Introduction to Factorial Designs

ERSH 8310

Lecture 8

October 21, 2009

Today's Class

- Old business:
  - Midterm discussion

- New business:
  - Chapter 10 (conceptual):
    - An introduction to factorial designs
      - More than one manipulated factor in an experiment
    - Basic information available from factorial designs
    - The concept and of interaction
  - Chapter 11 (practical):
    - Two-way ANOVA
      - How to compute values
      - How to test main effects statistically
      - How to test interactions statistically

MIDTERM DISCUSSION

BASIC INFORMATION AVAILABLE FROM FACTORIAL DESIGNS

Basic Information Available from Factorial Designs

- An experiment may be designed to focus attention on a single independent variable or factor

- An alternative approach is to study the influence of one independent variable in conjunction with variations in one or more additional independent variables
  - Factorial designs

- Factorial designs allow us to study:
  - The effects of two or more IVs separately on the DV
  - How two or more IVs combine to influence the DV

Simple Effects of an Independent Variable

- A factorial design consists of a set of single-factor experiments
  - Table 10.1 shows an example of a factorial design that investigates the format of the books (i.e., three line lengths and three letter-paper contrasts) on the reading speed.

- The results of the component single-factor experiments are called the simple effects of an independent variable
  - The means (or effects) of each group without consideration of the other IV(s)
  - In the example, the simple effects are the results of three separate studies

Factorial Design Example (Table 10.1)

Table 10.1: A factorial experiment (left) and its interpretation as a set of single-factor experiments (right).

A Factorial Design

	Line Length		
Contrast	3 in.	5 in.	7 in.
Low			
Medium			
High			

Three Single-Factor Designs

	3 in.	5 in.	7 in.
Low Contrast			

	3 in.	5 in.	7 in.
Medium Contrast			

	3 in.	5 in.	7 in.
High Contrast			

Interaction Effects

- An interaction is a comparison among the simple effects of the component experiments (i.e., differences in the simple effects)

- Same outcomes across all experiments = NO interaction

- Different outcomes in one or more experiments = INTERACTION

Main Effects

- The main effects of an independent variable refer to the average of the component single-factor experiments making up the factorial design
- We obtain the main effect by combining the individual treatment means from each component experiment
- Note that main effects are most easily interpreted when an interaction is absent

So…To Summarize

- A factorial design produces three important pieces of information:
  - The simple effects
  - The interaction effects
  - The main effects

THE CONCEPT OF INTERACTION

An Example of No Interaction

- Table 10.3 presents some hypothetical results for the experiment on reading speed
  - Equal numbers of children in each of the nine conditions
  - The values represent the average reading scores

- Factor A is the line length variable (a_1, a_2, a_3) and factor B is the contrast variable (b_1, b_2, b_3)

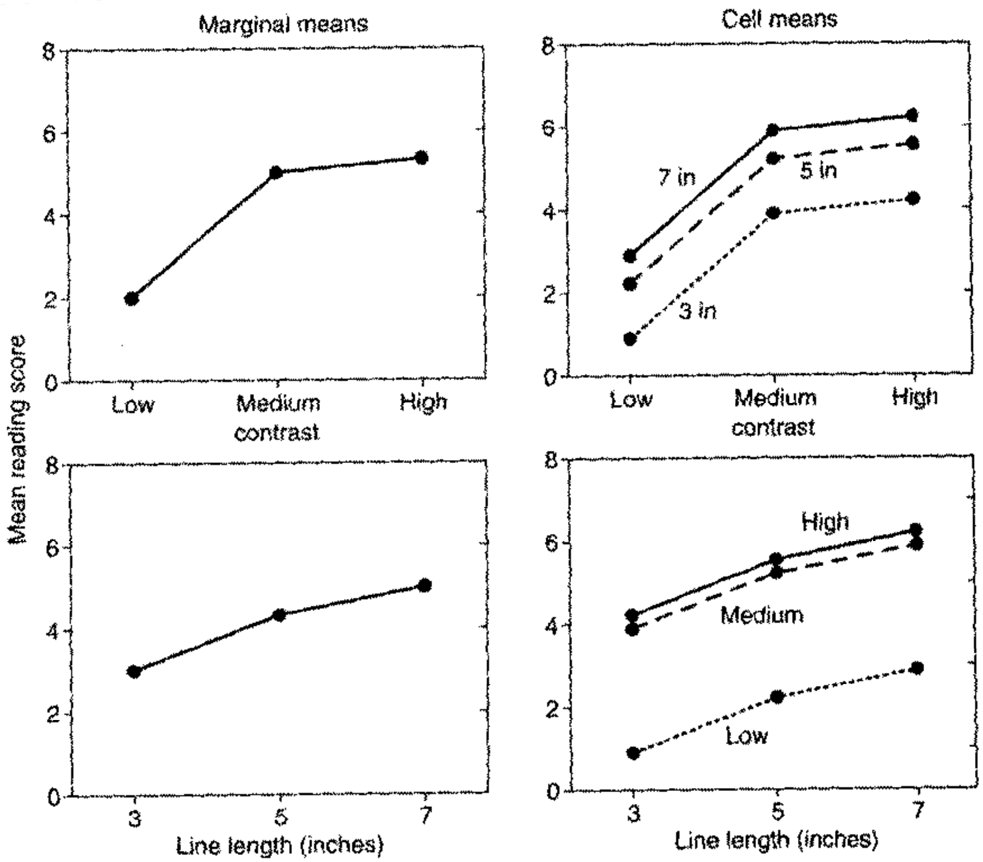
- If the plot of the marginal means and the plot of the cell means of the component single-factor experiments are parallel, there is no interaction (see Figure 10.1)

