ANALYSIS OF TREND
CHAPTER 5

ERSH 8310 • Lecture 7 • September 13, 2007

Today's Class

- Analysis of trends…
  - Using contrasts to do something a bit more practical.

- Linear trends.
- Quadratic trends.
- Trends in SPSS.

Today's Example Data

Training Times

Yet Another Experiment

- From p. 89:

"Consider an experiment designed to test the proposition that subjects learn better when training is distributed or spaced over a period of time than when the training is massed all at once. We could investigate this question with just two groups, one group that receives massed training and another that receives training at spaced intervals. However, this study would allow only a restricted glimpse of the phenomenon. Instead we could conduct a more comprehensive study that included conditions with several different spacings. It would provide information about the *form* of the relationship between learning and the degree of distributed training, giving a much clearer picture of the spacing phenomenon."

The Task…

- P. 89, continued…

"The subject's task is to track a spot moving in a random path around a computer screen, using a control stick to position a circle over it. Each subject has 10 one-minute practice trials learning how to track the spot. The independent variable is the time between practice trials: For one group the trials immediately follow another (0 seconds spacing – the massed condition); for a second group there is a 20-second pause between trials, for a third group there is a 40-second pause; and for the fourth group there is a 60-second pause. Twenty minutes after the last practice trial, each subject is given a 30-second test trial, and the dependent variable is the number of seconds that the subject is able to keep on the target. Suppose n=5 subjects are randomly assigned to the four groups. [The scores are given in the table on the next slide.]"

Analysis of Linear Trend

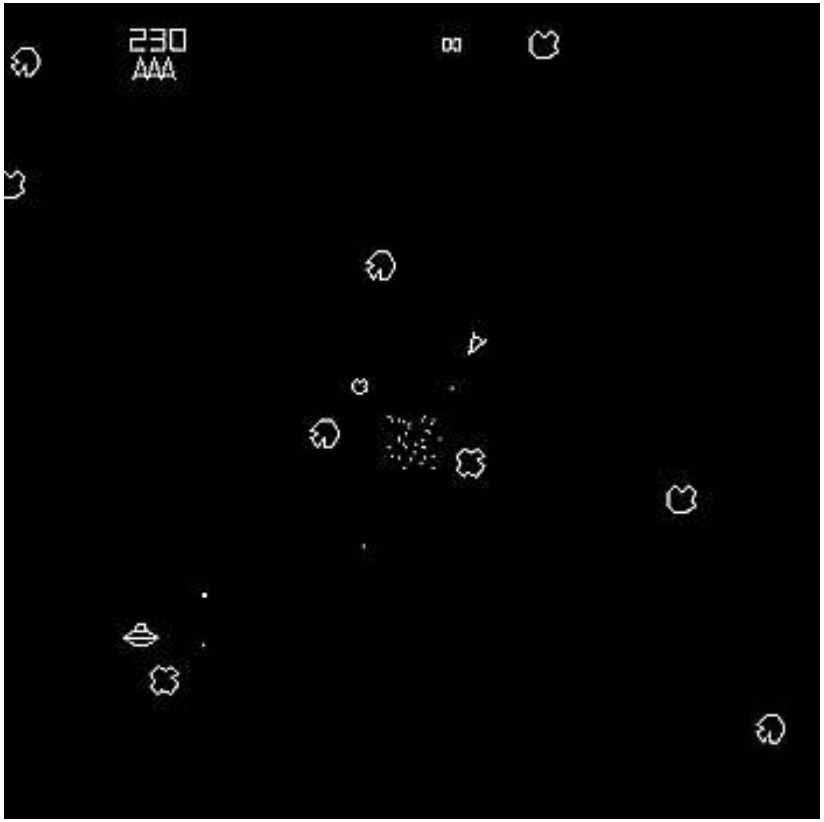
Trend Analysis

- Trend analysis is a specialized form of single-df comparisons when a quantitative independent variable is manipulated.
- The treatment levels represent different amount of a single common variable (e.g., the number of hours of food deprivation, different dosage levels of particular drug, rates of stimulus presentation, and the intensity of the unconditional stimulus in a conditioning experiment).
- We usually plot the entire set of treatment means on a graph, connect the points, and examine the display for any underling shape or trend.

A Numerical Example

- Assume the following experiment:
  - Subjects are given some material to learn with the independent variable, the trial interval of 0, 20, 40, and 60 seconds (i.e., $a = 4$).
  - The dependent variable is the number of seconds that the subject is able to keep on the target (i.e., tracking score).
  - Suppose that $an = 20$ and the hypothetical results are presented in Table 5.1.

