THE SINGLE-FACTOR WITHIN-SUBJECT DESIGN

ERSH 8310

Keppel and Wickens Chapter 16

Today's Class

- The simplest within-subject design with a single within-subject factor is discussed.
- The basic analysis of variance, tests of contrasts, effect-size calculations, and power determinations are presented.

The Analysis of Variance – Design and Notation

- Table 16.1 shows the basic structure of the scores in a single-factor within-subject design.
- The scores in the data table are denoted by Y_{ij}, where the first subscript i indicates the subject, and the second subscript j indicates the particular level of factor A.
- There are a levels of factor A and n subjects.
- The subjects are nested within the groups, and this fact is symbolized by denoting the subject factor S/A.
- It may be convenient and accurate to refer to this design as an A ×S design.

The Analysis of Variance – Design and Notation

Table 16.1: The notation system for an $A \times S$ design with $a = 4$ conditions and $n = 3$ subjects.

Subjects	a_1	a_2	a_3	a_4	Sum
	Levels of factor A				
s_1	Y_{11}	Y_{12}	Y_{13}	Y_{14}	S_1
s_2	Y_{21}	Y_{22}	Y_{23}	Y_{24}	S_2
s_3	Y_{31}	Y_{32}	Y_{33}	Y_{34}	S_3
Sum	A_1	A_2	A_3	A_4	T

Formulae

Note in Table 16.1,

$$\bar{Y}_j = A_j/n,$$

$$\bar{Y}_{S_i} = S_i/a,$$

and

$$\bar{Y}_T = T/an.$$

Partitioning the Variability

- The amount of variability among the scores in a one-way between-subject design is measured by the total variability, as expressed by the total sum of squares,

$$SS_T = \sum_{ij} (Y_{ij} - \bar{Y}_T)^2.$$

- Just as the between-subjects variability in the two-factor A $\times$B design with crossed factors id divided into two effects and an interaction, the total variability in an A $\times$S design is divided into SS_A, SS_S, and $SS_{A \times S}$.
- There is no within-cell term comparable to $SS_{S/AB}$ and $SS_{A \times S}$ will be treated as error.

Computational Formulas

- The computational formulas for the A ×S design are presented in Table 16.2.
  - Note that df_T = an - 1 that is one less than the total number of observations.

Table 16.2: Computational formulas for the $A \times S$ design.

Formulas for the analysis of variance

Source	df	SS	MS	F
A	$a-1$	$[A]-[T]$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{A\times S}}$
S	$n-1$	$[S]-[T]$	$\frac{SS_S}{df_S}$	
$A\times S$	$(a-1)(n-1)$	$[Y]-[A]-[S]+[T]$	$\frac{SS_{A\times S}}{df_{A\times S}}$	
Total	$an-1$	$[Y]-[T]$		

Formulas for the bracket terms

$$[A] = \frac{\sum A_j^2}{n} = n\sum \bar{Y}_A^2 \qquad [S] = \frac{\sum S_i^2}{a} = a\sum \bar{Y}_S^2$$

$$[Y] = \sum Y_{ij}^2 \qquad [T] = \frac{T^2}{an} = an\bar{Y}_T^2$$

Numerical Example

Searching for Letters

- Consider an experiment in which college students search for a particular letter in a string of letters on a computer screen. Half of the time the letter occurs in the string, and half of the time it does not.
- On one third of the trials the letter string is a word (condition a_1), on one third it is a pronounceable nonword (a_2), and on one third it is an unpronounceable set of random letters (a_3).
- The response measure is the average speed with which subjects correctly detect the target letter, measured in milliseconds.
- The experiment is a single-factor AxS design with a=3 types of letter strings.

The Old Style of Analysis

- Prior to running the repeated measures analysis, let's imagine we tried using what we already know about ANOVA:
  - Let's run the analysis as if we have subjects nested within each factor (not crossed).