Table 10.3

Table 10.3: A hypothetical set of means for the reading experiment that shows no interaction.

Contrast (Factor A)	Line Length (Factor B) 3 in. (b_1)	5 in. (b_2)	7 in. (b_3)	Mean
Low (a_1)	0.89	2.22	2.89	2.00
Medium (a_2)	3.89	5.22	5.89	5.00
High (a_3)	4.22	5.55	6.22	5.33
Mean	3.00	4.33	5.00	4.11

Figure 10.1



'igure 10.1: Reading scores from Table 10.3 plotted as a function of contrast (upper 'anel) and as a function of line length (lower panel). No interaction is present.

An Example of Interaction

- Table 10.4 presents a second set of hypothetical results using the same experimental design
- The same main effects are present – the interaction now is present
- The patterns of differences reflected by the simple effects are not the same at all levels of the other independent variable
- A simple way to describe the situation where interaction exists is that the cell mean lines are not parallel

Table 10.4

Table 10.4: A hypothetical set of means for the reading experiment that shows an interaction.

Contrast (Factor A)	Line Length (Factor B) 3 in. (b_1)	5 in. (b_2)	7 in. (b_3)	Mean
Low (a_1)	1.00	2.00	3.00	2.00
Medium (a_2)	3.00	5.00	7.00	5.00
High (a_3)	5.00	6.00	5.00	5.33
Mean	3.00	4.33	5.00	4.11