Tracking Task…Bringing You To Our Data



The Data

Intertrial Interval (A)			
0 seconds	20 seconds	40 seconds	60 seconds
a_1	a_2	a_3	a_4
4	18	24	16
6	13	19	17
10	15	21	13
9	11	16	23
11	13	15	21

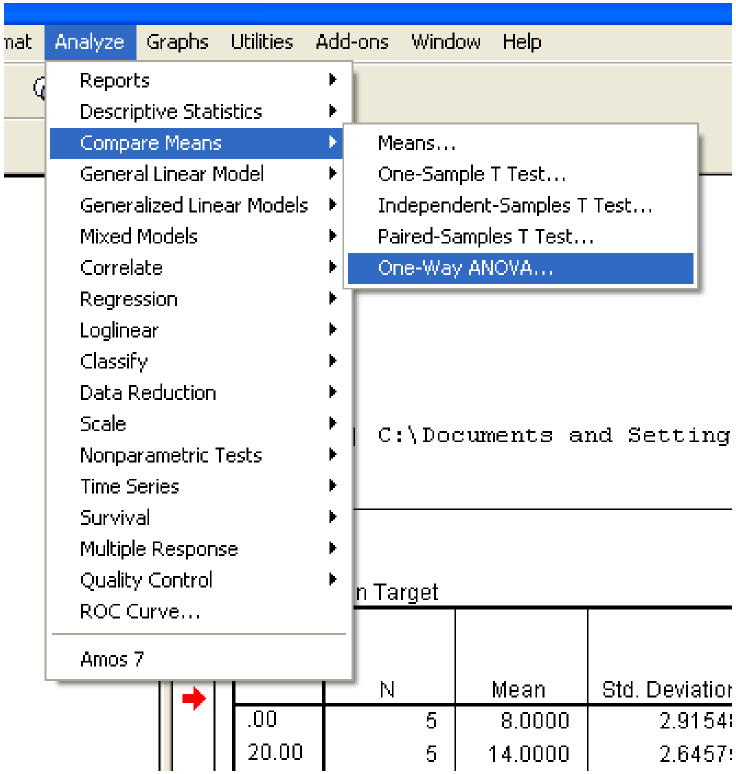
Testing for Linear Trend

- A way of assessing linear trend is to use a set of coefficients that represents an idealized version of a straight line (see Appendix A.3 on p. 577).
- Note that there are a-1 number of the orthogonal polynomials.
- For our example, this would be:

c_1	c_2	c_3	c_4
-3	-1	1	3

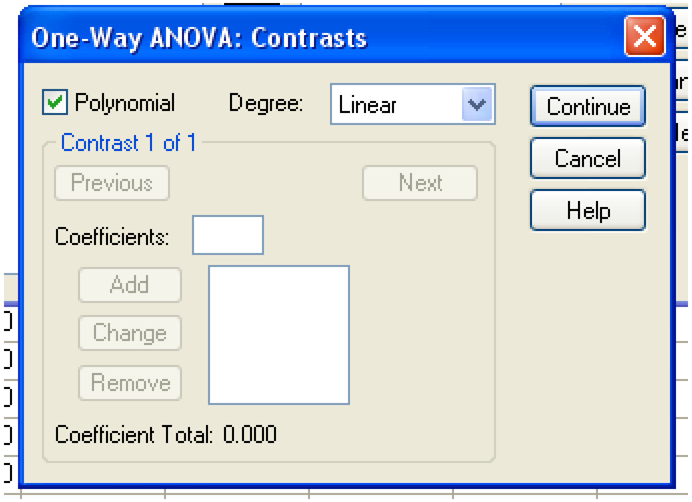
Example in SPSS

- Run the ANOVA:
  - Analyze…Compare Means…One-Way ANOVA



Example in SPSS

- Click on the “Contrasts…” button…
  - Be sure the checkbox by polynomial is checked.
  - Select “linear” in the degree listbox.



