The LCDM

NCME 2009
Workshop

Introduction

- Now we want to talk about a specific Diagnostic Model for classification
  - The LCDM
- I will provide a brief motivation
- Define the LCDM
- A brief discussion of interpretation and the Compensatory RUM

Introduction

- So previously, we have discussed
  - The purpose of Diagnostic Classification Models
  - Definition of Attributes
  - Definition of the Q-matrix
- We also noted that the combination of an examinee's attribute and the Q-matrix of an item help determine whether a person should correctly respond to an item

Introduction

- Although there are several models in the literature (to be discussed later) we will discuss a general model
- The LCDM (Log-Linear Cognitive Diagnosis Model) has its roots in a typical linear model (Log-Linear)
- Therefore, we first motivate the LCDM using a basic example rooted in a 2-Way ANOVA
- We will then use this example to introduce the LCDM

ANOVA

- Specifically, lets imagine that I am interested in measuring a work output. As a researche,r I believe that *work output* may be
  - Effected based on the lighting of the office
    - High or Low
  - Based on the temperature
    - Cold or Warm
- So I set up an experimental design for a 2-Way ANOVA

ANOVA

- So, I set up a 2 x 2 Factor design such that:

	Low Lights	High Lights
Cold	$\bar{Y}_{Cold,Low}$	$\bar{Y}_{Cold,High}$
Warm	$\bar{Y}_{Warm,Low}$	$\bar{Y}_{Warm,High}$

$$Y_{ijk} = \mu_0 + \alpha_j + \beta_k + \gamma_{jk} + e_{ijk}$$

ANOVA

- In doing this we would assess whether there was
  - A main effect associated with *Temperature* (α_j)
  - A main effect associated with *Lights* (β_k)
  - An interaction effect associated with *Temperature* and *Lights* (γ_{jk})

$$Y_{ijk} = \mu_0 + \alpha_j + \beta_k + \gamma_{jk} + e_{ijk}$$

ANOVA

- We note that this could be re-written another way because there are only two categories of lights and two categories of Temperature

- We could redefine two dummy variables

- D_{temp} where $D_{temp} = 0$ for cold and $D_{temp} = 1$ for warm

- D_{light} where $D_{light} = 0$ for Low and $D_{light} = 1$ for high

ANOVA

- So, I set up a 2 x 2 Factor design such that:

	$D_{light}=0$ Low Lights	$D_{light}=1$ High Lights
$D_{temp}=0$ Cold	$\bar{Y}_{Cold,Low}$	$\bar{Y}_{Cold,High}$
$D_{temp}=1$ Warm	$\bar{Y}_{Warm,Low}$	$\bar{Y}_{Warm,High}$

$$Y_{ijk} = \beta_0 + \beta_{temp} D_{temp} + \beta_{light} D_{light} + \beta_{t*l} D_{temp} D_{light} + e_{itl}$$

ANOVA

$$Y_{ijk} = \beta_0 + \beta_{temp} D_{temp} + \beta_{light} D_{light} + \beta_{t*l} D_{temp} D_{light} + e_{itl}$$

- β_0 is the mean for the cold and low light condition
- β_{temp} is the change of the mean when comparing cold to warm temperature for a business with low lights (Main Effect)
- β_{light} is the change of the mean when comparing low to high lights for a business with a cold temperature (Main Effect)
- β_{t*l} is additional mean change that is not explained by the shift in temperature and shift and lights, when both occur (Interaction)

The LCDM

- Obviously this is not quite a diagnostic model, but the basic idea can be used to define a diagnostic model
- That is, we can define the change in a particular outcome as a function of a set of main effects and interactions
  - Where possible, we may look for way to reduce the model (i.e., eliminate interactions)
- So now we focus on the ways this is different and then the similarities to a diagnostic model

The LCDM

- Specifically, the diagnostic models that we are focusing on differ in two characteristics
  - Instead of a continuous outcome such as work output we would like to model the probability of a correct response
  - Instead of "factors" that are described by observed characteristics such as high and low lights we focus on discrete *latent* variables that are out attributes.