Old Analysis – No Repeated Measures

Between-Subjects Factors

		N
condition	1.00	6
	2.00	6
	3.00	6

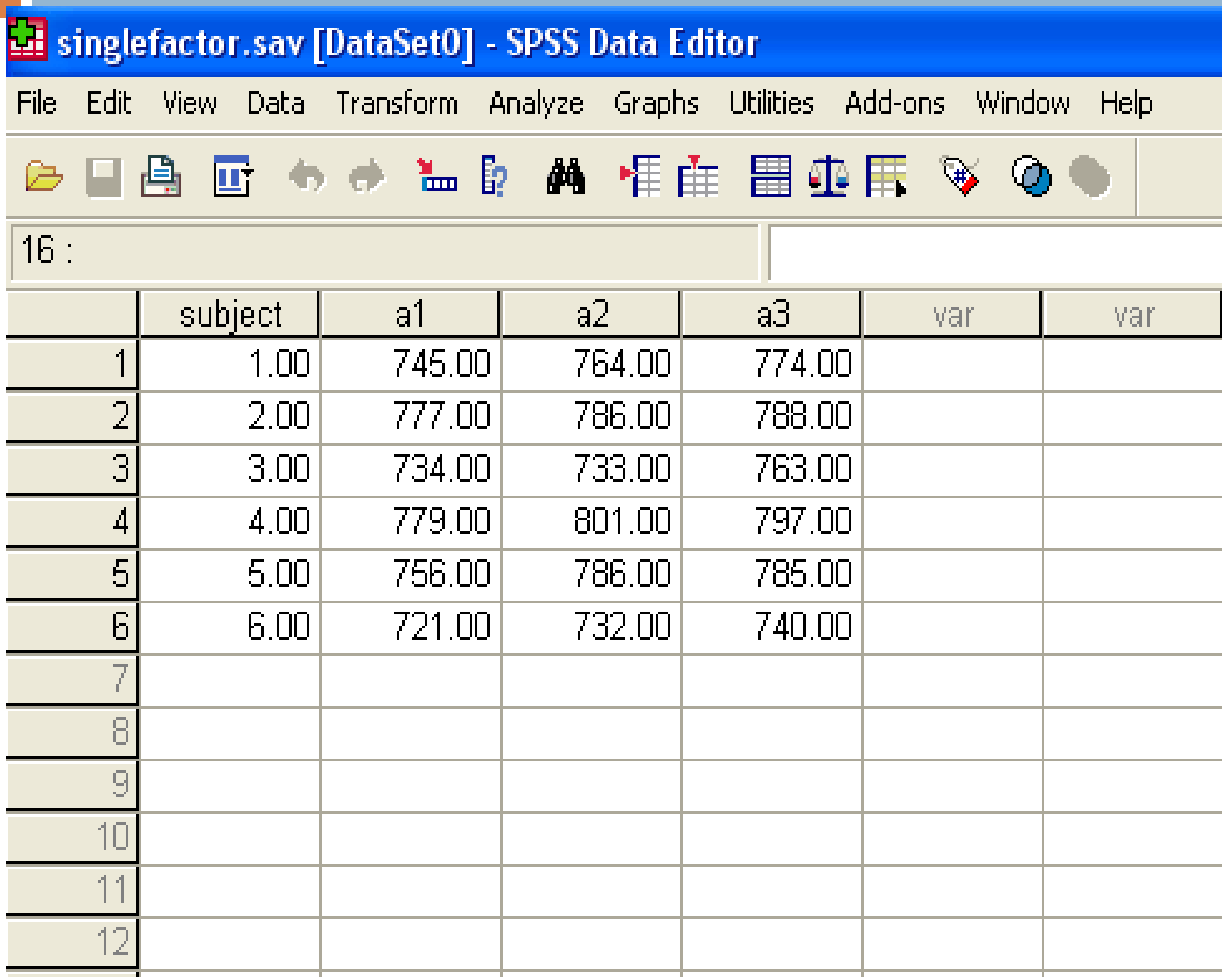
Tests of Between-Subjects Effects

Dependent Variable: time

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1575.000[a]	2	787.500	1.299	.302
Intercept	10520284.5	1	10520284.50	17353.524	.000
condition	1575.000	2	787.500	1.299	.302
Error	9093.500	15	606.233		
Total	10530953.0	18			
Corrected Total	10668.500	17			

a. R Squared = .148 (Adjusted R Squared = .034)

