VARIANCE ESTIMATES AND THE F RATIO

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Today's Class

- Completing the analysis (the ANOVA table).
- Evaluating the F ratio.
- Errors in hypothesis testing.
- A complete numerical example.

Completing the Analysis

The ANOVA Table

Completing the Analysis

- Table 3-1 presents a summary table of the analysis of variance that contains entries of the source, the basic ratio, the sum of squares, the degree of freedom, the mean squares, and the F value.

- Note that variance is SS/df.

Table 3.1: Summary of the analysis of variance

Source	Bracket Term	SS	df	MS	F
A	$[A] = \frac{\sum A_j^2}{n}$	$[A] - [T]$	$a - 1$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{S/A}}$
S/A	$[Y] = \sum Y_{ij}^2$	$[Y] - [A]$	$a(n-1)$	$\frac{SS_{S/A}}{df_{S/A}}$	
Total	$[T] = \frac{T^2}{an}$	$[Y] - [T]$	$an - 1$		

Degrees of Freedom

- The degree of freedom, df, associated with a sum of squares corresponds to the number of scores with independent information that enter into the calculation of the sum of squares.

- The general rule for computing the df of any sum of square is
  - df = (number of independent observations) -(number of restraints)

    or

  - df = (number of independent observation) -(number of population estimates).

Degrees of Freedom

- Note that $df_T = df_A + df_{S/A}$ and $(a)(n)-1 = (a-1)+a(n-1)$.

- For example, for data in Table 2-2, SS_T has $df_T = df_A+df_{S/A} = (3-1)+3(5-1) = 14$ given the estimate of the population mean is [Y] = 10 and, consequently, there is one constraint.

Mean Squares

- The mean squares are given by: MS = SS/df.

- So far, we have two mean squares:

$$MS_A = \frac{SS_A}{df_A} \qquad MS_{S/A} = \frac{SS_{S/A}}{df_{S/A}}$$

Confidence Intervals for Treatment Means

- The confidence interval for a population mean is given by the formula:

$$\bar{Y}_j - t \times s_{M_j} \leq \mu \leq \bar{Y}_j + t \times s_{M_j}$$

- Where $\bar{Y}_j$ is the mean of the treatment group
- t is a critical value with the degrees of freedom of n_j-1
- s_{M_j} is the estimated standard error of the mean or $s_j/\sqrt{n_j}$.
- For the pooled estimate, the degrees of freedom will be a(n-1) and s_M will be $\sqrt{MS_{S/A}/n_j}$.

The F Ratio

- The formula of the F ratio is $F = MS_A / MS_{S/A}$
- The degrees of freedom are:
  - $df_A = df_1 = a-1$
  - $df_{S/A} = df_2 = a(n-1)$.
- The F ratio is approximately 1.0 when the null hypothesis is true and is greater than 1.0 when the null hypothesis is false.

Evaluating the F Ratio

The Sampling Distribution of F

- A frequency distribution of a statistic (e.g., F) is called a sampling distribution of the statistic.
- Suppose that for an experiment a = 3, n = 5 (or N = 15), and the null hypothesis is true so that $\mu_1 = \mu_2 = \mu_3$.
- Assume that we draw a very large number of such experiments each consisting of three groups of 5 scores, and that we compute the value of F for each case.
- We can construct a graph relating F and frequency of occurrence [see Figure 3-1 for F(2,12)].

The F Table

- The exact shape of the F distribution is determined by the number of df's associated with the numerator and denominator mean squares in the F ratio.
- We will use $F(df_{num.}, df_{denom.})$ or $F_{df1,df2}$.
- An F table is found in Appendix A.1 (see pp. 571-575).

Using the F Table

- A particular value of F in this table is specified by three factors:
  - (1) the numerator df
  - (2) the denominator df
  - (3) the value of α
    - Where α refers to the proportion of area to the right of an ordinate drawn at F_α.
    - The α levels, α = .10, .05, .025, .01, and .001, are ones most commonly encountered, and these are listed in the table.

The Distribution of F When the Null Hypothesis Is False

- The theoretical distribution when H_0 is false is called the noncentral F (i.e., F') distribution.

Testing the Null Hypothesis

- The two hypotheses are:

H_0: $\mu_1 = \mu_2 = \mu_3$.

H_1: not all μ's are equal.

- Assume that we have conducted an experiment and that we have computed the value of F.
- Suppose we could agree on a dividing line for any F distribution, where values of F falling above the line are considered to be unlikely (i.e., incompatible with H_0) and values of F falling below the line as considered to be likely (i.e., compatible with H_0).

Decision Rules

- One decision rule is to reject the null hypothesis when the observed F falls within the region of incompatibility.
- In practice, there is fairly common agreement on a probability of $\alpha = .05$ to define the region of incompatibility for the F distribution.

Decision Rules

- This probability may be called the significance level. The rejection rule is:
- Reject H_0 when $F_{observed} \geq F_{(\alpha)}(df_{num.}, df_{denom.})$;
  retain H_0 otherwise.

- A researcher sometimes reports that an F is significant at the one percent level of significance.
- Occasionally a researcher reports the exact probability of $F_{observed}$, a value that is provided automatically by most statistical computer programs.
- The exact probability refers to the proportion of the sampling distribution of the F statistic fall at or above (i.e., more extreme) the F found in an experiment.
- We do not need to consult an F table to determine significance, but instead we simply compare the exact probability with the chosen significance level and reject H_0 if the exact probability is smaller than the chosen significance level.
- For example, provided that $\alpha = .05$, reject H_0 if $p \leq .05$ and retain H_0 otherwise.

Avoiding Common Misuses of Hypothesis Testing (p. 45-46)

- Never lose site of your data.
- Remember that not all null hypotheses represent interesting or plausible outcomes of the experiment and that their rejection is not inevitably informative.
- Remember that the null-hypothesis test only gives information about how likely the sampling operations are to have produced your effect.
- When you find a significant effect, ask what it means.
- Always interpret your results in the context of other studies in the field.

Errors in Hypothesis Testing

Errors in Hypothesis Testing

- If we reject H_0 when it is true, then we make the type I error.
- If we retain H_0 when it is false, then we make the type II error.
- Note that α is the probability of type I error, and that β is the probability of type II error (see Table 3-3 on page 47).
- Power refers to the probability of rejecting the null hypothesis when an alternative hypothesis is true (i.e., power = $1 - \beta$).

A Complete Numerical Example

Vigilance Task While Sleep Deprived

- There are a = 4 conditions, namely, 4, 12, 20, and 28 hours without sleep.
- There are n = 4 subjects randomly assigned to each of the different treatment conditions.
- The vigilance task score represents the number of failures to spot objects on a radar screen during a 30-minute test period.

Data (computation p. 51)

Hours without sleep			
4 hr	12 hr	20 hr	28 hr
a_1	a_2	a_3	A_4
37	36	43	76
22	45	75	66
22	47	66	43
25	23	46	62

Final Thought

- The F-Ratio is the vehicle to test the null hypothesis (es) of the experiment.
- The F-Ratio is composed of two variances: between groups and within groups.

- The ratio follows the idea of variance partitioning that will be present throughout the class.
- Don’t get too wrapped up into the formulae presented today: most every calculation will be done using a computer.

Next Class

- SPSS Introduction