The LCDM (Response)

- The LCDM is still a linear model but instead it is a special case of a log-linear model with *latent classes* (Hagenaars, 1993) and thus is also a special case of the General Diagnostic Model (GDM, von Davier, 2005).

- The LCDM defines the Logit of the probability of a correct response as a linear function of the attributes that have been mastered.

The LCDM

- The Logit of a probability is used because the items are dichotomous (correct or incorrect).
  - A linear model is not reasonable for probabilities because of the bounds, so we transform the probability.

$$Logit = \ln\left(\frac{P(X_{ij}=1)}{1-P(X_{ij}=1)}\right)$$

- The Logit ranges from a negative infinity to a positive infinity when using probabilities and so can be modeled as a linear function.

The LCDM (Response)

- So, based on the differences in responses we are focusing on:

		$D_{light}=0$ Low Lights	$D_{light}=1$ High Lights
$D_{temp}=0$	Cold	$\text{Logit}(p_{tl})$	$\text{Logit}(p_{tl})$
$D_{temp}=1$	Warm	$\text{Logit}(p_{tl})$	$\text{Logit}(p_{tl})$

- So we can now think about the way that our effects change the log-odds/

The LCMD (Latent Variables)

- Secondly, we no longer have observable variables, but instead we are interested in the effect of mastery (or nonmastery) on the log-odds
  - Notice that if the log-odds increases then the probability of a correct response must also increase
  - So by modeling the *logit* we are really focusing on the change in the probability of a correct response
- Thus our original dummy variables change to α
  - Mastery of nonmastery of each attribute (measured by the item)

The LCMD (Latent Variables)

- So, replacing our conceptual example with the attributes we get

	$\alpha_{sub}=0$ Nonmaster of Subtraction	$\alpha_{sub}=1$ Master of Subtraction
$\alpha_{add}=0$ Nonmaster of Addition	$\text{Logit}(p_{tl})$	$\text{Logit}(p_{tl})$
$\alpha_{add}=1$ *Mastery of Addition*	$\text{Logit}(p_{tl})$	$\text{Logit}(p_{tl})$

- Here we model how the logit of the probability of a correct response depends on master or nonmastery of two different attributes

The Q-matrix and the LCDM

- The last portion of the LCDM that was not discussed in our conceptual example is the Q-matrix
- The Q-matrix defines which attributes are measured by any given item
- If the Q-matrix equals zero, the attribute is not measured and so mastery (or nonmastery) of that attribute does not have an effect on the probability of a correct response

The Q-matrix and the LCDM

- Therefore, the effects of any attribute that is not measured by an attribute must have a zero effect

- So for an item such as 4+1-2 the LCDM would be:

$$\ln\left(\frac{P(X_{ij}=1|\alpha)}{1-P(X_{ij}=1|\alpha)}\right)=\lambda_{i,0}+\lambda_{i,(add)}\alpha_{j,add}+\lambda_{i,(sub)}\alpha_{sub}+\lambda_{i,(add*sub)}\alpha_{add}\alpha_{sub}$$

- Notice here we have changed our notation, but the model is still the same

The Q-matrix and the LCDM

- Where as, if an item only required addition, such as 4+1=? then the LCDM would be

$$\ln\left(\frac{P(X_{ij}=1|\alpha)}{1-P(X_{ij}=1|\alpha)}\right)=\lambda_{i,0}+\lambda_{i,(add)}\alpha_{j,add}$$

- Often the LCDM is written in terms of the probability

$$P(X_{ij}=1|\alpha)=\frac{\exp(\lambda_{i,0}+\lambda_{i,(add)}\alpha_{j,add})}{1+\exp(\lambda_{i,0}+\lambda_{i,(add)}\alpha_{j,add})}$$

The LCDM

- So generally speaking the LCDM is defined as:

$$P(X_{ij} = 1 | \alpha_i) = \frac{e^{\boldsymbol{\lambda}_j^T \mathbf{h}(q_j, \alpha_i)}}{1 + e^{\boldsymbol{\lambda}_j^T \mathbf{h}(q_j, \alpha_i)}}$$

