Experimental Design Sources of Variability and Sums of Squares

ERSH 8310

Lecture 2

Today's Class

- Chapter 1:
  - Basics of experimental design
- Chapter 2:
  - The logic of hypothesis testing
  - Component deviations
    - Sums of squares
  - Computational formulas for sums of squares

Chapter 1

EXPERIMENTAL DESIGN

Experimental Design Basics

- Experiments consist of plans for
  - Data collection
  - Data analysis

- A well-designed experiment permits the inference of causation
  - Random assignment
  - Large sample size

- Chapter 1 provides the basics in terms of design
  - The rest of the book provides the details of analysis

Variables in Experimental Research

- Independent variable
  - Manipulated variable
  - Also called treatment or factor

- Types of independent variables:
  - Qualitative (categorical)
    - i.e., Treatment type
  - Quantitative (continuous)
    - i.e., Amount of heat
  - Intrinsic (not manipulated – attribute of subjects)
    - i.e. gender

More Variables in Experimental Research

- Dependent variable
  - The outcome collected in an experiment
  - One DV – univariate procedure
  - Multiple DVs – multivariate procedure
  - Cautions:
    - Ceiling or floor effects

- Nuisance variables
  - Confounding factors
  - Helped by:
    - Design (holding constant)
    - Counterbalancing
    - Statistical modeling
    - Randomization

Control in Experimentation

- Control by design
  - Carefully designed experiments have controls
  - Blocking factors
  - Statistical control – Analysis of Covariance (ANCOVA)

- Control by randomization
  - Randomly accounts for nuisance variables by evenly distributing them across experimental groups

Populations and Generalizing

- Goal of research is to extend findings beyond subjects in an experiment

- Usually, a sample is taken from larger population

- To extent sample mimics population of interest, the results may generalize
  - Statistical generalization
  - Non-statistical generalization

Basic Experimental Designs

- Between subjects designs
  - Subjects receive only one combination of treatment conditions
  - Cross-sectional

- Within subjects designs
  - Subjects receive multiple treatment condition combinations
  - Repeated measures
  - Longitudinal

- Factorial design
  - Each factor is crossed with each other factor
  - Have subjects in all combinations of factors
  - Can be for within subjects or between subjects experiments

Chapter 2

SOURCES OF VARIABILITY AND SUMS OF SQUARES

THE LOGIC OF HYPOTHESIS TESTING

Differences Between Groups

- Differences observed among treatment means:
  - Influenced jointly by the actual differences in the treatments administered to the different groups
  - …and by chance factors introduced by randomization

- Experimenter must decide whether the differences associated with the treatment conditions are entirely or just partly due to chance

The Logic of Hypothesis Testing

- Main goal: to make inferences about the behavior of subjects in population
    - Inferences based on subjects in the sample

- Summary descriptions calculated from the data of a sample are called statistics

- Measures calculated from all the observations with the population are called parameters

- Usually Roman letters designate statistics and Greek letters designate parameters

Statistical Hypotheses

- A research hypothesis asserts that the treatments will produce an effect on behavior

- Statistical hypotheses are about the parameters of the different treatment populations

Null Hypothesis

- There are two statistical hypotheses, the null and the alternative
- The statistical hypothesis to be tested is called the null hypothesis H_0
- For example, the null hypothesis of the analysis of variance with three treatment conditions can be:

$$H_0: \mu_1 = \mu_2 = \mu_3$$

Alternative Hypothesis

- The alternative hypothesis specifies the values for the parameter that are incompatible with the null hypothesis

- For example:

$$H_1\text{: not all } \mu\text{'s are equal}$$

Experimental Error

- All nuisance variables are considered potential contributors to experimental error
  - Attempt to control for using random assignment
- Types of experimental error:
  - Individual differences (typically in behavioral sciences)
  - Measurement error
- We describe the contribution of all the different components of experimental error as unsystematic
  - Their influence is independent of the treatment effects