Figure 10.2

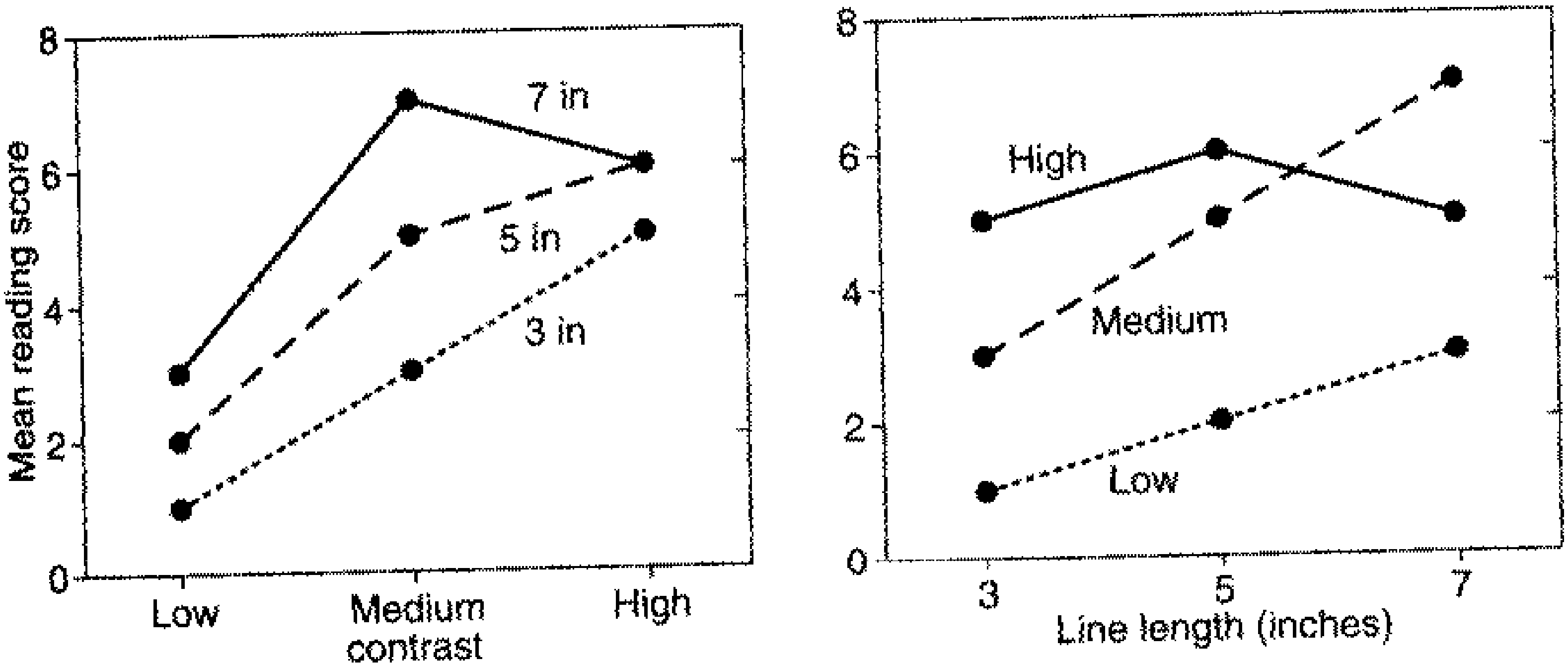


Figure 10.2: Reading scores from Table 10.4. An interaction is present.

THE DEFINITION OF AN INTERACTION

The Definition of an Interaction

1. An interaction is present when the effects of one independent variable on behavior change at the different levels of the second independent variable
2. An interaction is present when the values of one or more contrasts in one independent variable changes at the different levels of the other independent variable
3. An interaction is present when the simple effects of one independent variable are not the same at all levels of the second independent variable

The Definition of an Interaction

4. An interaction is present when the main effect of an independent variable is not representative of the simple effects of that variable
5. An interaction is present when the differences among the cell means representing the effect of factor A at one level of factor B do not equal the corresponding differences at another level of factor B
6. An interaction is present when the effects of one of the independent variables are conditionally related to the levels of the other independent variable

Interaction and Theoretical Analysis

- When interaction is found, there exist a complexity of post hoc explanations of a set of data

- If behavior is complexly determined, we need factorial experiments to isolate and to tease out the complexities

- The factorial design allows us to manipulate two or more independent variables concurrently and to obtain some idea of how the variables combine to produce the behavior

- An assessment of the interaction provides a hint to the rules of combination

Further Examples of Interaction

- Examples using a 2 ×2 factorial design are presented in Table 10.5 and Figure 10.3.

- The following are some the basic definitions:
  - A *factorial design* contains the conditions formed by combining each level of one independent variable with each level of another
  - A *simple effect* expresses the differences among the means of one independent variable at a fixed level of the other independent variable
  - The *main effect* expresses the difference among the means for one independent variable averaged over the levels of the other independent variable
  - An *interaction* is present when the simple effects of one independent variable are not the same at all levels of the other independent variable

Table 10.5

Table 10.5: *Eight different outcomes of a two-factor experiment.*

(1)

	b_1	b_2	$\bar{Y}_A$
a_1	5	5	5
a_2	5	5	5
$\bar{Y}_B$	5	5	

(2)

	b_1	b_2	$\bar{Y}_A$
a_1	4	4	4
a_2	6	6	6
$\bar{Y}_B$	5	5	

(3)

	b_1	b_2	$\bar{Y}_A$
a_1	7	3	5
a_2	7	3	5
$\bar{Y}_B$	7	3	

(4)

	b_1	b_2	$\bar{Y}_A$
a_1	6	2	4
a_2	8	4	6
$\bar{Y}_B$	7	3	

(5)

	b_1	b_2	$\bar{Y}_A$
a_1	6	4	5
a_2	4	6	5
$\bar{Y}_B$	5	5	

(6)

	b_1	b_2	$\bar{Y}_A$
a_1	5	3	4
a_2	5	7	6
$\bar{Y}_B$	5	5	

(7)

	b_1	b_2	$\bar{Y}_A$
a_1	8	2	5
a_2	6	4	5
$\bar{Y}_B$	7	3	

(8)

