

Detailed Analysis of Main Effects and Simple Effects



ERSH 8310
CHAPTER 12, KEPPEL AND WICKENS

Today's Class



- Interpreting a Two-Way Design
- Comparing the Marginal Means
- Interpreting the Interaction
- Testing the Simple Effects
- Simple Comparisons
- Effect Sizes and Power for Simple Effects
- Controlling Familywise Type-I Error

Detailed Analysis of Main Effects and Simple Effects



Detailed Analysis of Main Effects and Simple Effects



- The test for interaction is usually an effective way to begin the analysis, because its outcome influences all the analyses that follow.
 - If the interaction is significant, then less attention is paid to the two main effects, and the analysis tends to focus on the individual cell means and the joint variation of the two independent variables.
 - If the interaction is not significant or is relatively small in size, then attention is directed to the marginal means and the variation of each independent variable considered without reference to the other.

Detailed Analysis of Main Effects and Simple Effects



- The analysis of any study must return eventually to the actual pattern of means.
- It never suffices to assert that one factor is significant and another is not, nor that an interaction is or is not present.
 - A detailed description of the substance of the study is always necessary.

Interpreting a Two-Way Design



A PICTURE IS WORTH A THOUSAND WORDS.

Interpreting a Two-Way Design

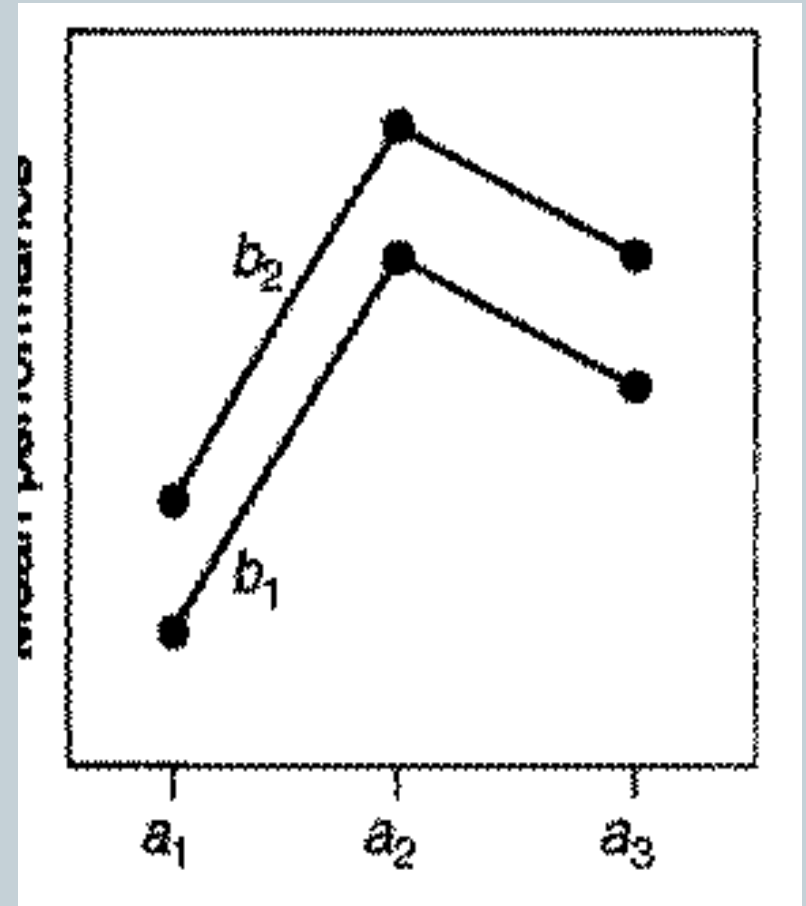


- The first step in examining data from a factorial study is to plot the means.
- Line graphs are usually clearer than bargraphs, particularly when exploring the data.
 - It is often necessary to try several plots before finding a good representation.
- The pattern of means from a factorial design can be expressed as main effects, simple effects, interaction components, or various special patterns implied by a theory.