Repeated Measures: Data File in SPSS

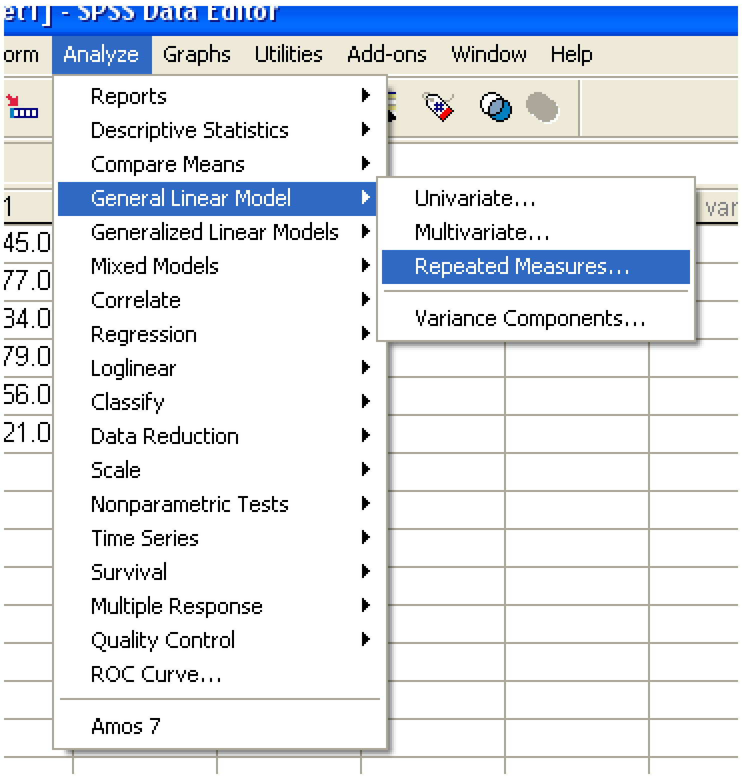


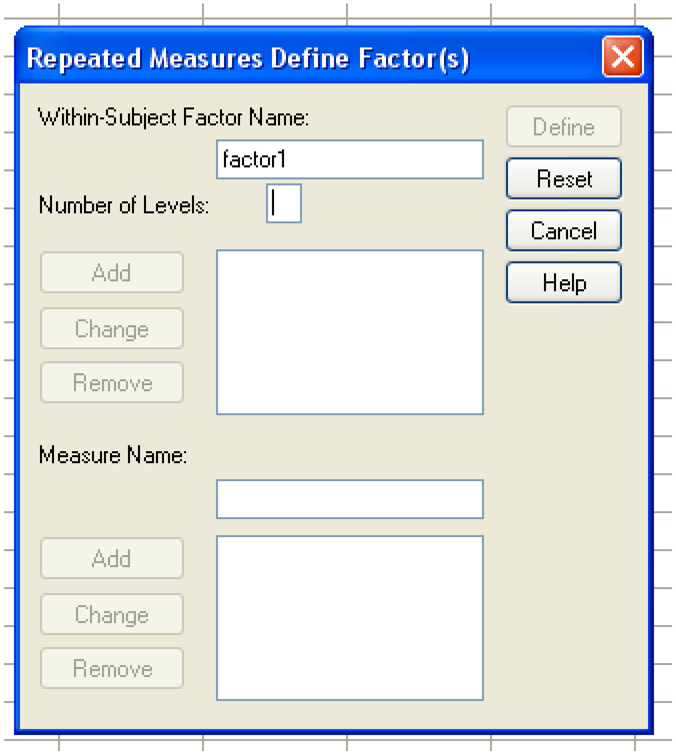
	subject	a1	a2	a3	var	var
1	1.00	745.00	764.00	774.00		
2	2.00	777.00	786.00	788.00		
3	3.00	734.00	733.00	763.00		
4	4.00	779.00	801.00	797.00		
5	5.00	756.00	786.00	785.00		
6	6.00	721.00	732.00	740.00		
7						
8						
9						
10						
11						
12						

The data are arranged in a wide-format:

