda_{i,1,(2)}$	2	Subtraction Simple Main Effect
$\lambda_{i,1,(3)}$	1	Multiplication Simple Main Effect
$\lambda_{i,2,(2,3)}$	0	No Interaction

Logit Response Function



Probability Response Function



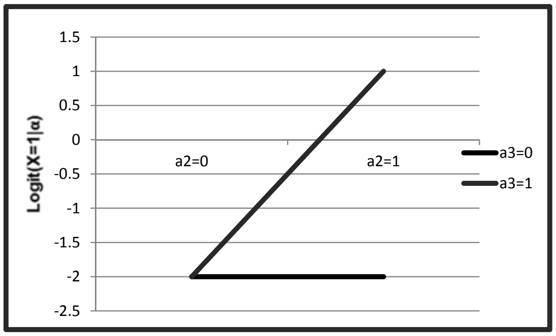
- One sub-model of the LCDM, the **Compensatory RUM** (Hartz, 2002), assumes that every item functions like this one, with no interaction

Strong Positive Interactions

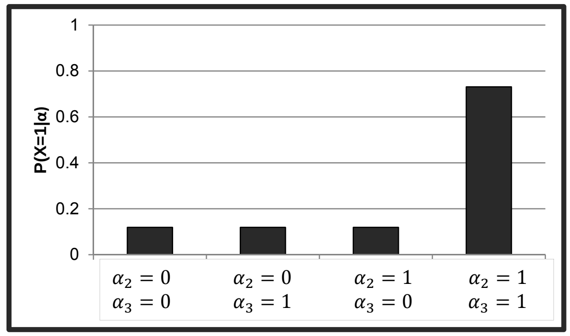
- **Positive interaction:** over-additive model
  - » No main effects
  - » Conjunctive model
    - All or none

Parameter	Estimate	Effect Name
$\lambda_{i,0}$	-2	Intercept
$\lambda_{i,1,(2)}$	0	Subtraction Simple Main Effect
$\lambda_{i,1,(3)}$	0	Multiplication Simple Main Effect
$\lambda_{i,2,(2,3)}$	3	Positive Interaction

Logit Response Function



Probability Response Function



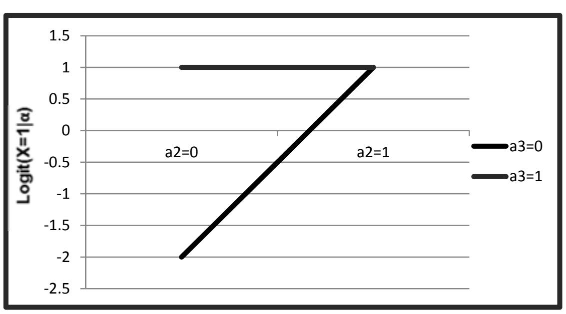
- Another sub-model of the LCDM, the **DINA** model (Haertel, 1989; Junker & Sijtsma, 1999), assumes that every item functions like this one, with a positive interaction and zero main effects

Strong Negative Interactions

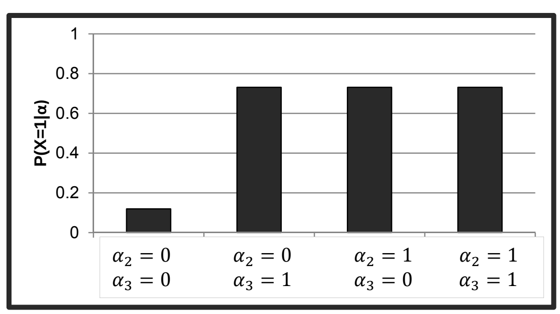
- **Negative interaction**: Under-additive logit model
  - » Disjunctive model
  - » One or more

Parameter	Estimate	Effect Name
$\lambda_{i,0}$	-2	Intercept
$\lambda_{i,1,(2)}$	3	Subtraction Simple Main Effect
$\lambda_{i,1,(3)}$	3	Multiplication Simple Main Effect
$\lambda_{i,2,(2,3)}$	-3	Negative Interaction

Logit Response Function



Probability Response Function



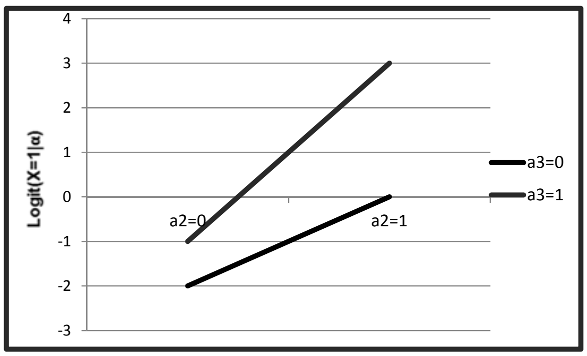
- Another sub-model of the LCDM, the **DINO** model (Templin & Henson, 2006), assumes that every item functions like this one.

