Estimation of Cognitive Diagnosis Model Parameters: A Software Demonstration

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Introduction

- Cognitive diagnosis models are latent class models with a set equality constraints placed on class probabilities.
  - Latent classes are defined by a set of dichotomous skills.
- Because they are constrained versions of latent class models, software (such as Mplus) exists that is capable of estimating a wide range of such models.
- In this session, you will learn how to use Mplus (and an auxiliary program, CDM) to estimate common models for cognitive diagnosis.

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Latent Class Analysis

- Latent class models are commonly attributed to Lazarsfeld and Henry (1968).
  - Latent class models are but one type of Finite Mixture Models (FMM) where the data is categorical and item responses are independent given class.
- Use of latent class models in educational measurement date to Macready and Dayton (1977).
- Commonly used cognitive diagnosis models are constrained versions of latent class models.
  - Understanding the basics of latent class analysis will be useful in understanding how we can estimate cognitive models using software that estimates latent class models (Mplus).

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Latent Class Analysis as a FMM

Using some notation of Bartholomew and Knott (1999), a latent class model for the response vector of individual i ($i = 1, \ldots, I$), with J variables ($j = 1, \ldots, J$) and C classes ($c = 1, \ldots, C$):

$$P(\mathbf{x}_i) = \sum_{c=1}^{C} \overset{\text{Structural}}{\eta_c} \overbrace{\prod_{j=1}^{J} \pi_{jc}^{x_{ij}} \left(1 - \pi_{jc}\right)^{1-x_{ij}}}^{\text{Measurement}}$$

- η_c is the probability that any individual is a member of class c (must sum to one).
- x_{ij} is the observed response of individual i to item j.
- π_{jc} is the probability of a positive response to item j from an individual from class c, or $P(x_{ij} = 1|c) = \pi_{jc}$

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LCA Example

- To illustrate the process of LCA, we will use an example presented in Bartholomew and Knott (p. 142).
  - The data are from a four-item test analyzed with an LCA by Macready and Dayton (1977).
- Recall, we have three pieces of information we can gain from an LCA:
  - Sample information - proportion of people in each class.
  - Item information - probability of correct response for each item from examinees from each class.
  - Examinee information - posterior probability of class membership for each examinee in each class.

```
TITLE:
    This is an analysis of Macready
    and Dayton’s data (1977).;
DATA:
    FILE IS mcdata.dat;
    FORMAT is 4I1;
VARIABLE:
    NAMES ARE x1-x4;
    CLASSES = c(2);
    CATEGORICAL = x1-x4;
ANALYSIS:
    TYPE = MIXTURE;
PLOT:
    type=plot3;
    series is x1(1) x2(2) x3(3) x4(4);
OUTPUT:
    TECH1 TECH8;
```

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LCA Parameter Estimates

- Structural parameters (representing the probability any given examinee falls into a given class):

Class	Parameter	Estimate
1	η_1	0.587
2	η_2	0.413

- Item parameters (representing the probability of getting an item correct, conditional on being in a given class):

	Item			
Class	1	2	3	4
1	$\pi_{11} = 0.753$	$\pi_{21} = 0.780$	$\pi_{31} = 0.432$	$\pi_{41} = 0.708$
2	$\pi_{12} = 0.209$	$\pi_{22} = 0.068$	$\pi_{32} = 0.018$	$\pi_{42} = 0.052$