Descriptives…

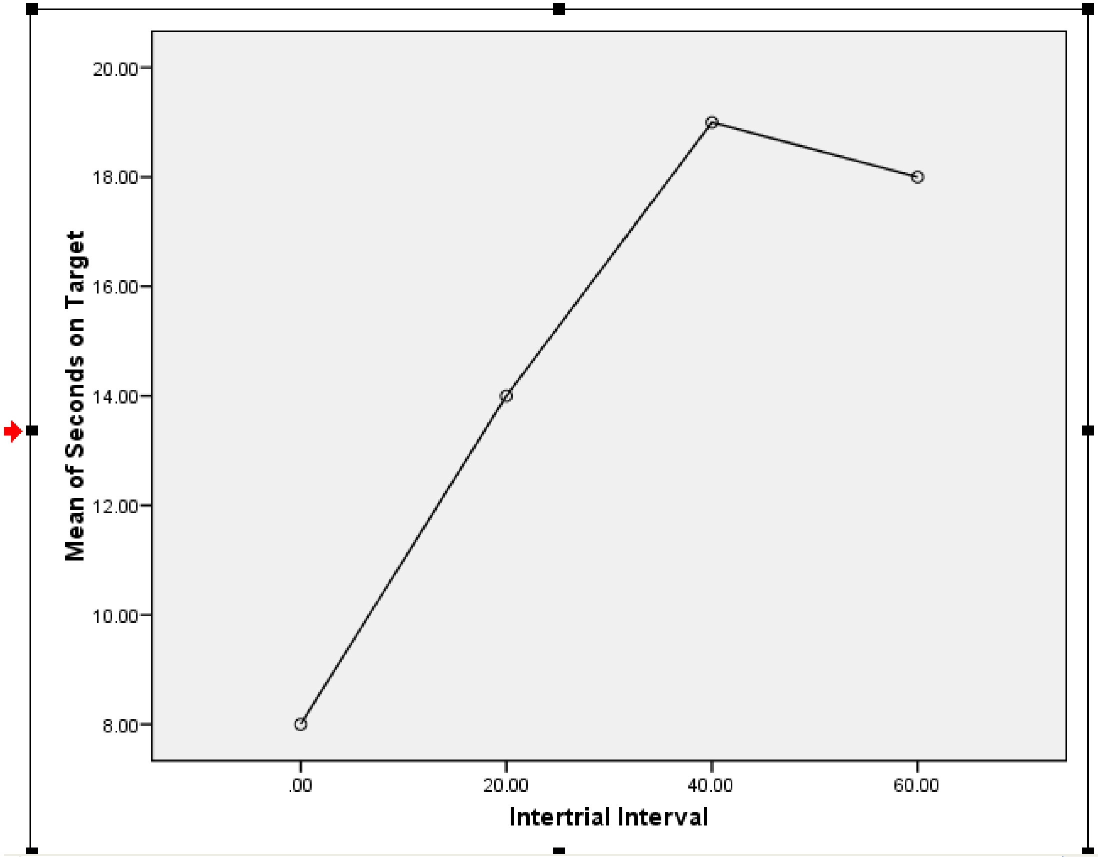
Descriptives

Seconds on Target

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
.00	5	8.0000	2.91548	1.30384	4.3800	11.6200	4.00	11.00
20.00	5	14.0000	2.64575	1.18322	10.7149	17.2851	11.00	18.00
40.00	5	19.0000	3.67423	1.64317	14.4378	23.5622	15.00	24.00
60.00	5	18.0000	4.00000	1.78885	13.0333	22.9667	13.00	23.00
Total	20	14.7500	5.39859	1.20716	12.2234	17.2766	4.00	24.00

Means Plot (Is There A Trend?)

Means Plots



The Single-df Linear Trend Test

- The single-df comparison for the testing of linear trend is performed with:

$$\hat{\psi}_{\text{linear}} = \sum_{j=1}^{a} c_{1j}\,\bar{Y}_j \,,$$

$$SS_{\text{linear}} = \frac{n\,\hat{\psi}^2_{\text{linear}}}{\sum_{j=1}^{a} c_{1j}^2} \,,$$

$$F = \frac{MS_{\text{linear}}}{MS_{S/A}} \,,$$

We Can Formulate Our Contrast

	A	B	C	D	E	F
1	Level	Mean	Coefficient	(Coefficient*Mean	Coefficient^2	
2	0 sec	8	-3	-24	9	
3	20 sec	14	-1	-14	1	
4	40 sec	19	1	19	1	
5	60 sec	18	3	54	9	
6						
7	Sum			35	20	
8						
9	Contrast SS = n*sum(Coefficient*Mean)^2/sum(Coefficient^2)					
10	306.25					
11						

Contrast Test of Hypothesis

- We can test the hypothesis:
- $H_{0:}\ \Psi = 0$

This is the p-value for the contrast hypothesis test

ANOVA

Seconds on Target

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		373.750	3	124.583	11.074	.000
	Linear Term	Contrast	306.250	1	306.250	27.222	.000
		Deviation	67.500	2	33.750	3.000	.078
Within Groups			180.000	16	11.250		
Total			553.750	19			

Contrast Coefficients

- The c_{1j} are the linear coefficients in Appendix A.3 (e.g., -3, -1, 1, 3, for a = 4) and $df_{num} = 1$ and $df_{denom} = a(n-1)$.
- The four means can be plotted and a straight line can also be drawn.
- The straight line (i.e., the linear function) does not fit the data perfectly.