Less Extreme Interactions

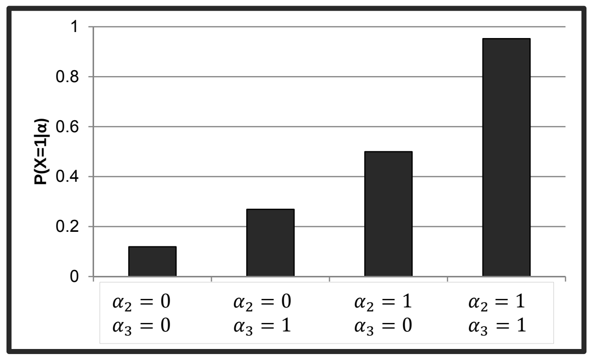
- Extreme interactions may be less likely in practice
- **Positive interaction with positive main effects:**

Parameter	Estimate	Effect Name
$\lambda_{i,0}$	-2	Intercept
$\lambda_{i,1,(2)}$	2	Subtraction Simple Main Effect
$\lambda_{i,1,(3)}$	1	Multiplication Simple Main Effect
$\lambda_{i,2,(2,3)}$	2	Positive Interaction

Logit Response Function



Probability Response Function

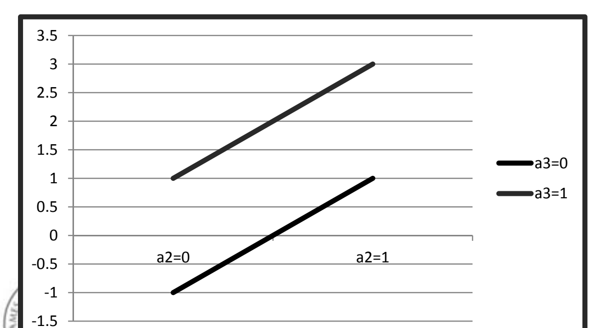


LCDM Sudoku: Beginner

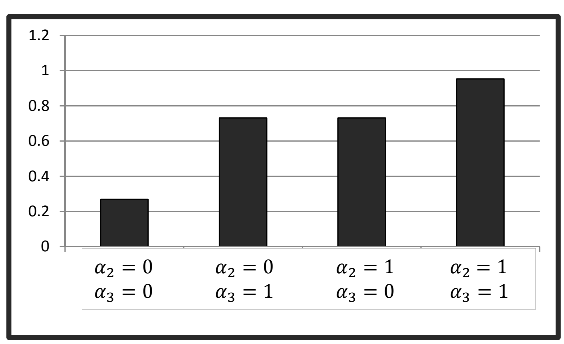
- Can you fill in the effects?

Parameter	Estimate	Effect Name
$\lambda_{i,0}$		Intercept
$\lambda_{i,1,(2)}$		Subtraction Simple Main Effect
$\lambda_{i,1,(3)}$		Multiplication Simple Main Effect
$\lambda_{i,2,(2,3)}$		Interaction

Logit Response Function



Probability Response Function

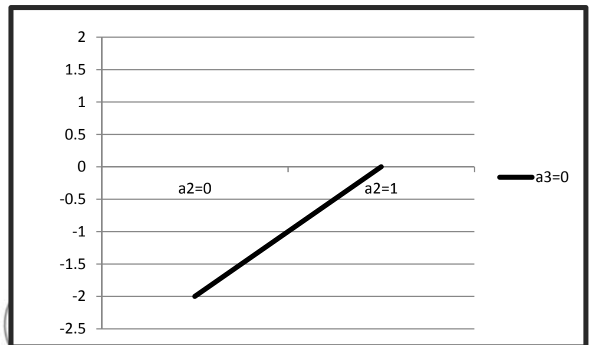


LCDM Sudoku: Expert

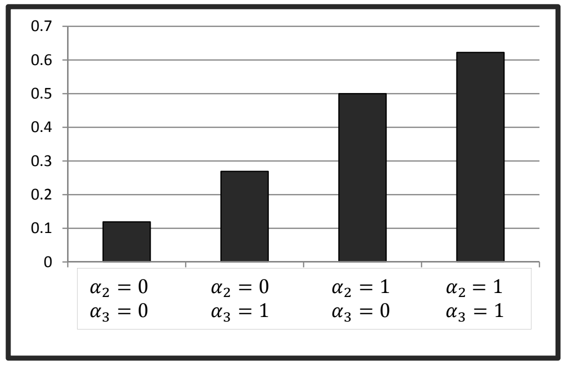
- Can you fill in the effects?