Interpreting Classes

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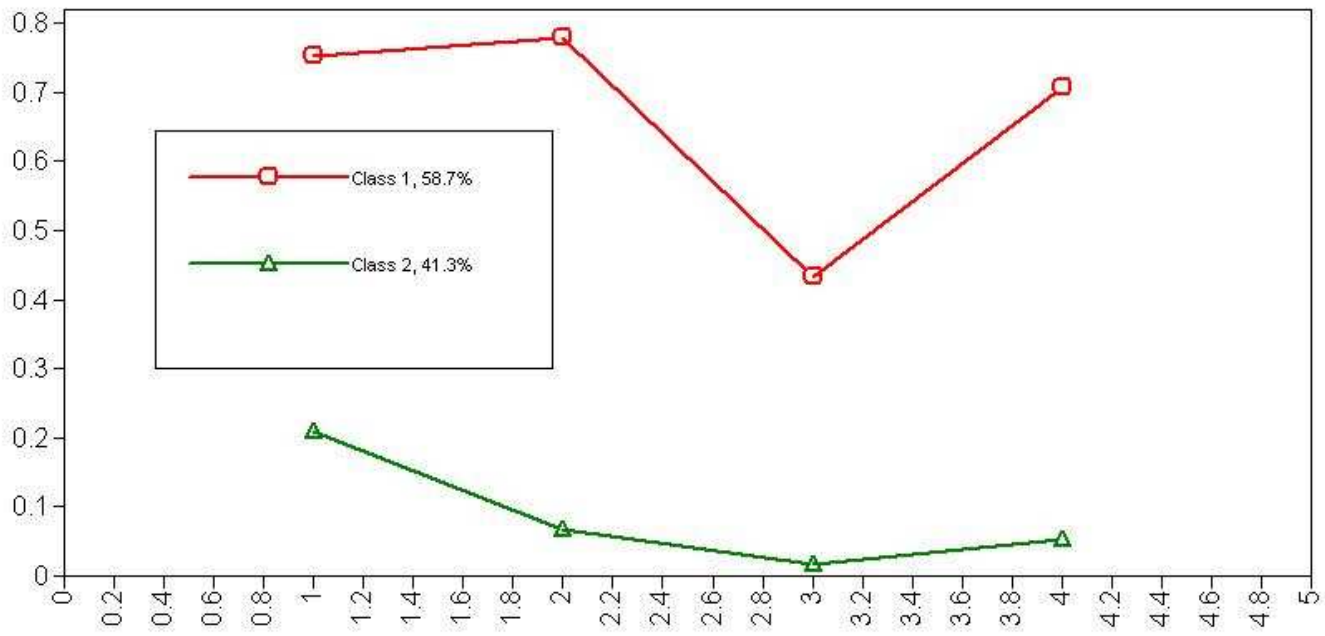

- After the analysis is finished, we need to examine the item probabilities to gain information about the characteristics of the classes.
- An easy way to do this is to look at a chart of the item response probabilities by class.

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Examine Parameter Estimates

- Examinee estimates are found in the form of posterior probabilities of class membership (denoted by $\hat{\alpha}_c$):

Examinee	Responses	$\hat{\alpha}_1$	$\hat{\alpha}_2$	ML Class	Max $\hat{\alpha}$
73	1000	0.177	0.823	2	0.823
76	0111	0.999	0.001	1	0.999
80	0110	0.974	0.026	1	0.974
82	0101	0.975	0.025	1	0.975
87	0100	0.474	0.526	2	0.526

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LCA Limitations

- LCA has limitations which make its application to educational measurement difficult:
  - Classes not known prior to analysis.
  - Class characteristics not know until after analysis.
- Both of these problems are related to LCA being an exploratory procedure for understanding data.
- Some models for cognitive diagnosis are more confirmatory versions of LCA:
  - Specifying what our classes mean prior to analysis (skill patterns).
  - Placing constraints on the class item probabilities according to the types of skills needed to correctly answer each item.

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Concepts

To demonstrate CDM concepts, imagine a test covering basic math:

1.) $2 + 3 - 1$ 2.) $4 \div 2$ 3.) $3 \times (4 - 2)$

- Using traditional assessment methods, an individual's score, or general math ability, could be estimated.
- Instead, math ability can be expressed as a set of basic skills (commonly called attributes):
  - Add
  - Subtract
  - Multiply
  - Divide
- Cognitive diagnosis models estimate a profile of the skills an individual has mastered.

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Example Q-Matrix

Math Test Example Q-matrix

	Add	Sub	Mult	Div
$2+3-1$	1	1	0	0
$4/2$	0	0	0	1
$3\times(4-2)$	0	1	1	0

- Unlike IRT, not every item measures each attribute.
- A Q-matrix indicates which attributes are measured by each item.
- Notice that the Q-matrix defines the nature of each attribute.

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Notation

I denotes the total number of examinees.

J denotes the total number of items.

K denotes the total number of attributes.

Q has elements q_{jk} that indicate whether mastery of the kth attribute is required by the jth item

$q_{j\cdot}$ denotes the number of attributes measured by item j:

$$q_{j\cdot} = \sum_{k=1}^{K} q_{jk}$$

$\boldsymbol{\alpha_i}$ is a set of indicators of attribute mastery for examinee i for all K attributes.