- Where

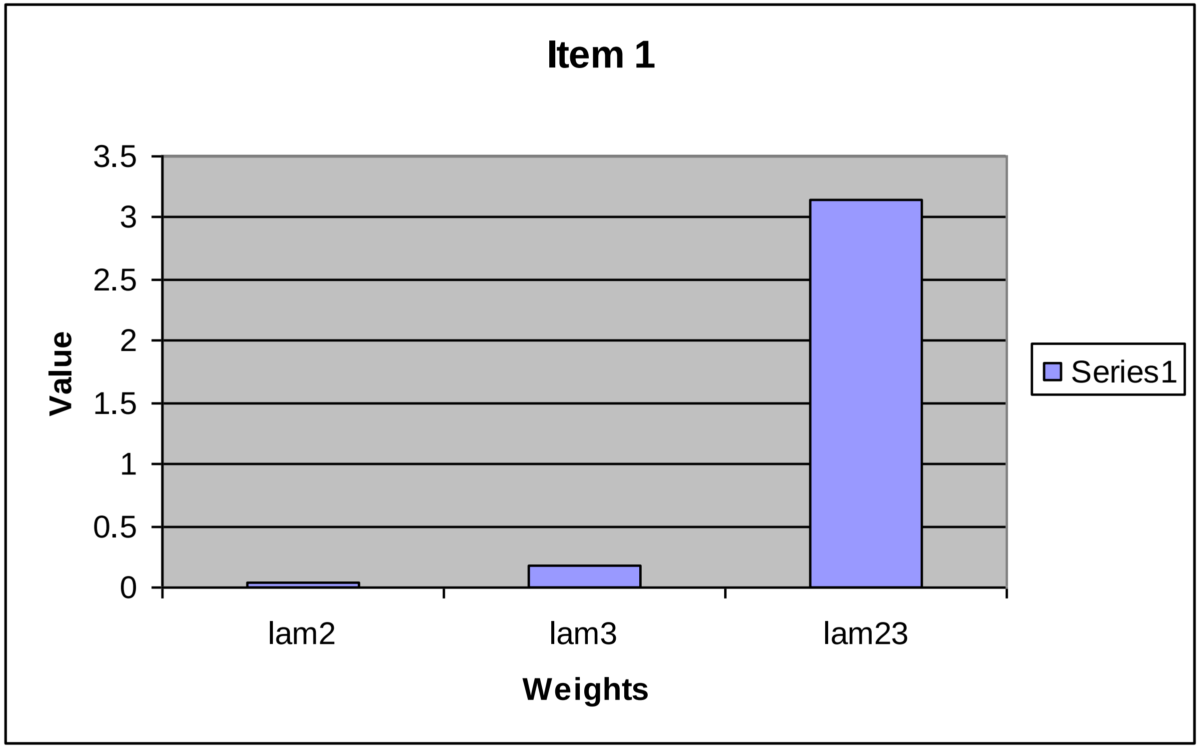
$$\boldsymbol{\lambda}_j^T \mathbf{h}(q_j, \alpha_i) = \lambda_0 + \sum_{u=1}^{K} \lambda_{ju} \left(\alpha_u q_{ju} \right) + \sum_{u=1}^{K} \sum_{v>u} \lambda_{juv} \left(\alpha_{iu} \alpha_{iv} q_{ju} q_{jv} \right) + \ldots$$

The LCDM

- In general, the LCDM is a linear model that predicts the change the in the logit of the probability of a correct response as a function of mastery or nonmastery of the required attributes