Possible Outcomes

Three possible outcomes that might dictate the subsequent analyses are:

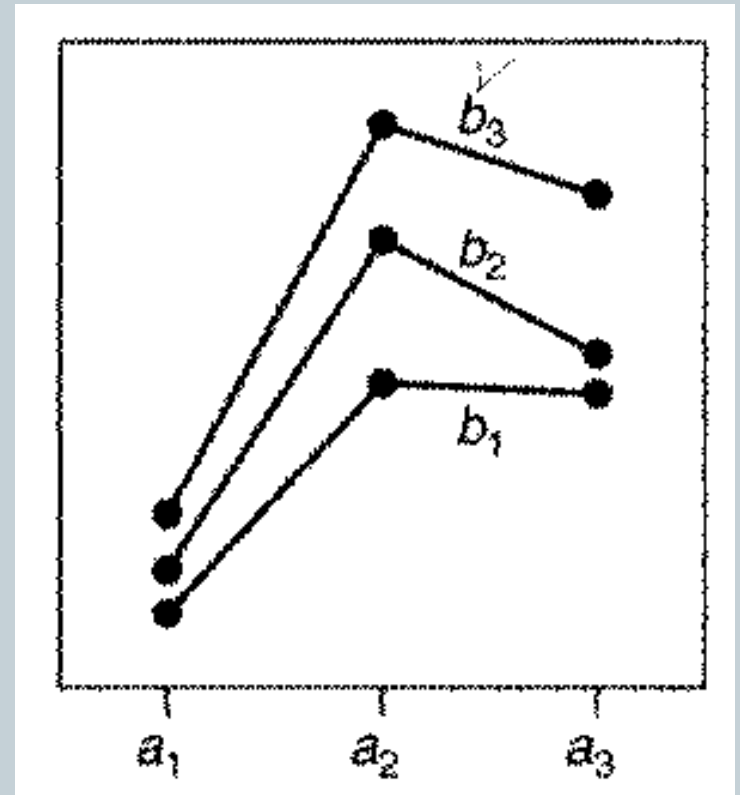
1. No interaction is present.
 - The two-way design is reduced to multiple one-way effects, and out attention is directed at follow-up tests that investigate analytical questions about the marginal means.



Possible Outcome #2

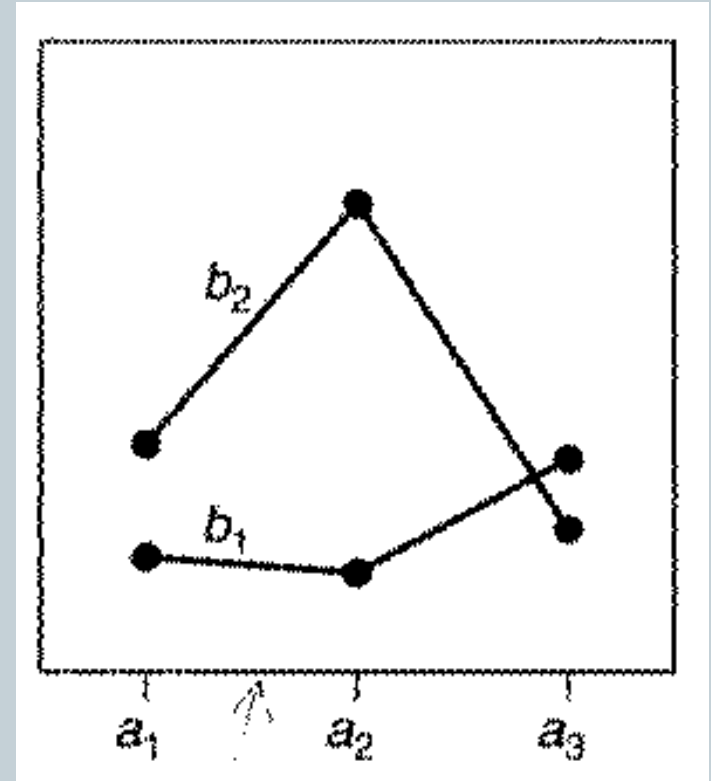
2. An interaction is present, but it is dominated by the main effects.