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CDM are a Subset of Latent Class Models

A latent class model for the response vector of individual i:

$$P(\mathbf{x}_i) = \sum_{c=1}^{C} \overset{\text{Structural}}{\eta_c} \overbrace{\prod_{j=1}^{J} \pi_{jc}^{x_{ij}} \left(1 - \pi_{jc}\right)^{1-x_{ij}}}^{\text{Measurement}}$$

Cognitive Diagnosis Models are constrained latent class models:

- For K skills, total of 2^K classes are defined.
- The Q-matrix specifies the set of latent traits necessary for each item (like the factor pattern matrix in CFA).
- Equality constraints (measurement portion of model - π_{jc}) determined by:
  - Choice of model.
  - Q-matrix specifications.

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Important Distinctions in CDM

- An important distinction in commonly used CDM is that of the model being either conjunctive or disjunctive.
- **Conjunctive models** (such as the to-be-presented DINA, NIDA, and RUM) model item responses by stating that the probability that an examinee correctly responds to an items that requires a skill an examinee has not mastered cannot be improved by some other required skill that an examinee has mastered.
  - This type of model works well with strictly defined skills, such as types of mathematics skills.
- **Disjunctive models** state the opposite, that the response probability of an item can be equally high if another skill has been mastered (can be thought of as having another skill be able to compensate for the lacked skill).
  - This type of model works well with more coarsely defined skills, such as reading or writing skills.

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Class Number Conversions

- To use Mplus to estimate common CDM, we must first make one small note about class numbering.
- In CDM classes consist of all possible combinations of skill patterns an examinee may possess.
- In traditional LCA, a set of classes (numbered $c = 1, \ldots, C$) are defined.
- We note that every given integer can be converted to a binary sequence (representing a skill pattern).

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Class Number Conversions

- For example, consider a Q-matrix with $K = 4$ skills (which translates into $2^K = 16$ possible skill patterns - or latent classes):

C	$\boldsymbol{\alpha}_c$		C	$\boldsymbol{\alpha}_c$
1	0000		9	1000
2	0001		10	1001
3	0010		11	1010
4	0011		12	1010
5	0100		13	1100
6	0110		14	1101
7	0110		15	1110
8	0111		16	1111

LCA Item Response Probabilities

- Imagine an LCA where an exponential number of classes is defined:

  Total Attributes: 4

  Total Classes: 16

  Model: LCA

  Q-matrix: None

  Equivalence Classes Per Item: None

- There is nothing to keep some skill pattern response patterns from exceeding others.

Equal $P(X_{ij} = 1|c)$ Denoted By Symbol

		Item		
C	$\boldsymbol{\alpha}$	1	2	3...
1	[0000]	$\pi_{1,1}$	$\pi_{2,1}$	$\pi_{3,1}$
2	[0001]	$\pi_{1,2}$	$\pi_{2,2}$	$\pi_{3,2}$
3	[0010]	$\pi_{1,3}$	$\pi_{2,3}$	$\pi_{3,3}$
4	[0011]	$\pi_{1,4}$	$\pi_{2,4}$	$\pi_{3,4}$
5	[0100]	$\pi_{1,5}$	$\pi_{2,5}$	$\pi_{3,5}$
6	[0101]	$\pi_{1,6}$	$\pi_{2,6}$	$\pi_{3,6}$
7	[0110]	$\pi_{1,7}$	$\pi_{2,7}$	$\pi_{3,7}$
8	[0111]	$\pi_{1,8}$	$\pi_{2,8}$	$\pi_{3,8}$
9	[1000]	$\pi_{1,9}$	$\pi_{2,9}$	$\pi_{3,9}$
10	[1001]	$\pi_{1,10}$	$\pi_{2,10}$	$\pi_{3,10}$
11	[1010]	$\pi_{1,11}$	$\pi_{2,11}$	$\pi_{3,11}$
12	[1011]	$\pi_{1,12}$	$\pi_{1,12}$	$\pi_{1,12}$
13	[1100]	$\pi_{1,13}$	$\pi_{1,13}$	$\pi_{1,13}$
14	[1101]	$\pi_{1,14}$	$\pi_{1,14}$	$\pi_{1,14}$
15	[1110]	$\pi_{1,15}$	$\pi_{2,15}$	$\pi_{3,15}$
16	[1111]	$\pi_{1,16}$	$\pi_{2,16}$	$\pi_{3,16}$

CDM Item Response Probabilities

- By contrast, a CDM equates certain cells:

Total Attributes: 4

Total Classes: 16

Model: DINA

Q-matrix:

	α_1	α_2	α_3	α_4
Item	Add	Sub	Mult	Div
$1.)\ 2+3-1$	1	1	0	0
$2.)\ 4/2$	0	0	0	1
$3.)\ 3\times(4-2)$	0	1	1	0