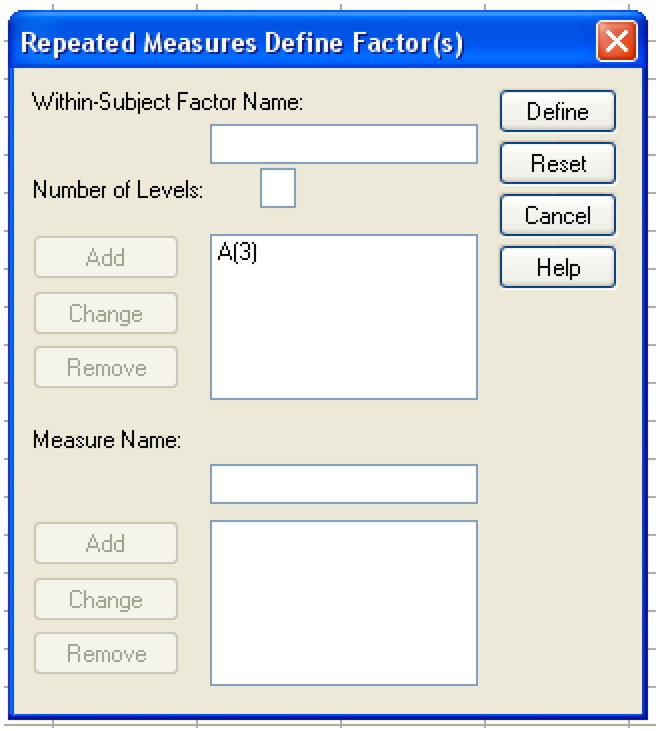
- One variable per column.

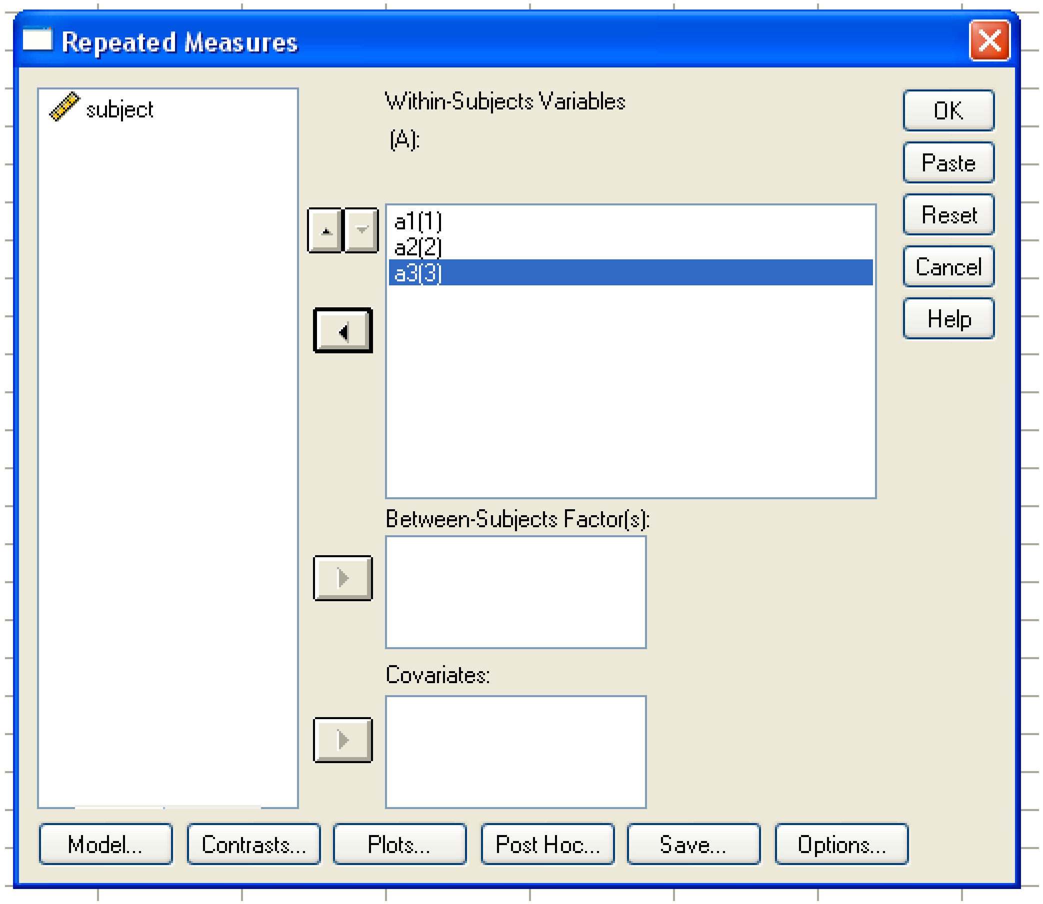
Running the Analysis





Running the Analysis





Analysis Output: SPSS

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
A	Sphericity Assumed	1575.000	2	787.500	14.432	.001
	Greenhouse-Geisser	1575.000	1.610	978.061	14.432	.003
	Huynh-Feldt	1575.000	2.000	787.500	14.432	.001
	Lower-bound	1575.000	1.000	1575.000	14.432	.013
Error(A)	Sphericity Assumed	545.667	10	54.567		
	Greenhouse-Geisser	545.667	8.052	67.771		
	Huynh-Feldt	545.667	10.000	54.567		
	Lower-bound	545.667	5.000	109.133		