- The effect of either factor changes with the levels of the other.
- The simple picture of two main effects is not appropriate, and the two factors cannot be treated completely separately.
- To investigate the finding, we must consider how the simple effects of one factor differ with the levels of the other.
- Main effects that dominate the interaction usually represent solid, well-known and often-replicated manipulations.



Possible Outcome #3

3. The interaction dominates the main effects.
- It can be deceptive to look at the marginal effects as all.
 - We would be justified in ignoring the main effects altogether.



Comparing the Marginal Means



Comparing the Marginal Means



- We have two sets of marginal means, one for each of the two factors in the design. The significance of comparisons is evaluated with the error term from the overall analysis, namely, $MS_{S/AB}$.

Computational Formulas

- We may use:

- Where:

$$SS_{\psi_A} = \frac{\sum_{j=1}^a c_j^2 \bar{Y}_{A_j}^2}{n},$$

$$\hat{\psi}_A = \sum_{j=1}^a c_j \bar{Y}_{A_j},$$

More Computational Formulas



$$\hat{\psi}_B^2$$

$$SS_{\psi_B} = \frac{\sum_{k=1}^b c_k^2}{b},$$

$$\hat{\psi}_B = \sum_{k=1}^b c_k \bar{Y}_{B_k},$$

$$F_{\psi_B} = \frac{MS_{\psi_B}}{MS_{A/AB}}.$$

Interpreting the Interaction



Interpreting the Interaction



- Interaction can be analyzed by means of two procedures;
 - (1) the analysis of the simple effects and
 - (2) the analysis of interaction comparison (see Chapter 13).

Selecting a Set of Simple Effects for Analysis



- We will choose to analyze the set of simple effects that is the most natural, useful, or potentially revealing-the manipulation that will be the easiest to explain.
 - Choose the factor with the greater number of levels.
 - Choose a quantitative factor.
 - Choose the factor with the greater main-effect sum of squares.
 - Choose a manipulated factor.

Testing the Simple Effects



Testing the Simple Effects



- Simple effects are based on the differences among the cell means within a particular row or column of the matrix of means.

Computational Formulas



- See Table 12.1 for the layout. We can use:

$$SS_{A \text{ at } b_k} = \frac{\sum_{j=1}^a (AB_{ij})^2}{n} - \frac{B_k^2}{an}$$

with $df_{A \text{ at } b_k} = a - 1$ and

$$F = \frac{MS_{A \text{ at } b_k}}{MS_{S/AB}}$$

Variance Heterogeneity



- The safest solution when variance heterogeneity appears is to base the error term only on the groups that actually contribute to the simple effect.

Partitioning of the Sums of Squares



Note that

$$\sum_{k=1}^b SS_{A \text{ at } b_k} = SS_A + SS_{A \times B}$$

and

$$\sum_{j=1}^a SS_{B \text{ at } a_j} = SS_B + SS_{A \times B}.$$

Partitioning of the Sums of Squares



- The analysis of simple effects is especially useful when theory predicts the nature of interaction.
- Testing the significance of simple effects under these circumstances often helps us establish the details of the theoretical prediction.

Simple Comparisons



Simple Comparisons



- The simple comparison takes the form of linear combinations of means, for example,

$$\psi_A \text{ at } b_k = \sum_{j=1}^a c_j \mu_{jk},$$

where $\sum_j c_j = 0$.

Computational Formulas



- The observed value of the contrast is:

$$\hat{\psi}_{A \text{ at } b_k} = \sum_{j=1}^a c_j \bar{Y}_{jk}.$$

- The corresponding sum of squares is:

$$SS_{\psi_{A \text{ at } b_k}} = \frac{n \hat{\psi}_{A \text{ at } b_k}^2}{\sum_j c_j^2}.$$

Computational Formulas



- The F ratio, for example, is

$$F = \frac{MS_A \text{ at } b_1}{MS_{S/AB}}.$$

Simple Contrasts and Interactions



- Once we have completed the analysis of the simple effects, we usually need to conduct an analysis that focus on interaction again (see Chapter 13).