Equivalence Classes Per Item: 2

Item	1	2	3
Probability Parameters	2	2	2

Equal $P(X_{ij}=1|c)$ Denoted By *Color*

C	$\boldsymbol{\alpha}$	Item 1	Item 2	Item 3...
1	[0000]	$\pi_{1,1}$	$\pi_{2,1}$	$\pi_{3,1}$
2	[0001]	$\pi_{1,2}$	$\pi_{2,2}$	$\pi_{3,2}$
3	[0010]	$\pi_{1,3}$	$\pi_{2,3}$	$\pi_{3,3}$
4	[0011]	$\pi_{1,4}$	$\pi_{2,4}$	$\pi_{3,4}$
5	[0100]	$\pi_{1,5}$	$\pi_{2,5}$	$\pi_{3,5}$
6	[0101]	$\pi_{1,6}$	$\pi_{2,6}$	$\pi_{3,6}$
7	[0110]	$\pi_{1,7}$	$\pi_{2,7}$	$\pi_{3,7}$
8	[0111]	$\pi_{1,8}$	$\pi_{2,8}$	$\pi_{3,8}$
9	[1000]	$\pi_{1,9}$	$\pi_{2,9}$	$\pi_{3,9}$
10	[1001]	$\pi_{1,10}$	$\pi_{2,10}$	$\pi_{3,10}$
11	[1010]	$\pi_{1,11}$	$\pi_{2,11}$	$\pi_{3,11}$
12	[1011]	$\pi_{1,12}$	$\pi_{1,12}$	$\pi_{1,12}$
13	[1100]	$\pi_{1,13}$	$\pi_{1,13}$	$\pi_{1,13}$
14	[1101]	$\pi_{1,14}$	$\pi_{1,14}$	$\pi_{1,14}$
15	[1110]	$\pi_{1,15}$	$\pi_{2,15}$	$\pi_{3,15}$
16	[1111]	$\pi_{1,16}$	$\pi_{2,16}$	$\pi_{3,16}$

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Common Models

- Throughout the rest of the talk, we will cover several of the common conjunctive cognitive diagnosis models (and their disjunctive counterparts):

Conjunctive Model	Disjunctive Model
DINA	DINO
NIDA	NIDO
RUM	Compensatory RUM

- Each of these models can be estimated with Mplus.
- Because of the number of constraints involved, the Mplus syntax gets rather lengthy (click here for an example).
- Because of this, a program called CDM was written.

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CDM Program

- The CDM program writes the Mplus code and parses the Mplus output for each of these models.
- The basic control file for the CDM program is shown on the next page.
- The CDM program and user guide can be found at http://jtemplin.myweb.uga.edu/cdm/ncme2008.html.

```
*Qmatrix
file = Qmatrix File Name
attributes = Number of Attributes
[format = *]

*DataFile
file = Data File Name
items = Number of Items
[format = *]

*CDM
model = Model name
[startmethod = random]
[randomseed = 0]

*Program
mpluspath = Path to Mplus
[analysis = all]
[inputfile = cdm_DATE_TIME.imp]
[itemfile = cdm_DATE_TIME_out.dat]
[examfile = cdm_DATE_TIME_exam.dat]
[covfile = cdm_DATE_TIME_cov.dat]
```

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Demonstration Files

- For each demonstration, the follow files and settings will be used:
  - A data file from a 20-item test (simulated data).
    - Filename: data.csv.
    - Format: Free (comma delimited).
  - A four-skill Q-matrix.
    - Filename: qmatrix.in.
    - Format: 4I1.
  - Starting method: subscore (for some models this may be dangerous).

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The DINA Model

- Deterministic Input; Noisy "And" Gate
  (Macready and Dayton, 1977; Haertel, 1989; Junker and Sijstma, 2001)

$$P(X_{ij} = 1|\xi_{ij}) = (1 - s_j)^{\xi_{ij}} g_j^{(1-\xi_{ij})}.$$

where

$$\xi_{ij} = \prod_{k=1}^{K} \alpha_{ik}^{q_{jk}}$$

$s_j = P(X_{ij} = 0|\xi_{ij} = 1)$ - "slip" parameter

$g_j = P(X_{ij} = 1|\xi_{ij} = 0)$ - "guess" parameter

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DINA Characteristics

- Separates examinees into two classes per item:
  - Examinees who have mastered *all* necessary attributes.
  - Examinees who are lacking mastery of *one or more* necessary attributes.
- The DINA model ensures all attributes missed are treated equally, resulting in the same chance of “guessing” correctly.
- For each item, the DINA model specifies two parameters to be estimated.
  - For a test of J items, a total of $2 \times J$ item parameters are modeled.