	b_1	b_2	$\bar{Y}_A$
a_1	7	1	4
a_2	7	5	6
$\bar{Y}_B$	7	3	

Figure 10.3

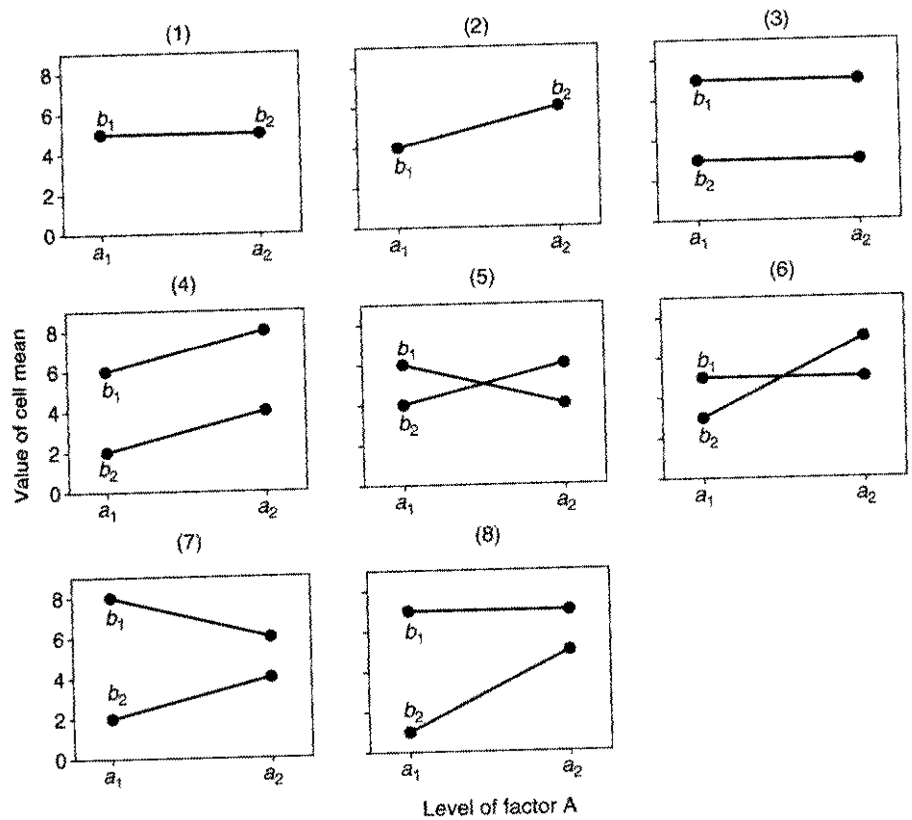


Figure 10.3: Plot of the cell means from Table 10.5.

Chapter 11

THE OVERALL TWO-FACTOR ANALYSIS

The Overall Two-Factor Analysis

- In Chapter 2 the total sum of squares was partitioned into the between-group sum of squares ($SS_{between}$ or SS_A)and the within-group sum of squares (SS_{within} or $SS_{S/A}$)

- In the analysis of the factorial design the $SS_{between}$ is divided into:
  - The sum of squares reflecting the main effect of factor A (SS_A)
  - The sum of squares reflecting the main effect of factor B (SS_B)
  - The sum of squares reflecting the A ×B interaction effect ($SS_{A \times B}$)

- The same number of subjects in each of the treatment conditions will be considered in the current chapter (n)

Understanding the Book

- The notational system is summarized in Table 11.1
  - We won't be doing two-way ANOVA by hand

- The basic observation or score in the two-way factorial design is denoted as Y_{ijk}
  - Subject: i = 1,…,n
  - Level of factor A: j = 1,…,a
  - Level of factor B: k = 1,…,b

Partitioning the Sums of Squares

- In Chapter 2, it was shown that:

$$SS_T = SS_{between} + SS_{within}$$

- In the two-way factorial design, $SS_{between}$ is further decomposed:

$$SS_{between} = SS_A + SS_B + SS_{A\times B}$$

- Of course, this changes the ANOVA table computations

$$SS_T = SS_{between} + SS_{within} = SS_A + SS_B + SS_{A\times B} + SS_{within}$$

Degrees of Freedom

- The degrees of freedom are:
  - $df_A = a - 1$
  - $df_B = b - 1$
  - $df_{A \times B} = (a - 1)(b - 1)$
  - $df_{S/AB} = ab(n - 1)$
  - $df_T = abn - 1 = df_A + df_B + df_{A \times B} + df_{S/AB}$

- **Rules for calculating DF:**
  - $df_T = N - 1$ (always)
  - Main effect df = number of levels minus one (always)
  - Interaction df = product of main effect df (always)
  - Sum of all df must equal df_T
  - Remainder can go into error term (term with slash)

Mean Squares

- The mean squares are:
  - $MS_A = SS_A / df_A$
  - $MS_B = SS_B / df_B$
  - $MS_{A \times B} = SS_{A \times B} / df_{A \times B}$
  - $MS_{S/AB} = SS_{S/AB} / df_{S/AB}$
- Mean Squares are always formed by dividing by the respective component's degrees of freedom
- SS_A, SS_B and $SS_{A \times B}$ are <u>*mutually orthogonal*</u>
  - Meaning that they sum to SS_T

F Ratios

- F-ratios are formed the same way:
  - Dividing the MS for an effect by the Ms_{within}
- Here MS_{within} is also called $MS_{S/AB}$
  - Pronounced Mean Squares for Subjects within A and B