Plotting the Regression Line

- We may express the linear relationship using the liner regression equation,

$$\bar{Y}_j' = b_0 + b_1 X_j,$$

where $[\bar{Y}]_j'$ is the predicted mean for level a_j, $b_0 = [\bar{Y}]_T - b_1 [\bar{X}]$, and $b_1 = (\sum_{j=1}^{a} x_j [\bar{Y}]_j) / (\sum_{j=1}^{a} x_j^2)$, where x_j are the deviations based on the X_j (e.g., -30, -10, 10, 30 instead of -3, -1, 1, 3).

Evaluating the Linear Fit

- It can be determined if the linear function provides a complete summary of the relationship between X and Y. The test is:

$$F_{failure} = \frac{MS_{failure}}{MS_{S/A}},$$

$$MS_{failure} = \frac{SS_{failure}}{df_{failure}},$$

$$SS_{failure} = SS_{A} - SS_{linear},$$

$$df_{failure} = df_{A} - df_{linear}.$$

Analysis of Quadratic Trend

Analysis of Quadratic Trend

- A quadratic trend is one that displays concavity, a single bend either upward or downward.
- The coefficients of orthogonal polynomials in Appendix A.3 can be used to test quadratic trend.
- The testing of quadratic trend can be accomplished by the formulae on the next slide.

Our Example

- Quadratic contrast coefficients:

c_1	c_2	c_3	c_4
1	-1	-1	1

	A	B	C	D	E
1	Level	Mean	Coefficient	(Coefficient*Mean	Coefficient^2
2	0 sec	8	1	8	1
3	20 sec	14	-1	-14	1
4	40 sec	19	-1	-19	1
5	60 sec	18	1	18	1
6					
7	Sum			-7	4
8					
9	Contrast SS = n*sum(Coefficient*Mean)^2/sum(Coefficient^2)				
10	61.25				
11					

Testing Quadratic Trend

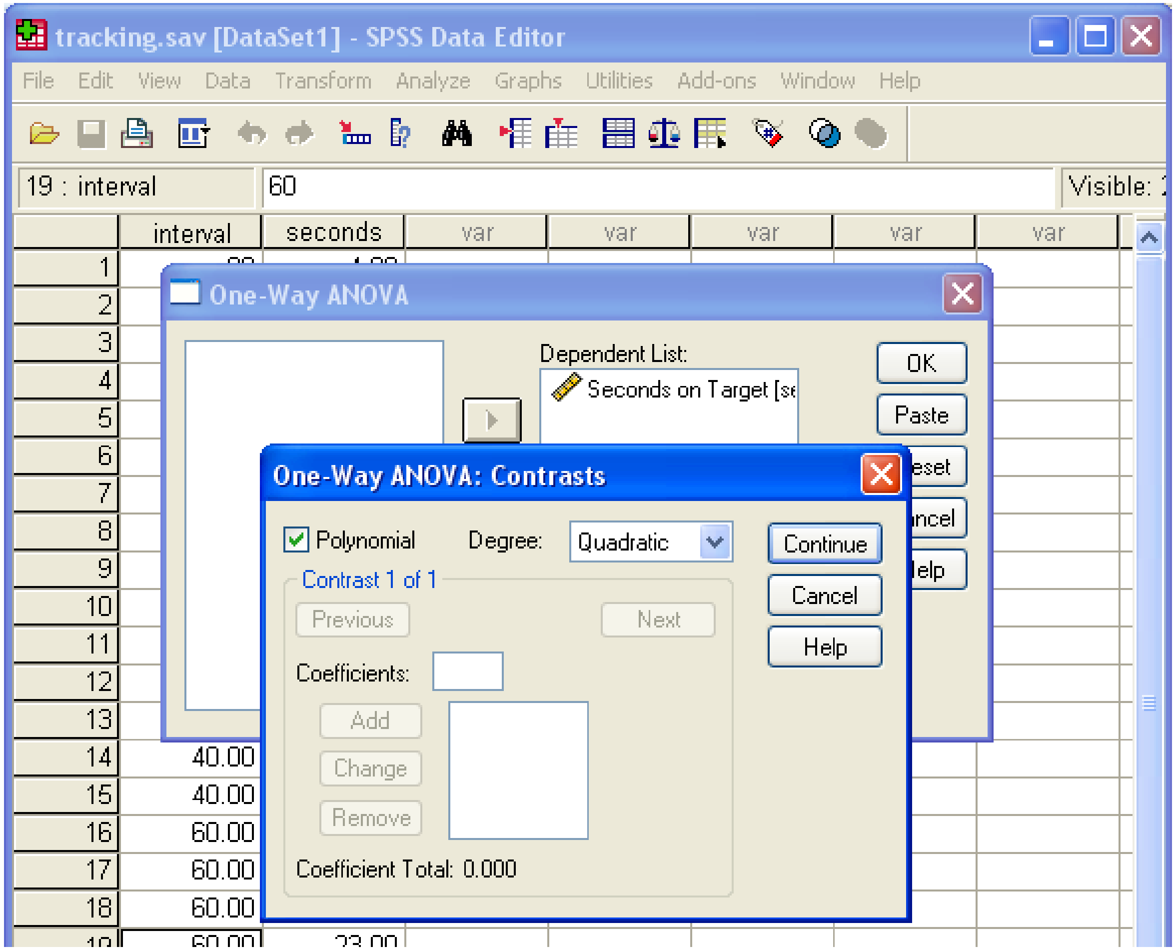
$$\hat{\psi}_{quadratic} = \sum_{j=1}^{a} c2_j \bar{Y}_j ,$$