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DINA Demonstration

- Consider the first item: $2 + 3 - 1$
- Addition and subtraction are attributes needed to correctly answer this problem.
- Imagine those examinees who have mastered both addition and subtraction ($\xi_{i1} = 1$).
  - If 85% of those examinees correctly responded to this item, then $s_1 = 0.15$.
- Now consider those examinees who have not mastered either addition *or* subtraction ($\xi_{i1} = 0$).
  - If 30% of those examinees correctly responded to this item, then $g_1 = 0.30$.

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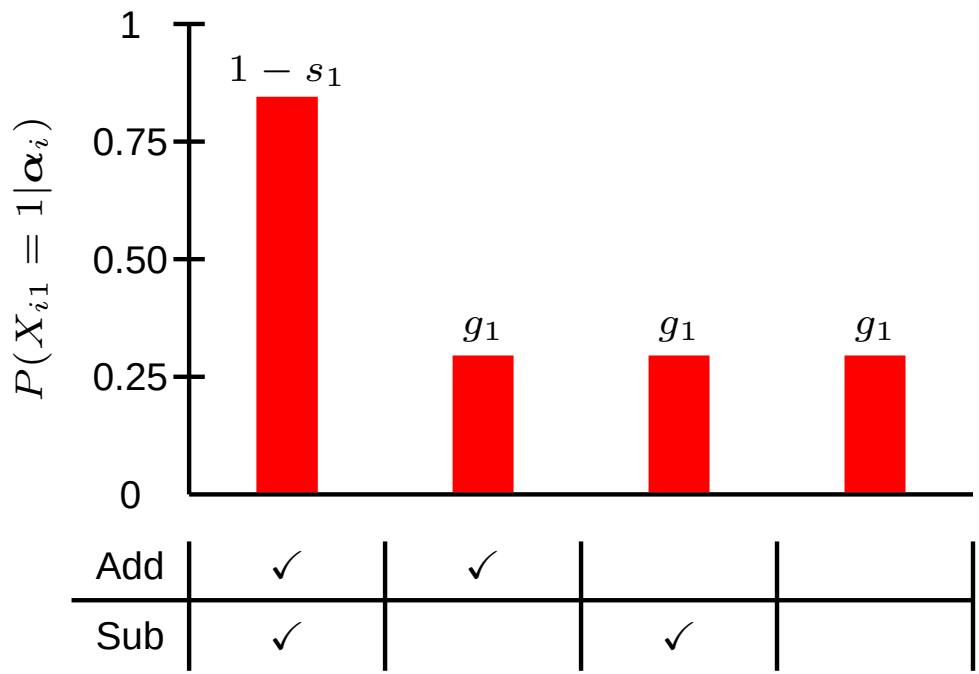
DINA Demonstration

Item response function for $2 + 3 - 1$:
$s_1 = 0.15$ and $g_1 = 0.30$.



DINA Item Response Constraints

Total Attributes: 4

Total Classes: 16

Model: DINA

Q-matrix:

	α_1	α_2	α_3	α_4
Item	Add	Sub	Mult	Div
$1.)\ 2+3-1$	1	1	0	0
$2.)\ 4/2$	0	0	0	1
$3.)\ 3\times(4-2)$	0	1	1	0

Equivalence Classes Per Item: 2

Item	1	2	3
Probability Parameters	2	2	2

Equal $P(X_{ij}=1|\boldsymbol{\alpha}_i)$ Denoted By Color

	Item		
$\boldsymbol{\alpha}$	1	2	3
[0000]	g_1	g_2	g_3
[0001]	g_1	$1-s_2$	g_3
[0010]	g_1	g_2	g_3
[0011]	g_1	$1-s_2$	g_3
[0100]	g_1	g_2	g_3
[0101]	g_1	$1-s_2$	g_3
[0110]	g_1	g_2	$1-s_3$
[0111]	g_1	$1-s_2$	$1-s_3$
[1000]	g_1	g_2	g_3
[1001]	g_1	$1-s_2$	g_3
[1010]	g_1	g_2	g_3
[1011]	g_1	$1-s_2$	g_3
[1100]	$1-s_1$	g_2	g_3
[1101]	$1-s_1$	$1-s_2$	g_3
[1110]	$1-s_1$	g_2	$1-s_3$
[1111]	$1-s_1$	$1-s_2$	$1-s_3$

```
*Qmatrix
file = qmatrix.in
format = 4i1
attributes = 4

*DataFile
file = data.csv
items = 20

*CDM
model = DINA
startmethod = subscore

*Program
mpluspath=C:\Program Files\Mplus\Mplus.exe
```