- $F_A = MS_A/MS_{S/AB}$
- $F_B = MS_B/MS_{S/AB}$
- $F_{AxB} = MS_{AxB}/MS_{S/AB}$

THE STATISTICAL MODEL

The Linear Model

- The model underlying the two-way ANOVA is:

$$Y_{ijk} = \mu_T + \alpha_j + \beta_k + (\alpha\beta)_{jk} + E_{ijk}$$

- Where:
  - μ_T is the grand mean (mean of all scores)
  - $\alpha_j = \mu_j - \mu_T$ is the average treatment effect at level a_j
  - $\beta_k = \mu_k - \mu_T$ is the average treatment effect at level b_k
  - $(\alpha\beta)_{jk} = \mu_{jk} - \mu_j - \mu_k + \mu_T$ is the interaction effect at cell a_jb_k
  - $E_{ijk} = Y_{ijk} - \mu_{jk}$ is the experimental error for each score

Null Hypotheses

- The null hypothesis for the A main effect:

$$H_0: \alpha_1 = \alpha_2 = \dots = \alpha_a = 0$$

- The null hypothesis B main effect:

$$H_0: \beta_1 = \beta_2 = \dots = \beta_b = 0$$

- The null hypothesis for the A ×B interaction:

$$H_0: (\alpha\beta)_{11} = (\alpha\beta)_{12} = \dots = (\alpha\beta)_{ab} = 0$$

AN EXAMPLE

A Numerical Example

- A hypothetical investigation of the role of certain drugs [factor A-Control (a1), Drug X (a2), Drug Y (a3)] and drive level [factor B-1 hour of food deprivation (b1), 24 hour of food deprivation (b2)] on learning performance (Y) of monkeys
- The animals are given a series of 20 ``oddity'' problems and the response measure Y is the number of errors in the 20 training trials
- The design is a 3×2 factorial with a cell sample size of n = 4

Data…

10_04data.SAV [DataSet1] - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Uti

34 : deprive

	errors	drug	deprive
1	1.00	1.00	1.00
2	4.00	1.00	1.00
3	.00	1.00	1.00
4	7.00	1.00	1.00
5	13.00	2.00	1.00
6	5.00	2.00	1.00
7	7.00	2.00	1.00
8	15.00	2.00	1.00
9	9.00	3.00	1.00
10	16.00	3.00	1.00
11	18.00	3.00	1.00
12	13.00	3.00	1.00
13	15.00	1.00	2.00
14	6.00	1.00	2.00
15	10.00	1.00	2.00
16	13.00	1.00	2.00
17	6.00	2.00	2.00
18	18.00	2.00	2.00
19	9.00	2.00	2.00
20	15.00	2.00	2.00
21	14.00	3.00	2.00
22	7.00	3.00	2.00
23	6.00	3.00	2.00
24	13.00	3.00	2.00
25			

Y has a column

Factor A has a column

Factor B has a column

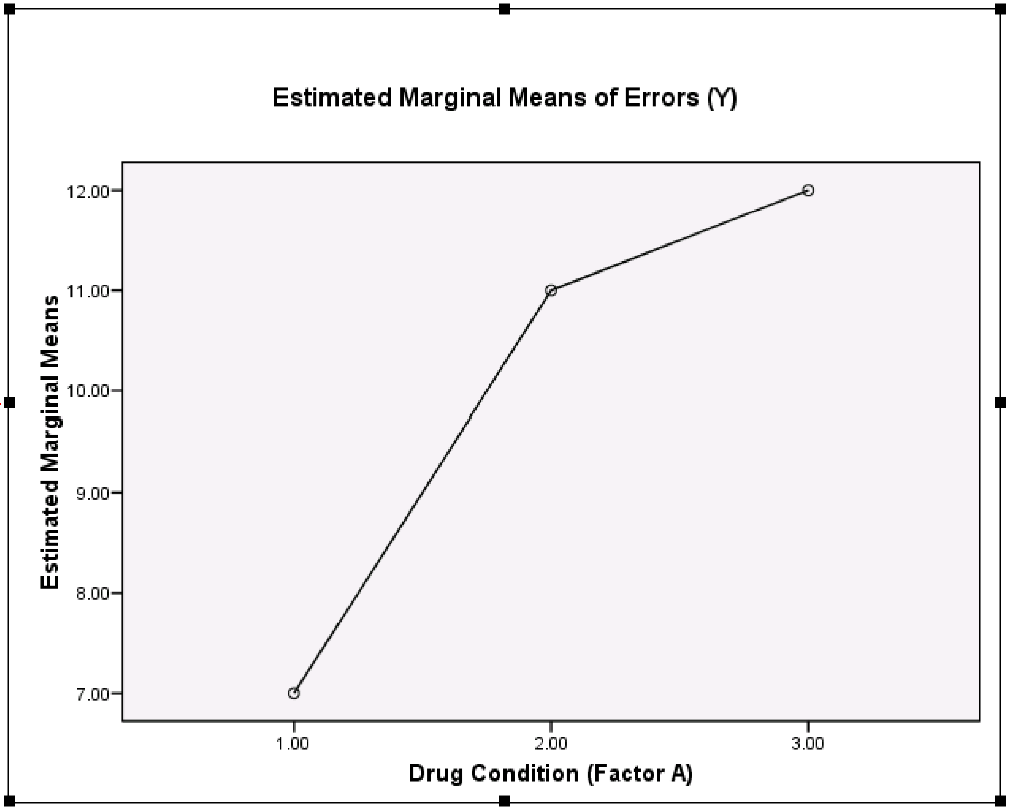
Multifactor ANOVA in SPSS

- As you may recall, up to this point we have only run analyses using the Analyze…Compare Means…One-Way ANOVA option