Parameter	Estimate	Effect Name
$\lambda_{i,0}$		Intercept
$\lambda_{i,1,(2)}$		Subtraction Simple Main Effect
$\lambda_{i,1,(3)}$		Multiplication Simple Main Effect
$\lambda_{i,2,(2,3)}$		Interaction

Logit Response Function



Probability Response Function



Previously Popular DCMs

- Because the advent of the GDM and LCDM has been fairly recent, other earlier DCMs are still in use

- Such DCMs are much more restrictive than the LCDM
  - » Not discussed at length here
  - » It is anticipated that field will adapt to more general forms

- Each of these models can be fit using the LCDM
  - » Fixing certain model parameters

- Shown for reference purposes
  - » See Henson, Templin, & Willse (2009) for more detail

Other DCMs with the LCDM

- The Big 6 - DCMs with latent variables:
  - » **DINA** (Deterministic Inputs, Noisy 'AND' Gate)
    - Haertel (1989); Junker and Sijtsma (1999)
  - » **NIDA** (Noisy Inputs, Deterministic 'AND' Gate)
    - Maris (1995)
  - » **RUM** (Reparameterized Unified Model)
    - Hartz (2002)
  - » **DINO** (Deterministic Inputs, Noisy 'OR' Gate)
    - Templin & Henson (2006)
  - » **NIDO** (Noisy Inputs, Deterministic 'OR' Gate)
    - Templin (2006)
  - » **C-RUM** (Compensatory Reparameterized Unified Model)
    - Hartz (2002)

Other DCMs with the LCDM

LCDM Parameters	Non-compensatory Models			Compensatory Models		
	DINA	NIDA	NC-RUM	DINO	NIDO	C-RUM
Main Effects	Zero	Positive	Positive	Positive	Positive	Positive
Interactions	Positive	Positive	Positive	Negative	Zero	Zero
Parameter Restrictions	Across Attributes	Across Items	---	Across Attributes	Across Items	---

Adapted from: Rupp, Templin, and Henson (2010)

GENERAL FORM OF THE LCDM

More General Versions of the LCDM

- The LCDM is based on the General Diagnostic Model by von Davier (GDM; 2005)
  - » The GDM allows for both categorical and continuous latent variables
  - » The GDM does not specify interactions among latent variables
- The LCDM appears in the psychometric literature in a more general form (Henson, Templin, & Willse, 2009)
  - » For items measuring more than two attributes, higher level interactions are possible
    - Difficult to estimate in practice

General Form of the LCDM

- The LCDM specifies the probability of a correct response as a function of a set of attributes and a Q-matrix:

$$P(X_{ri} = 1 \mid \boldsymbol{\alpha}_r) = \frac{e^{\lambda_{i,0} + \boldsymbol{\lambda}_i^T \mathbf{h}(\mathbf{q}_i, \boldsymbol{\alpha}_r)}}{1 + e^{\lambda_{i,0} + \boldsymbol{\lambda}_i^T \mathbf{h}(\mathbf{q}_i, \boldsymbol{\alpha}_r)}}$$

- The term in the exponent is the logit we have been using all along

Logit($X_{ri} = 1 \mid \boldsymbol{\alpha}_r$)

$$\lambda_{i,0} + \boldsymbol{\lambda}_i^T \mathbf{h}(\mathbf{q}_i, \boldsymbol{\alpha}_r) =$$

$$\lambda_{i,0} + \sum_{u=1}^{A} \lambda_{i,1,(u)} (\alpha_{ru} q_{iu}) + \sum_{u=1}^{A-1} \sum_{v=u+1}^{A} \lambda_{i,2,(u,v)} (\alpha_{ru} \alpha_{rv} q_{iu} q_{iv}) + \cdots$$