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CDM Output Files

- cdmplus_output.csv (click to open in Excel):
  - Model fit statistics.
  - Attribute pattern probability estimates.
  - Cognitive diagnosis model item parameter estimates.
- cdmexam_out.csv (click to open in Excel)
  - Examinee estimates
  - Attribute pattern probability estimates.
  - Most likely pattern.

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Reparameterized Unified Model (Hartz, 2002)

- To parsimoniously parameterize the Unified Model (DiBello, Stout, & Roussos, 1995), the Reparameterized Unified Model (or RUM; Hartz, 2002) was developed.
- The RUM incorporates two features not parameterized with most other models:
- The RUM is also called the Fusion Model.
  - Parameterization of multiple response classes per item.
  - Incorporation of completeness parameter.

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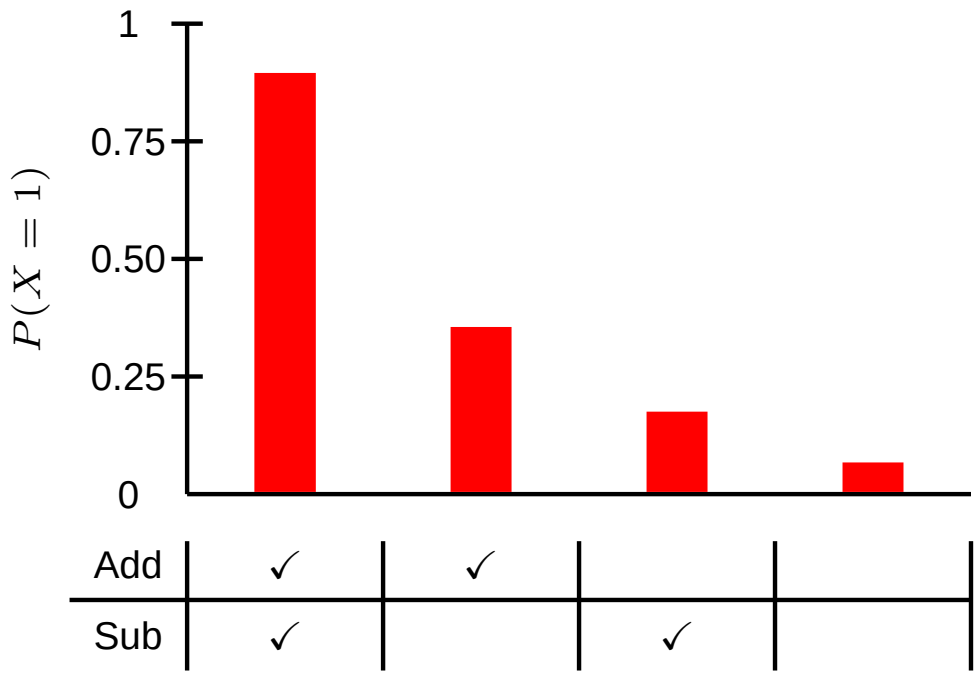
RUM Demonstration

- Consider the item:

$$2 + 3 - 1$$

- Addition and Subtraction are skills needed to correctly answer this problem.



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Item Completeness

- The quality of cognitive diagnosis model estimates are partially determined by the accuracy of the entries in the Q-matrix.
- In practice, situations occur where the Q-matrix is not accurately specified.
- For each item, the RUM maps the misspecified Q-matrix entries onto a single latent continuum.
- The RUM incorporates a continuous examinee parameter, denoted by θ_i, that represents the examinee's latent "ability" across the misspecified attributes.
- Because of the conjunctive nature of completeness, Mplus will not estimate the full RUM.