Estimates of Experimental Error

- An estimate of experimental error is provided by the variability of subjects treated alike

- We assume that experimental error is the same for the different treatment conditions
  - Giving us more stable estimate of this quantity by pooling and averaging these separate estimates of variability

- Assume that the null hypothesis is true
  - The sample means may be different
    - If so, the only reasonable explanation that we can offer for these differences is the operation of experimental error

Estimates of Treatment Effects

- If the null hypothesis is false, there are real differences among the means of the treatment populations

- A systematic component that contributes to the differences among the means is called the treatment effect
  - Differences among treatment means may reflect two different quantities

Estimates of Treatment Effects

- When the population means are equal the differences among the group means will reflect the operation of experimental error alone

- When the population means are not equal, the differences among the group means will reflect the operation of:
  - An unsystematic component (i.e., experimental error), and
  - A systematic component (i.e., treatment effects)

Evaluation of the Null Hypothesis

- When the null hypothesis is true, two estimates of experimental error are available from the experiment

- The ratio of the two estimates can produce a useful statistic

$$\frac{\text{differences among treatment means}}{\text{differences among subjects treated alike}} = \frac{\text{experimental error}}{\text{experimental error}}$$

- We would expect to find an average value of this ratio of approximately 1.0

Evaluation of the Null Hypothesis

- Consider now the same ratio when the null hypothesis is false
- The ratio becomes:

$$\frac{\text{treatment error} + \text{experimental error}}{\text{experimental error}}$$

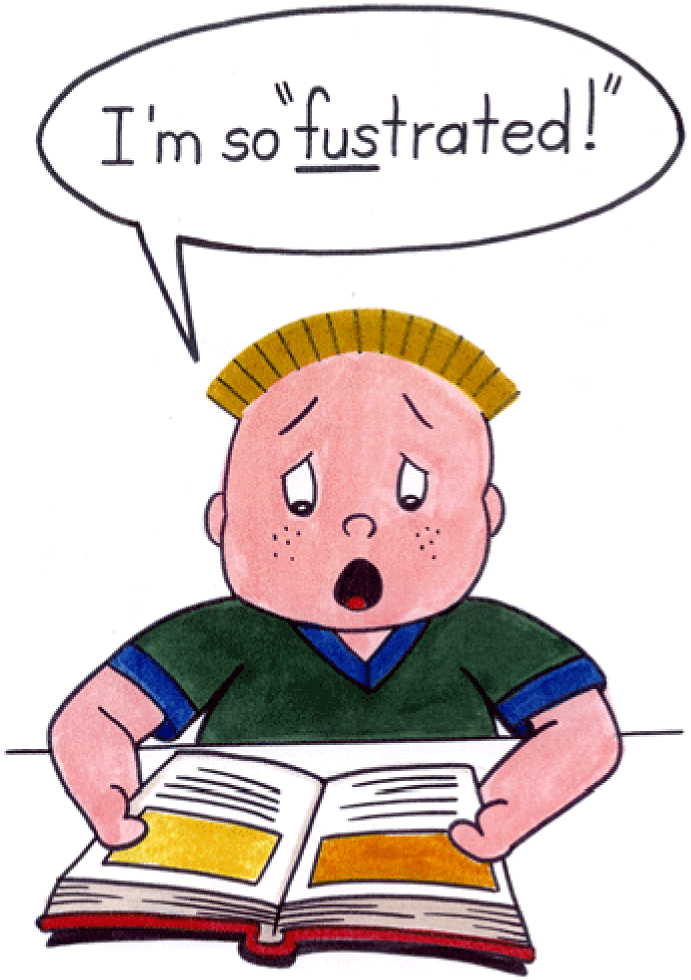
- Under replication, we would expect to find an average value of this ratio that is greater than 1.0
- We make decisions about the null hypothesis as a reflection of how likely the result is due to chance
- If the probability of obtaining ratio of this size or larger by chance is reasonably low, we will reject the null hypothesis