Analysis Output: Table 16.3

The AS data table

Subjects	a_1	a_2	a_3	Sum
	Types of strings			
s_1	745	764	774	2,283
s_2	777	786	788	2,351
s_3	734	733	763	2,230
s_4	779	801	797	2,377
s_5	756	786	785	2,327
s_6	721	732	740	2,193
Sum	4,512	4,602	4,647	13,761
$\sum_i Y_{ij}^2$	3,395,728	3,534,002	3,601,223	
Mean	752.00	767.00	774.50	
s_j	23.26	29.22	20.60	
s_{M_j}	9.50	11.93	8.41	

Calculation of the bracket terms

$$[A] = \frac{\sum A_j^2}{n} = \frac{4{,}512^2 + 4{,}602^2 + 4{,}647^2}{6} = 10{,}521{,}859.5$$

$$[S] = \frac{\sum S_i^2}{a} = \frac{2{,}283^2 + \cdots + 2{,}193^2}{3} = 10{,}528{,}832.3$$

$$[Y] = \sum Y_{ij}^2 = 745^2 + 764^2 + \cdots + 732^2 + 740^2 = 10{,}530{,}953.0$$

$$[T] = \frac{T^2}{an} = \frac{13{,}761^2}{(3)(6)} = 10{,}520{,}284.5$$

Summary of the analysis of variance

Source	SS		df	MS	F
A	$[A] - [T] =$	1,575.0	2	787.50	14.43*
S	$[S] - [T] =$	8,547.8	5	1,709.56	
$A \times S$	$[Y] - [A] - [S] + [T] =$	545.7	10	54.57	
Total	$[Y] - [T] =$	10,668.5	17		

* $p < .05$

Analytical Comparisons

Within-Subject Contrasts and Error Variability

- The sum of squares for the contrast is

$$SS_{\psi_A} = \frac{n\,\hat{\psi}_A^2}{\sum_j c_j^2},$$

- And $SS_{\psi_A} = MS_{\psi_A}$ because the degree of freedom is on

Within-Subject Contrasts and Error Variability

- The error term $MS_{S/A}$ is not appropriate. Instead the F ratio is

$$F_{\psi_A} = \frac{MS_{\psi_A}}{MS_{\psi_A \times S}},$$

where

$$MS_{\psi_A \times S} = \frac{\sum_i \hat{\psi}_i^2 - n\,\hat{\psi}_A^2}{n - 1}$$

and

$$\hat{\psi}_i = \sum_i c_j Y_{ij}.$$

Testing a Within-Subject Contrast

- The term $[\hat{}(\psi)]_i$ is called a contrast variable.
- We may use a t procedure to test the contrast.
- The t and F procedures are equivalent because $t_{df}^2 = F_{df1 = 1,\ df2 = df}$.
- When the number of contrasts being tested is not large, the Bonferroni or Sidák-Bonferroni corrections are useful.
  - These procedures control familywise error by adopting a more stringent level of significance for the additional tests (e.g., α_{FW}/c for Bonferroni).

Effect Size and Power

Estimating Treatment Effects

- The estimate of the partial omega squared is:

$$\hat{\omega}_A = \frac{(a-1)(F_A-1)}{(a-1)(F_A-1)+an}.$$

- The squared correlation ratio (η^2 or R^2) is obtained by:

$$\eta^2 = \frac{SS_A}{SS_A + SS_{A \times S}}.$$

Power and Sample Size

- Power and sample-size calculations in the within-subject design are based on the measures of effect size.
- The example data (and the summary of the analysis of variance) contained in Table 16.7 can be used to illustrate the calculation of a required sample size to achieve a given power.

Final Thought

- The repeated measures ANOVA partitions variability due to a subject.
- Removing such variability aids in the power of the test.

- The repeated measures analysis described in this class was an initial first pass at the approach.
  - We will see assumptions of such an approach are very strong.
- Newer methods will relax some of these assumptions.

Next Time…

- Chapter 17: Within Subject Designs: Further Topics
- Final exam handed out.