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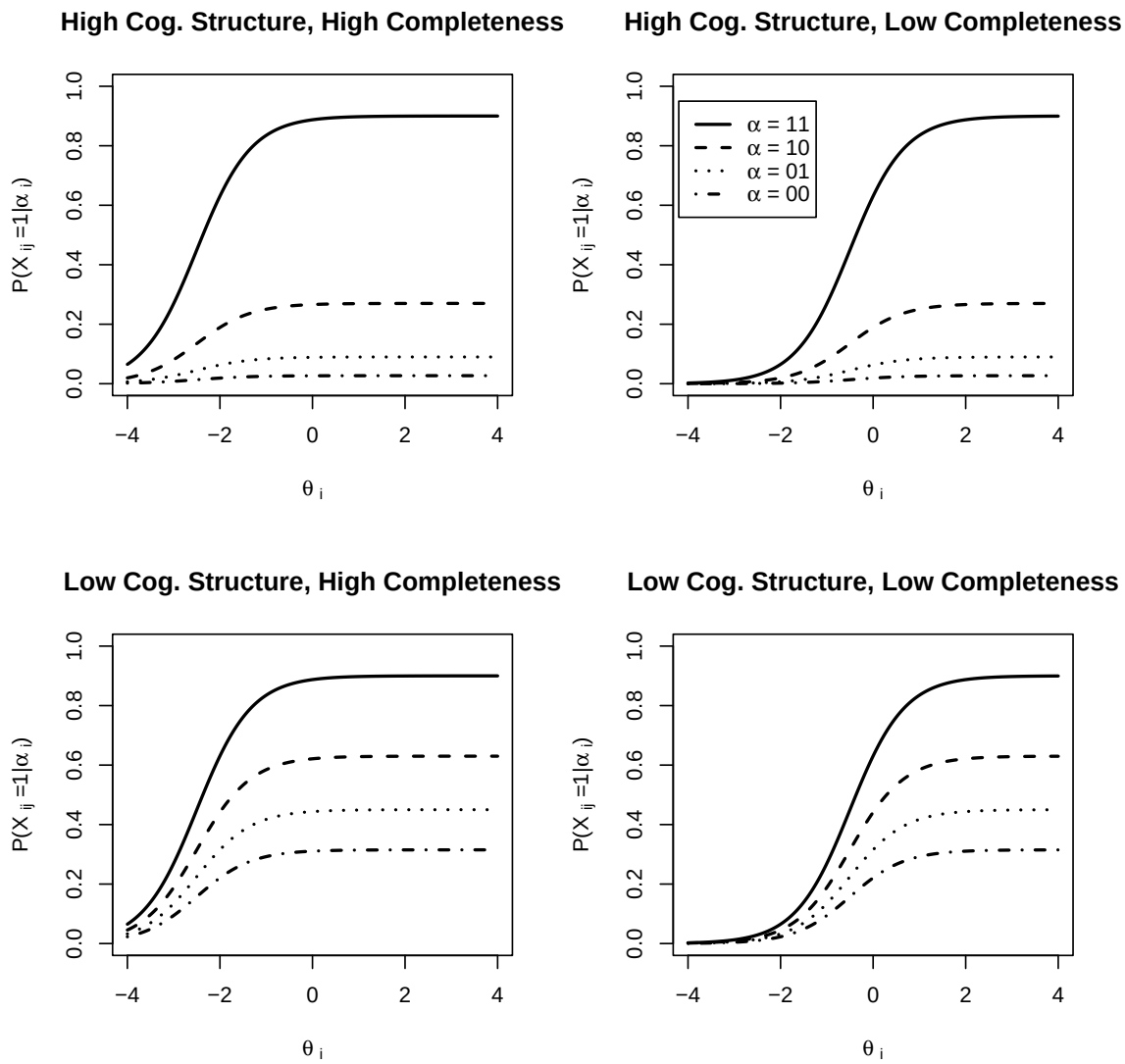
Reparameterized Unified Model (Hartz, 2002)

$$P\left(X_{ij}=1|\boldsymbol{\alpha}_i,\theta_i\right)=P_{c_j}(\theta_i)\left(\pi_j^*\prod_{k=1}^{K}r_{jk}^{(1-\alpha_{ik})\times q_{jk}}\right)$$

- $\mathbf{q}_j$ is the pre-specified row vector $(1\times K)$ of Q-matrix entries for item j.
- The "completeness" term $P_{c_j}(\theta_i)=\left(\frac{e^{1.701(c_j+\theta_i)}}{1+e^{1.701(c_j+\theta_i)}}\right)$.
- π_j^* is the maximum probability of correct response conditional on mastery of all Q-matrix attributes for item j.
- r_{jk}^* is the "penalty" imposed for missing attribute k.
- Let $q_{j\cdot}=\sum_{k=1}^{K}q_{jk}$. The RUM places $2^{q_{j\cdot}}$ equality constraints on the 2^K class item response probabilities.