THE COMPONENT DEVIATIONS

Component Deviations

- Let’s try an example:

- Suppose we were interested in the effect of therapeutic drugs on reading comprehension of hyperactive boys



Example

- Three conditions of a one-factor study:
  - One group of boys serves as a control or placebo condition
  - A second group is given one of the drugs
  - A third group is given the other
- The independent variable, types of drugs is referred to as factor A
- The three levels of factor A are denoted as a_1, a_2, and a_3
- The subjects are drawn from a fourth-grade class and randomly assigned to each of the levels of factor A
- There are five subjects in each level
- The response measure is the score for each subject, denoted by Y
  - The number of test items correctly answered by each student

Example Data

Level a_1	Level a_2	Level a_3
16	4	2
18	7	10
10	8	9
12	10	13
19	1	11

$$\bar{Y}_{a_1} = 15 \qquad \bar{Y}_{a_2} = 6 \qquad \bar{Y}_{a_3} = 9$$

$$\bar{Y}_T = 10$$

The Grand Mean

Deviations

- Let us use the notation $Y_{i,j}$ to represent
  - A score form the j^{th} level of factor A, and
  - The i^{th} subject

- Note we may also use Y_{ij}
  - For example, $Y_{5,2} = 1$

- The deviation of $Y_{i,j}$ from the grand mean $\bar{Y}_T$ can be seen as two components:

$$\left(Y_{i,j} - \bar{Y}_T\right) = \left(\bar{Y}_{a_i} - \bar{Y}_T\right) + \left(Y_{i,j} - \bar{Y}_{a_i}\right)$$

Deviations

- The total deviation is: $\left(Y_{i,j} - \overline{Y}_T\right)$
- The between groups deviation is: $\left(\overline{Y}_{a_i} - \overline{Y}_T\right)$
- The within groups deviation is: $\left(Y_{i,j} - \overline{Y}_{a_i}\right)$

Deviations Explained

- The score for each subject in an experiment can be expressed as:
  - A deviation from the grand mean

- This deviation can be partitioned into two components:
  - A between-groups deviation
  - A within-groups deviation

Deviations Explained

- The between and within deviations are central:
  - A quantity that will reflect **treatment effects** in the population in addition to experimental error (i.e., the between-groups deviation)
  - A quantity that will reflect **experimental error** alone (i.e., the within-groups deviation).

SUMS OF SQUARES: DEFINING FORMULAS

Sums of Squares

- To evaluate the null hypothesis, it is necessary to transform the between-groups and within-groups deviations into more useful quantities:
  - Variances

- How ANOVA gets its name

Variance

- A variance is defined as follows:

$$\text{variance} = \frac{\text{sum of squared deviations from the mean}}{\text{degrees of freedom}} = \frac{SS}{df}$$

- where df is approximately equal to the number of cases in the set

- This means that the variance is basically an average of the squared deviations

Handy Properties of Sums of Squares

- Note that:

$$SS_{total} = SS_{between\ groups} + SS_{within\ groups}$$

or, equivalently,

$$SS_T = SS_A + SS_{S/A}$$

- Where T denotes total
- A denotes the factor
- S/A denotes the subjects within the factor

Total Sum of Squares

- The total sum of squares is formed by squaring the total deviation for each subjects and summing the squares of the total deviations:

$$SS_T = \sum \left(Y_{ij} - \bar{Y}_T\right)^2$$

- Where:

$$\bar{Y}_T = \frac{\sum Y_{ij}}{N}$$

- In our example $SS_T = 390$

Between-Groups Sums of Squares

- The between-groups sums of squares:

$$SS_A = \sum_{j=1}^{a} n_j \left(\bar{Y}_{A_j} - \bar{Y}_T\right)^2$$

Where a is the total number of groups or treatment levels and:

$$\bar{Y}_{A_j} = \sum_{i=1}^{n_j} \frac{Y_{ij}}{n_j}$$

Between Groups Sums of Squares

- If a=3 then

$$SS_A = n_1\left(\overline{Y}_{A_1} - \overline{Y}_T\right)^2 + n_2\left(\overline{Y}_{A_2} - \overline{Y}_T\right)^2 + n_3\left(\overline{Y}_{A_3} - \overline{Y}_T\right)^2$$