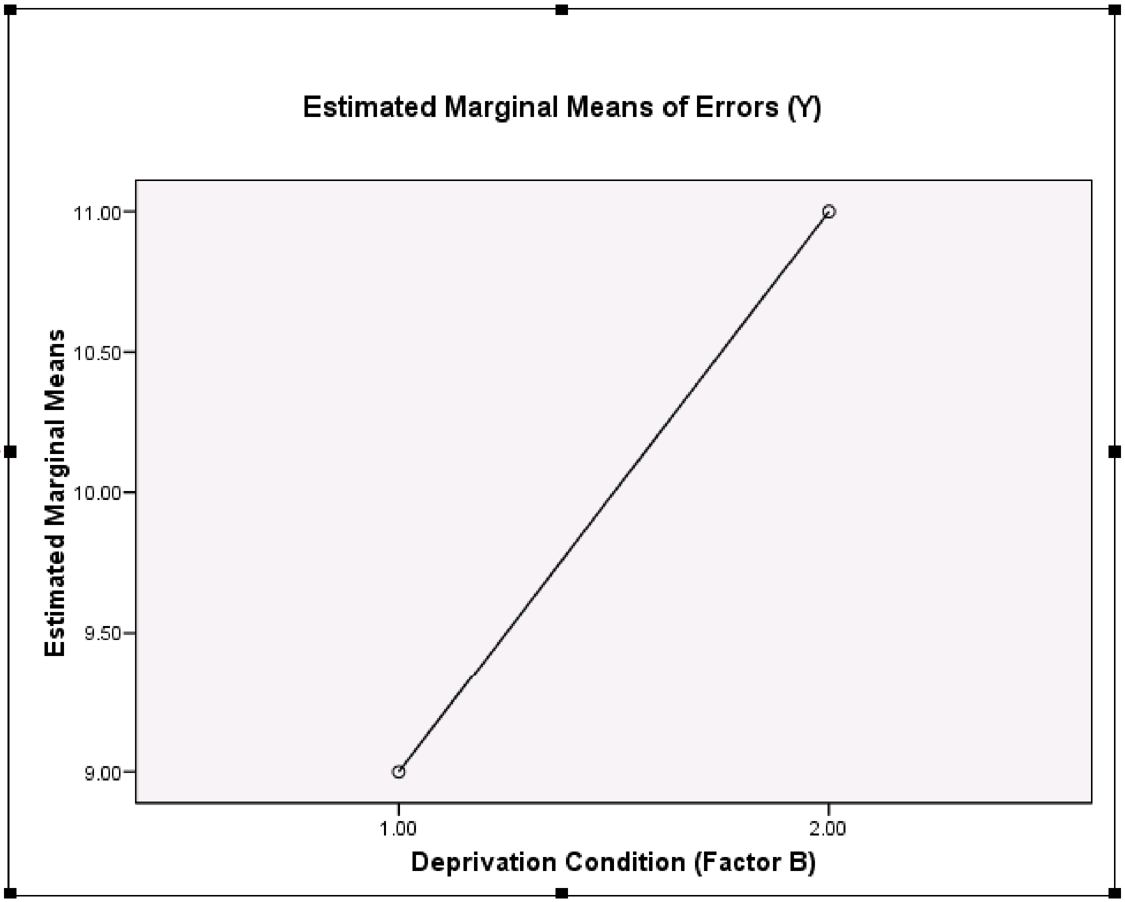
- For a more general approach (with multiple factors), we must now use a slightly different option
  - Analyze…General Linear Model…Univariate
  - More on this in Lab