$$SS_{quadratic} = \frac{n\, \hat{\psi}^2_{quadratic}}{\sum_{j=1}^{a} c2_j^2} ,$$

$$F = \frac{MS_{quadratic}}{MS_{S/A}}$$

with $df_{num} = 1$ and $df_{denom} = a(n-1)$.

In SPSS…


tracking.sav [DataSet1] - SPSS Data Editor
File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help
19 : interval
60
interval
seconds
var
One-Way ANOVA
Dependent List:
Seconds on Target [se
OK
Paste
One-Way ANOVA: Contrasts
Polynomial
Degree:
Quadratic
Continue
Cancel
Help
Contrast 1 of 1
Previous
Next
Coefficients:
Add
Change
Remove
Coefficient Total: 0.000
40.00
60.00

SPSS Output

- Is our quadratic trend contrast significant?

ANOVA

Seconds on Target

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		373.750	3	124.583	11.074	.000
	Linear Term	Contrast	306.250	1	306.250	27.222	.000
		Deviation	67.500	2	33.750	3.000	.078
	Quadratic Term	Contrast	61.250	1	61.250	5.444	.033
		Deviation	6.250	1	6.250	.556	.467
Within Groups			180.000	16	11.250		
Total			553.750	19			

Evaluating the Quadratic Fit

- The residual variation remaining beyond the linear and quadratic components can be evaluated with $F_{failure}$.
- The quadratic regression equation can be constructed, similarly to the linear regression equation:

$$\bar{Y}'_j = b_0 + b_1 X_j + b_2 X_j^2.$$

- Use a multiple regression program to obtain the regression coefficients.

Higher-Order Trend Components

- Testing for Higher-Order Trends:
  - A curve that has two reversals is called a cubic trend component, and one that has three reversals is called a quartic trend component.
  - The higher-order trend can be tested with the coefficients in Appendix A.3.
  - For example, we may calculate the cubic trend for $a = 4$.

Planning a Trend Analysis

Issues in Trend Analysis

- There are two questions that surround the choice of intervals, namely, the nature of the spacing between adjacent levels and the number of intervals.
  - It is better to use equally spaced intervals.
- Trend Coefficients for Unequal Intervals:
  - The coefficients can be found by simply picking numbers that agree with the actual spacing and adjust them to sum to zero.
- The Number of Intervals:
  - Seriously consider investing in a substantial experiment that includes a sufficient number of treatment conditions to provide convincing evidence of the trends.

Other Issues

- Other Functional Forms
  - Other functions (e.g., exponential or logarithm) can be used.
- Monotonic Trend Analysis
  - When we have only predicted the rank order of the conditions, we have a monotonic hypothesis based on a monotonic function.
  - We may apply the logic behind the trend coefficients in Appendix A.3 appropriately modified for a given problem.

Final Thought

- Trend analysis takes the idea of contrasts and maps it onto looking at trends.
- Trend analysis is typically conducted when the factor levels have some understandable scale.

- Contrasts are specific hypothesis tests that examine how each mean may differ from all the other means.

Next Class

- Chapter 6: Pairwise Comparisons