RUM Illustrated

Consider an item requiring two attributes $(q_{j\cdot} = 2)$:



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Completeness Revisited

- In practice, the completeness parameter is difficult (if not impossible) to estimate.
- For these reasons, a reduced version of the RUM is often used.
- The reduced version of the RUM sets $P_{c_j}(\theta_i) = 1$ for all examinees i and items j.
- The remaining portion of the RUM retains the diagnostic information for the model:

$$P\left(X_{ij} = 1 | \boldsymbol{\alpha}_i\right) = \pi_j^* \prod_{k=1}^{K} r_{jk}^{(1-\alpha_{ik}) \times q_{jk}}$$

Reduced Rum Item Response Constraints

Total Attributes: 4

Total Classes: 16

Model: Reduced RUM

Q-matrix:

	α_1	α_2	α_3	α_4
Item	Add	Sub	Mult	Div
$1.)\ 2+3-1$	1	1	0	0
$2.)\ 4/2$	0	0	0	1
$3.)\ 3\times(4-2)$	0	1	1	0

Equivalence Classes Per Item: $(2^{q_{j\cdot}})$

Item	1	2	3
Probability Parameters	4	2	4

Equal $P(X_{ij}=1|\boldsymbol{\alpha}_i)$ Denoted By Color

$\boldsymbol{\alpha}$	Item 1	Item 2	Item 3
[0000]	$\pi_1^* r_{11}^* r_{12}^*$	$\pi_2^* r_{24}^*$	$\pi_3^* r_{32}^* r_{33}^*$
[0001]	$\pi_1^* r_{11}^* r_{12}^*$	π_2^*	$\pi_3^* r_{32}^* r_{33}^*$
[0010]	$\pi_1^* r_{11}^* r_{12}^*$	$\pi_2^* r_{24}^*$	$\pi_3^* r_{32}^*$
[0011]	$\pi_1^* r_{11}^* r_{12}^*$	π_2^*	$\pi_3^* r_{32}^*$
[0100]	$\pi_1^* r_1^*$	$\pi_2^* r_{24}^*$	$\pi_3^* r_{33}^*$
[0101]	$\pi_1^* r_1^*$	π_2^*	$\pi_3^* r_{33}^*$
[0110]	$\pi_1^* r_1^*$	$\pi_2^* r_{24}^*$	π_3^*
[0111]	$\pi_1^* r_1^*$	π_2^*	π_3^*
[1000]	$\pi_1^* r_2^*$	$\pi_2^* r_{24}^*$	$\pi_3^* r_{32}^* r_{33}^*$
[1001]	$\pi_1^* r_2^*$	π_2^*	$\pi_3^* r_{32}^* r_{33}^*$
[1010]	$\pi_1^* r_2^*$	$\pi_2^* r_{24}^*$	$\pi_3^* r_{32}^*$
[1011]	$\pi_1^* r_2^*$	π_2^*	$\pi_3^* r_{33}^*$
[1100]	π_1^*	$\pi_2^* r_{24}^*$	$\pi_3^* r_{33}^*$
[1101]	π_1^*	π_2^*	$\pi_3^* r_{33}^*$
[1110]	π_1^*	$\pi_2^* r_{24}^*$	π_3^*
[1111]	π_1^*	π_2^*	π_3^*

```
*Qmatrix
file = qmatrix.in
format = 4i1
attributes = 4

*DataFile
file = data.csv
items = 20

*CDM
model = RUM
startmethod = subscore

*Program
mpluspath=C:\Program Files\Mplus\Mplus.exe
```

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- Model
- Completeness Revisited
- RUM Output

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CDM Output Files

- cdmplus_output.csv (click to open in Excel):
  - Model fit statistics.
  - Attribute pattern probability estimates.
  - Cognitive diagnosis model item parameter estimates.
- cdmexam_out.csv (click to open in Excel)
  - Examinee estimates
  - Attribute pattern probability estimates.
  - Most likely pattern.

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Wrapping Up

- Cognitive diagnosis models provide a method for categorical estimating latent skills.
- Mplus can estimate many common models for cognitive diagnosis.
- Because of the sheer volume of the constraints, the difference in parameterization, and the difference in the meaning of each class, the CDM program can augment Mplus in estimation of many common models.
- The CDM program is in it's beginning stages - more models and options will be added as time progresses.
- For information or bug reporting, please email me: jtemplin@uga.edu.