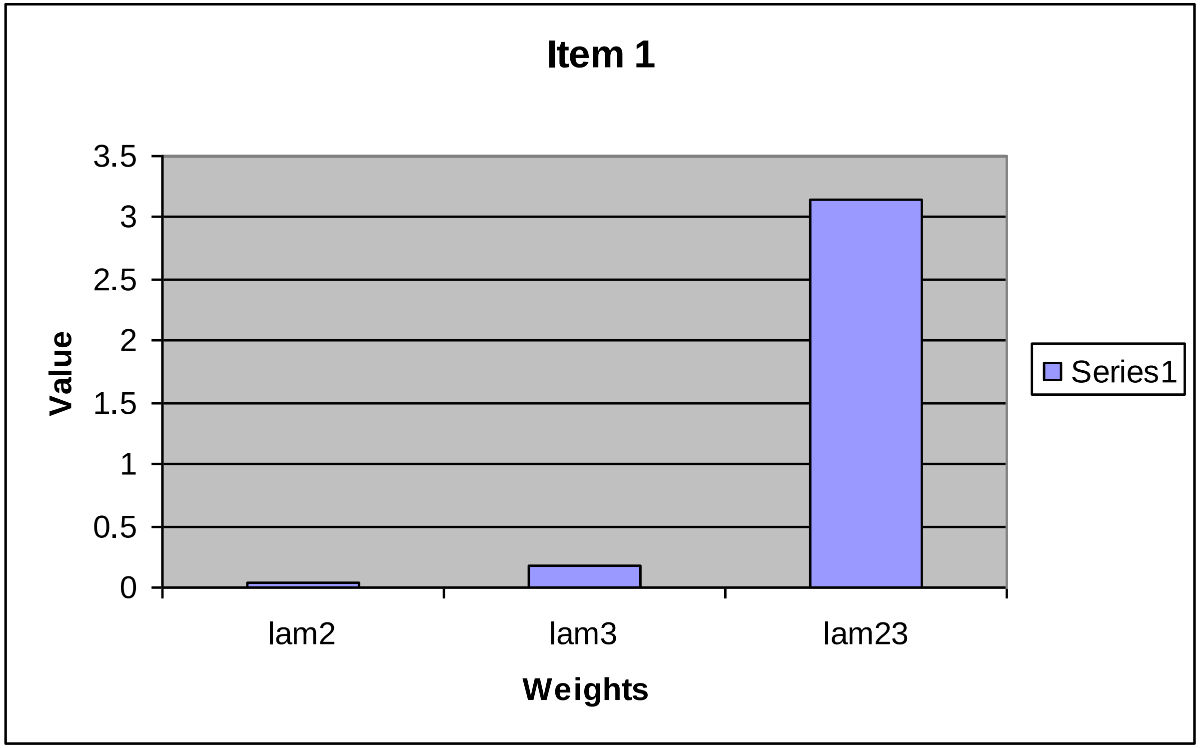
- Because of its similarities to any other linear model, the importance of any attribute can be interpreted in a similar way as with an ANOVA
  - First evaluate high-order interaction
  - Then evaluate in terms of change in log-odds or in terms odds ratios

LCDM Example



LCDM Example

- No strong main effects
- Strong positive interaction (exp(3.1)=22.2 times more likely to correct answer this item)



A Flexible Model

- In fact, because of its flexibility, the LCDM can fit many of the models that have been previously defined in the literature
  - DINA, DINO, and reduced RUM (to be discussed next)

- As a specific case, imagine that there are no interactions and so there are only main effects.

  - This is a case commonly referred to as the Compensatory RUM

The Compensatory RUM

- Because there are only main effects $\exp(\lambda_{i,(a)})$ describes the increase in the odds of correctly responding to the item.

$$P(X_{ij}=1|\alpha_i)=\frac{e^{\lambda_{i,0}+\sum_{k=1}^{K}\lambda_{i,(a)}\alpha_k q_{ik}}}{1+e^{\lambda_{i,0}+\sum_{k=1}^{K}\lambda_{i,(a)}\alpha_k q_{ik}}}$$

Summary

- By defining the LCDM in this way a general diagnostic model is defined that allows for any "type" of cognition to directly be modeled
- In addition, because it is rooted in a similar mind set as ANOVA, one could develop the model and interpret the model in a similar way
  - Try to reduce the model based on interactions
  - Interpret in a similar way as a logistic model
- Questions?