Analysis Output

- First, lets look at some mean plots…

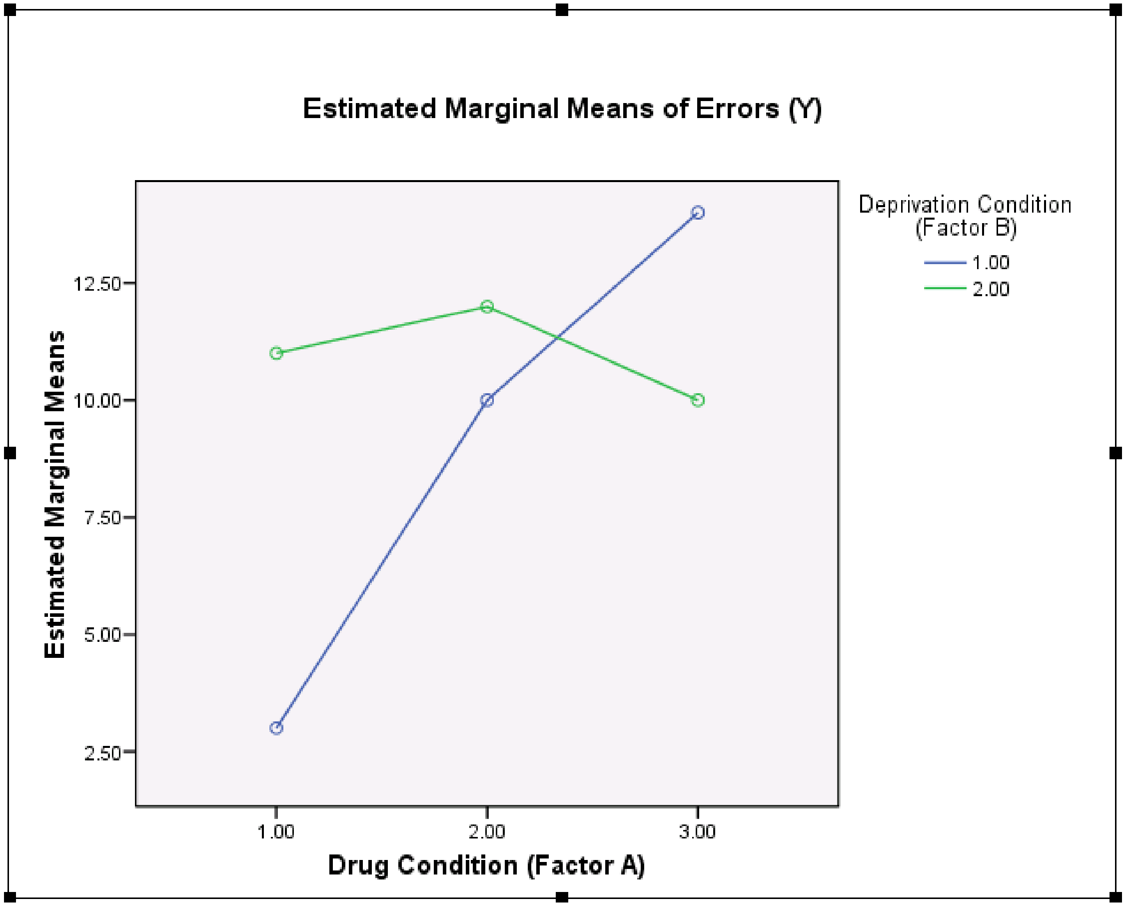


Factor B

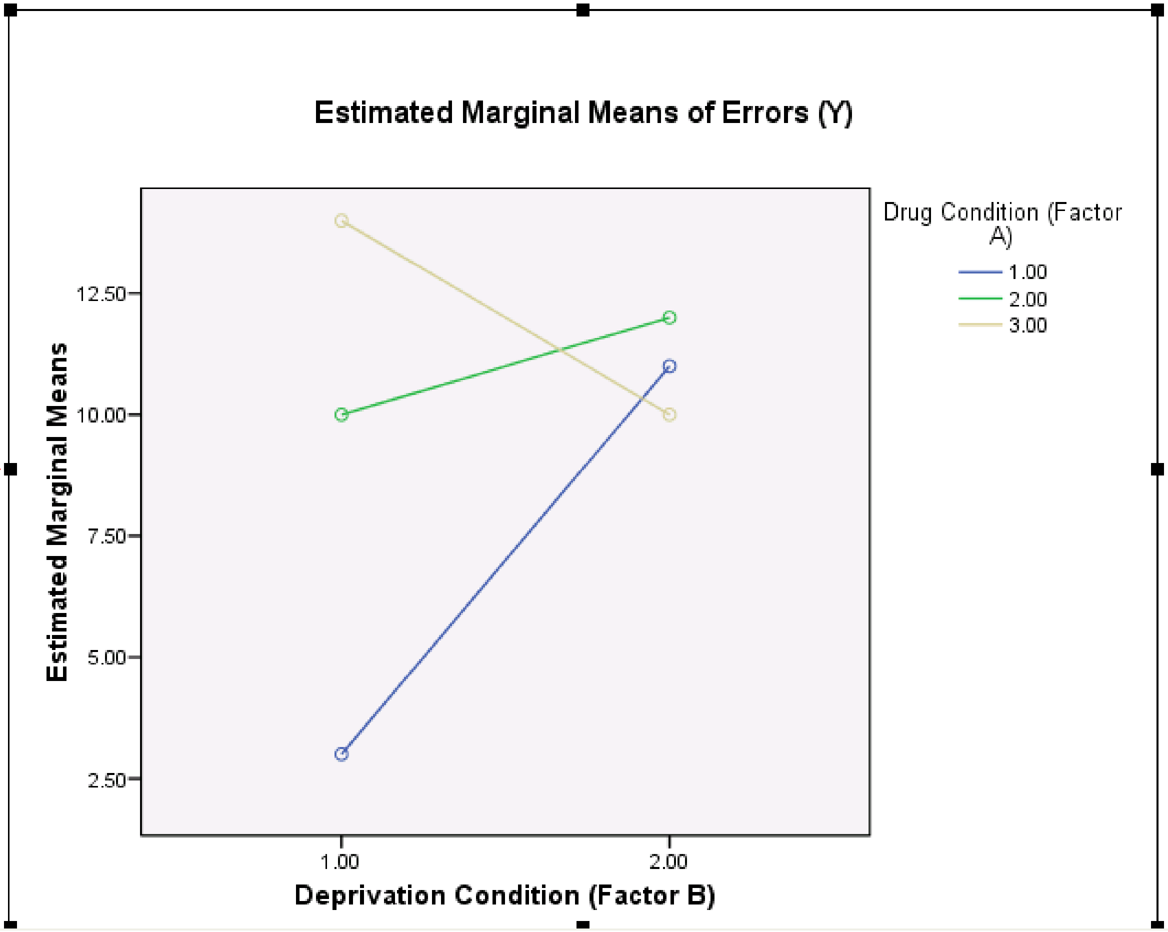


Estimated Marginal Means of Errors (Y)
Estimated Marginal Means
11.00
10.50
10.00
9.50
9.00
1.00
2.00
Deprivation Condition (Factor B)

Interaction (Plot #1)



Interaction (Plot #2)



Statistical Output

Between-Subjects Factors

		N
Drug Condition (Factor A)	1.00	8
	2.00	8
	3.00	8
Deprivation Condition (Factor B)	1.00	12
	2.00	12

Main Effect Test for Factor A

Main Effect Test for Factor B

Interaction Test for AxB

Tests of Between-Subjects Effects

Dependent Variable: Errors (Y)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	280.000[a]	5	56.000	3.055	.036
Intercept	2400.000	1	2400.000	130.909	.000
drug	112.000	2	56.000	3.055	.072
deprive	24.000	1	24.000	1.309	.268
drug * deprive	144.000	2	72.000	3.927	.038
Error	330.000	18	18.333		
Total	3010.000	24			
Corrected Total	610.000	23			

a. R Squared = .459 (Adjusted R Squared = .309)

So…What Do We Conclude?

- We have three "omnibus" hypotheses being tested in our two-way ANOVA example:
  - The null hypothesis for the A main effect:

    $$H_0: \alpha_1 = \alpha_2 = \ldots = \alpha_a = 0$$

    - P- Value = 0.072
  - The null hypothesis B main effect:

    $$H_0: \beta_1 = \beta_2 = \ldots = \beta_b = 0$$

    - P- Value = 0.268
  - The null hypothesis for the A ×B interaction:

    $$H_0: (\alpha\beta)_{11} = (\alpha\beta)_{12} = \ldots = (\alpha\beta)_{ab} = 0$$

    - P-value = 0.038

Means…