- If all n are equal:

$$SS_A = n\sum_{j=1}^{a}\left(\overline{Y}_{A_n} - \overline{Y}_T\right)^2$$

- In our example, $SS_A = 210$

Within-Groups Sums of Squares

- The within-groups sums of squares:

$$SS_{S/A} = \sum_{i=1}^{a} \sum_{j=1}^{n_i} \left(Y_{ij} - \bar{Y}_{A_i} \right)^2$$

- Because they sum to the total, $SS_{S/A} = SS_T - SS_A$

- So, for our example, $SS_{S/A} = 390 – 210 = 180$

COMPUTATIONAL FORMULAS

Computational Formulas

- We usually calculate the sum of squares with formulas that are equivalent algebraically but much simpler computationally
  - Because the book makes use of these formulae, we will highlight them here…

- In the analysis of the complete randomized single-factor design, we need to designate three basic quantities:
  - The individual scores or observations (the raw data)
  - The sum of these scores for each treatment condition (the treatment sums or subtotals)
  - The sum of all the scores or observations (the grand sum or grand total)

Individual Scores

- The individual scores are designated by Y
- Sometimes Y with a subscript i or subscripts i and j can be used
- For example, Y_{ij} or $Y_{i,j}$ can be used where the subscript j refers to the levels of the independent variable, factor A ($j = 1,\ldots,a$), and the subscript i refers to the subject within the level j ($i = 1,\ldots,n_j$)

Treatment Sums and Means

- The treatment sums, or subtotals, are

$$A_j = \sum_{i=1}^{n_j} Y_{ij}$$

- The treatment means are:

$$\bar{Y}_{A_j} = \frac{A_j}{n_j}$$

Grand Sum and Grand Mean

- The grand sum is the sum of all the scores in the experiment:

$$T = \sum_{i=1}^{n_j} \sum_{j=1}^{a} Y_{ij} = \sum_{j=1}^{a} A_j$$

- The grand mean is (with equal sample sizes):

$$\bar{Y}_T = \frac{T}{N} = \frac{T}{(a)(n)}$$

Basic Ratios

- Basic ratios represent a common step in the computational formulas for sums of squares in the analysis of variance

- There are three basic ratios:
  - One involving the individual observation Y $$[Y] = \sum_{i=1}^{n_j} \sum_{j=1}^{a} Y_{ij}^2$$
  - Another involving the treatment subtotals A $$[A] = \frac{\sum_{j=1}^{a} A_j^2}{n}$$
  - The other involving the grand total T $$[T] = \frac{T^2}{(a)(n)}$$

Sums of Squares

- The basic ratios are combined to produce the three required sums of squares:
- $SS_T = [Y] – [T]$
- $SS_A = [A] – [T]$
- $SS_{S/A} = [Y] – [A]$
- Also, note $SS_T = SS_A + SS_{S/A}$

Concluding Remarks

- Sums of squares play an important role in determining if group differences observed in an experiment are due to chance or not

- The formulae presented today provide the basics of how ANOVA works: by partitioning variance into distinct types:
  - Between groups
  - Within groups

- Such partitioning will be what is done throughout this class for nearly all techniques

Up Next…

- Lab:
  - A sample experiment
  - How to do ANOVA in SPSS

- Homework:
  - Complete by the beginning of class on Wednesday

- Reading:
  - Chapter 3 of Keppel:
    - Variance Estimates and the F Ratio