Questions and Sample Answers for Chapter 8

Question 1

In this chapter, four models are proposed to parameterize the structure components in DCMs. How should practitioners think about choosing one of those four models? Explain.

Question 2

What is the log-linear structural approach to modeling attribute space in the DCMs? How does it reduce the complexity of the estimated DCM? How is it both similar to and different from the general log-linear structure of the LCDM? Explain.

Question 3

This chapter focused on the structural component of DCMs. Which of the following descriptions about the structural component are supportable?

a. The structural component of a DCM models the probability of correct item responses conditional on latent class membership, while the measurement component models the probability of respondents belonging in each of the latent classes.

b. Interpretability and communicability of results are the primary criteria for selecting a structural model.

c. Attention must be paid to the parameterization of both the measurement and structural components of a DCM to ensure ensuring there is not a mismatch the specific form of the measurement component of the model and the parameterization of its structural component.

d. Specifying tailored structural models is an essential part of any modeling enterprise so practitioners need to learn about the details of these models first before they can apply DCMs to their response data.

e. None of the above.

f. All of the above.

Question 4

Given the following factor loading matrix for a higher-order structural model, which attributes load on Factor 2?

Section 2 – Sample Answers

Question 1

The choice of modeling approach for the structural part of a DCM depends as much on software capabilities (e.g., Mplus does not currently allow for the estimation of a tetrachoric structural model) as on practical needs (e.g., a higher-order structural model may be useful for reporting diagnostic profiles and proficiency scores).

To choose a particular structural model within a particular modeling approach, statistical tests are generally the preferred method. For example, within the log-linear structural modeling approach one can start with a full model that contains all relevant main and interaction effects and delete unnecessary structural parameters if their associated statistical hypothesis tests are non-significant. Similarly, for a higher-order structural factor model one can being with a model that contains a single factor and then successively add factors to see how many different factors can be empirically supported. As with measurement model choices, the final model may be a compromise between statistical efficiency and interpretational desirability.

Question 2

The log-linear parameterization of the structural models estimates the latent class membership probabilities υ_c using a log-linear model that includes main effects and interaction effects. The model is expressed via a kernel expression, μ_c, and uses this expression to predict the latent class membership probabilities. There is one less parameter estimate than there are latent classes because the last latent class membership probability is determined by all previous probabilities due to a sum-to-one constraint.

A saturated log-linear model contains main effects of attributes and interaction effects of attributes in all possible combinations. This parameterization is expressed as follows:

$$\mu_c = \sum_{a=1}^{A} \gamma_{1,(a)} \alpha_{ca} + \sum_{a=1}^{A-1} \sum_{a'=a+1}^{A} \gamma_{2,(a,a')} \alpha_{ca} \alpha_{ca'} + \ldots + \gamma_{A,(a,a',\ldots)} \prod_{a=1}^{A} \alpha_{ca}$$

In this formula, $\gamma_{1,(a)}$ is the main effect parameter for Attribute a; $\gamma_{2,(a,a')} \alpha_{ca} \alpha_{ca'}$ is the two-way interaction effect parameter associated with parameters a and a' ; and $\gamma_{A,(a,a',\ldots)}$ is the a-way interaction effect for all attributes. α_{ca} is the latent variable for Attribute a in latent class c.

Latent class membership probabilities are computed using the following formula:

$$\upsilon_c = \frac{\exp(\mu_c)}{\sum_{c=1}^{C} \exp(\mu_c)}$$

The saturated model is statistically identical to an unconstrained structural model. The model with main effects only is identical to a model in which all latent attribute variables are uncorrelated. Models with different levels of interaction effects represent the correlational structure between the latent attribute variables with different degrees of precision.

Thus, a log-linear structural model allows for conducting hypothesis tests for each parameter, which can be eliminated if the associated test is non-significant. As always, the goal is to find a parsimonious model that can sufficiently accurately predict the latent class membership probabilities vis-à-vis an unconstrained model.

Finally, the log-linear structural model and the LCDM framework share a logistic link function and a linear predictor. Technically, the LCDM contains two model components, a measurement component and a structural component. The log-linear formulation in the measurement component relates the item parameters to the response probabilities via a logistic link function whereas the log-linear formulation in the structural component relate respondent parameters to latent class membership probabilities via a logistic link function.

Question 3

Correct answer: e

a. The LCDM contains two model components, a measurement component and a structural component. The log-linear formulation in the measurement component relates the item parameters to the response probabilities via a logistic link function whereas the log-linear formulation in the structural component relate respondent parameters to latent class membership probabilities via a logistic link function. Thus, this option is incorrect as the relationships are the reverse of what is stated.

b. The structural model part of an LCDM formulation for DCMs is a rather technical aspect from a practical perspective and not necessarily something that practitioners learn about first. Unless a higher-order structural factor model is selected as the modeling approach, which can be useful in practice for providing higher-order proficiency scores alongside the diagnostic profiles, most modeling approaches are driven by a desire to simplify the estimation process through parameter elimination. Thus, this answer is overstating the role of practical considerations for choosing a particular structural model.

c. There is no need to match the modeling structure of the structural component of the DCM and the measurement component of the DCM within an LCDM formulation so this answer is incorrect.

d. While it can certainly be useful to specify tailored structural model specifications, it is debatable whether this is an “essential” part of any modeling enterprise – it is certainly a useful part but arguably not of primary importance given the current state of adoption of DCMs for real-life reporting and decision-making purposes. This answer seems to overstate the importance of structural model components.

Question 4

Correct answer: c

The factor loading matrix is organized so that items are listed as rows and factors are listed as columns. Thus, there are four items and three higher-order latent factor variables. Thus, answer (c) represents the correct loading structure because there are non-zero loadings for Item 2 and Item 4 on Factor 2.