Intercept

Main Effects

Two-Way Interactions

Higher Interactions

An Example of an Item Measuring Three Attributes: An Illustration of the LCDM

	Add	Subtract	Multiply	Divide
16-3x4÷6	0	1	1	1

$$\log \frac{P(X_{ri} = 1 \mid \alpha_r)}{P(X_{ri} = 0 \mid \alpha_r)} = \lambda_{i,0} + \boldsymbol{\lambda}_i^T \mathbf{h}(\mathbf{q}_i, \boldsymbol{\alpha}_r)$$

$$\lambda_{i,0} + \lambda_{i,1(2)}(\alpha_{r2}) + \lambda_{i,1(3)}(\alpha_{r3}) + \lambda_{i,1(4)}(\alpha_{r4}) + \lambda_{i,2(23)}(\alpha_{r2} \cdot \alpha_{r3}) +$$
$$\lambda_{i,2(24)}(\alpha_{r2} \cdot \alpha_{r4}) + \lambda_{i,2(34)}(\alpha_{r3} \cdot \alpha_{r4}) + \lambda_{i,3(234)}(\alpha_{r2} \cdot \alpha_{r3} \cdot \alpha_{r4})$$

- Let’s break it down.

An Example of an Item Measuring Three Attributes: An Illustration of the LCDM

	Add	Subtract	Multiply	Divide
16-3x4÷6	0	1	1	1

$$\log \frac{P(X_{ri}=1 \mid \alpha_r)}{P(X_{ri}=0 \mid \alpha_r)} =$$

$$\lambda_{i,0} +$$ Intercept

$$\lambda_{i,1(2)}(\alpha_{r2}) + \lambda_{i,1(3)}(\alpha_{r3}) + \lambda_{i,1(4)}(\alpha_{r4}) +$$ A main effect for each individual attribute

$$\lambda_{i,2(23)}(\alpha_{r2} \cdot \alpha_{r3}) + \lambda_{i,2(24)}(\alpha_{r2} \cdot \alpha_{r4}) + \lambda_{i,2(34)}(\alpha_{r3} \cdot \alpha_{r4})$$ Three 2-way interactions between all pairs of attributes

$$+\lambda_{i,3(234)}(\alpha_{r2} \cdot \alpha_{r3} \cdot \alpha_{r4})$$ One 3-way interaction between all three attributes

General Form of the LCDM

- Logit for general form of LCDM:

$$\lambda_{i,0} + \boldsymbol{\lambda}_i^T \mathbf{h}(\mathbf{q}_i, \boldsymbol{\alpha}_r) = \lambda_{i,0} + \sum_{u=1}^{A} \lambda_{i,1,(u)}(\alpha_{ru} q_{iu}) + \sum_{u=1}^{A-1} \sum_{v=u+1}^{A} \lambda_{i,2,(u,v)}(\alpha_{ru}\alpha_{rv} q_{iu} q_{iv}) + \ldots$$

- Consider a test that measures three attributes.

$\boldsymbol{\lambda}_i = (2^A - 1)\ x\ 1$ matrix

$\boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r) = (2^A - 1)\ x\ 1$ matrix

$$\boldsymbol{\lambda}_i = \begin{bmatrix} \lambda_{i,1(1)} \\ \lambda_{i,1(2)} \\ \lambda_{i,1(3)} \\ \lambda_{i,2(1,2)} \\ \lambda_{i,2(1,3)} \\ \lambda_{i,2(2,3)} \\ \lambda_{i,3(1,2,3)} \end{bmatrix} \qquad \boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r) = \begin{bmatrix} q_{i1}\alpha_{r1} \\ q_{i2}\alpha_{r2} \\ q_{i3}\alpha_{r3} \\ q_{i1}\alpha_{r1}q_{i2}\alpha_{r2} \\ q_{i1}\alpha_{r1}q_{i3}\alpha_{r3} \\ q_{i2}\alpha_{r2}q_{i3}\alpha_{r3} \\ q_{i1}\alpha_{r1}q_{i2}\alpha_{r2}q_{i3}\alpha_{r3} \end{bmatrix}$$

A cell equals one only if 1) the item measures the attribute(s) and 2) the respondent has mastered the attribute (s).

$\boldsymbol{\lambda}_i^T = 1\ x\ (2^A - 1)$ matrix

$\boldsymbol{\lambda}_i^T \boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r) = 1\ x\ 1$ matrix or a scalar

Concluding Remarks

Session2 – Take-Home Points

- The LCDM uses an ANOVA-like approach to map latent attributes onto item responses
  - » Uses main effects and interactions for each attribute
  - » Uses a logit link function
- Multiple diagnostic models are subsumed by the LCDM

After Lunch

- Session 3: Applications
  - » Demonstration of the LCDM in practice
    - Educational measurement example (English Language):
      - ♦ Estimation specifications
      - ♦ Parameter interpretation/model reduction
      - ♦ Inferences made
- Session 4: Diagnostic Classification Structural Models
- Session 5: Model Fit