Effect Sizes and Power for Simple Effects



Effect Sizes and Power for Simple Effects



- In most studies that use a factorial design, effect sizes are reported for the overall main effects and interaction (see Sections 11.6 and 11.7).

Effect Size



- The partial omega squared is:

$$\omega_{(\text{effect})}^2 = \frac{\sigma_{\text{effect}}^2}{\sigma_{\text{effect}}^2 + \sigma_{\text{error}}^2}.$$

Effect Sizes and Power for Simple Effects



- The estimates of the partial omega squared for factor A for the main comparison, the simple effect, and the simple comparison are:

$$\hat{\omega}_{(\psi_A)}^2 = \frac{F_{\psi_A} - 1}{(F_{\psi_A} - 1) + 2bn}$$

$$\hat{\omega}_{(A \text{ at } b_k)}^2 = \frac{df_{A \text{ at } b_k}(F_{A \text{ at } b_k} - 1)}{df_{A \text{ at } b_k}(F_{A \text{ at } b_k} - 1) + an}$$

and

$$\hat{\omega}_{(\psi_A \text{ at } b_k)}^2 = \frac{F_{\psi_A \text{ at } b_k} - 1}{(F_{\psi_A \text{ at } b_k} - 1) + 2n}$$

Sample-Size Calculations



- The power for a simple effect in a factorial design or its components rarely needs to be calculated.
 - How convenient...

Controlling Familywise Type I Error



Controlling Familywise Type I Error



- There is a general consensus that the three principal effects (i.e., the two main effects and the interaction) are planned tests and do not require error correction.
- They are evaluated at a conventional significance level, such as $\alpha = .05$.

Main-Effect Comparisons



- In the absence of an interaction, attention is usually drawn to one or both of the two main effects.
- The two factors are usually treated separately, each with an allotment of familywise error equal to the level of the original tests (e.g., $\alpha_{FW} = .05$).

Main-Effect Comparisons



- A comparison that is a central planned portion of the study is evaluated without error control.
- For a small set of c meaningful comparisons, such as a few trend components, the familywise error rate can be controlled with the Bonferroni method by taking $\alpha = \alpha_{FW}/c$ (cf. Sidák-Bonferroni correction).

Main-Effect Comparisons



- The set of all pairwise differences between means is most easily tested with the Tukey (or Fisher-Hayter) procedure. The critical differences for the two factors are:

$$D_{\text{FH-A}} = q_{a-1} \sqrt{\frac{MSS_{AB}}{bn}}$$

and

$$D_{\text{FH-B}} = q_{b-1} \sqrt{\frac{MSS_{AB}}{an}},$$

Main-Effect Comparisons



- The Scheffé procedure is used when the broadest error control is desired. The critical value of the F is increased in the factorial design to:

$$F_{\text{Scheffe-A}} = (a-1) F_{A, \alpha_{FW}}(df_A, df_{S/AB})$$

and

$$F_{\text{Scheffe-B}} = (b-1) F_{B, \alpha_{FW}}(df_B, df_{S/AB}).$$

Simple Effects



- There is no consensual standard for the level of familywise error to use with the simple effects.
 - The Bonferroni (or Sidák-Bonferroni) procedure is the most practical approach here.
- For example, by setting $\alpha_{FW} = .10$ in the 3×2 design, the two simple effects of A at b_k would be evaluated at per-test rate of $\alpha = .05$ or the three simple effects of B at a_j would be evaluated at per-test rate of $\alpha = .033$.
- The formulas from Chapter 6 apply directly, except that the error term $MS_{S/AB}$ from the factorial design is used instead of $MS_{S/A}$.

Final Thought

- Today's class delved into what to do following the overall two-way analysis.
- Main effects were the focus of the discussion.
- Note that most everything discussed today came in the presence of the possible interaction between independent variables.
- The nature of the interaction dictates the level to which you describe the main effects.



Next Class



- Chapter 13
 - Analysis of Interaction Components