Estimated Marginal Means

1. Grand Mean

Dependent Variable: Errors (Y)

Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
10.000	.874	8.164	11.836

2. Drug Condition (Factor A)

Dependent Variable: Errors (Y)

Drug Condition (Factor A)	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	7.000	1.514	3.820	10.180
2.00	11.000	1.514	7.820	14.180
3.00	12.000	1.514	8.820	15.180

3. Deprivation Condition (Factor B)

Dependent Variable: Errors (Y)

Deprivation Condition (Factor B)	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	9.000	1.236	6.403	11.597
2.00	11.000	1.236	8.403	13.597

4. Drug Condition (Factor A) * Deprivation Condition (Factor B)

Dependent Variable: Errors (Y)

Drug Condition (Factor A)	Deprivation Condition (Factor B)	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	1.00	3.000	2.141	-1.498	7.498
	2.00	11.000	2.141	6.502	15.498
2.00	1.00	10.000	2.141	5.502	14.498
	2.00	12.000	2.141	7.502	16.498
3.00	1.00	14.000	2.141	9.502	18.498
	2.00	10.000	2.141	5.502	14.498

Wrapping Up…

- The two-way ANOVA provides a very tool for making experiments more complex
  - Multiple IVs
  - Interactions and main effects

- Next week we will see how all of Chapters 1-8 apply to these types of designs
  - You may already be able to guess…power? Effect size? Contrasts? Post-hoc comparisons?

Up Next…

- In Lab:
  - How to do two-way ANOVA in SPSS
    - The General Linear Models box
- Homework:
  - Assigned tomorrow morning, due next week at start of class (10/21 at 4:40pm)
- Next week
  - Read chapters 12 and 13