THE SINGLE-FACTOR WITHIN-SUBJECT DESIGN: FURTHER TOPICS

ERSH 8310

Keppel and Wickens Chapter 17

Today's Class

- A within-subject design requires changes in the assumptions that underlie the tests, making it more likely that they will be violated.
- These issues and their correction are discussed in this chapter.

Advantages and Limitations

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- A study conducted with a within-subject design obtains more data from each subject than one conducted with a between-subjects design, and the analysis has a smaller error term.
- Repeated observations of a subject, however, cannot be collected under constant conditions, and any earlier observation has the potential to influence later ones.
- The assumptions that underlie the analysis are more complex than those of the between-subjects designs.

Advantages of the Within-Subject Design

- The three principal advantages of a within-subject design are:
  1. More efficient use of subject resources (i.e., the economy of the design).
  2. Greater comparability of the conditions (i.e., increased control of subject variability).
  3. Reduced error variance (i.e., the treatment-by-subject interaction variability is almost always less than the pooled within-group variability).

Limitations of the Within-Subject Design

- The within-subject design has both statistical and nonstatistical limitations.:
  1. The statistical problems mostly concern the sensitivity of the assumptions of the analysis.
     - The scores produced by a single subjects are more alike than are the scores produced by different subjects (i.e., the observations are not independent).
  2. The nonstatistical problems arise from the fact that the repeated observations must necessarily take place under somewhat different conditions, and some aspect of this difference, other than the treatment being investigated, can affect the scores (e.g., incidental effects: practice and fatigue, memory; carryover effect, contrast effect, context effect).

Limitations of the Within-Subject Design

- A carryover effect occurs when a treatment has a transient effect that carries over to affect whatever condition is administered immediately after it.
- A contrast effect is a carryover effect that occurs when two treatments interact in a way that depends on both conditions.
- A context effect occurs when a subject's behavior is influenced by the context provided by exposure to other conditions in an experiment.

The Statistical Model

Statistical Models

- The difference between the models for the between-subjects and within-subject designs lies in the assumption of independence of the scores.
  - Two different models have been applied to within-subject data (i.e., univariate and multivariate).

- In the univariate approach, each score Y_{ij} is viewed as a separate random variable made up of systematic and random components, including a component specific to the subject.

- In the multivariate approach, all the scores from a single subject are treated as a single statistical entity; fewer assumptions about the data are required.

- The authors emphasized the univariate approach (see p. 373).

The Univariate Model

- A score Y_{ij} is expressed by the equation:

$$Y_{ij} = \mu_T + \alpha_j + S_i + (S\alpha)_{ij} + E_{ij},$$

- Where:
  - μ_T is the grand mean.
  - α_j is the treatment effect.
  - S_i is the overall ability of the subject i.
  - $(S\alpha)_{ij}$ is the idiosyncratic response of the subject in a particular condition.
  - E_{ij} is the variability of the individual observations.
  - Note that $S_i \sim N(0,\sigma_S^2)$, $(S\alpha)_{ij} \sim N(0,\sigma_{A\times S}^2)$, and $E_{ij} \sim N(0,\sigma_{error}^2)$.

Expected Mean Squares

- The expected mean squares are

$$E(MS_A) = \frac{1}{n-1} \sum_j \alpha_j^2 + \sigma_{A\times S}^2 + \sigma_{error}^2,$$

$$E(MS_S) = a\,\sigma_S^2 + \sigma_{error}^2,$$

$$E(MS_{A\times S}) = \sigma_{A\times S}^2 + \sigma_{error}^2.$$

Assumptions…

- For the univariate model, the variances of all the treatment conditions are identical (i.e., homogeneity of variance) and the correlations between the scores are identical (i.e., homogeneity of correlation).
- When these restrictions hold, the data are said to show compound symmetry.

The Multivariate Model

- The vector of scores has a multivariate normal distribution.
- The multivariate model relaxes the assumption of compound symmetry.
- When the assumptions of the univariate model hold, however, the multivariate tests have less power.

The Sphericity Assumption

The Sphericity Assumption

- A slightly weaker assumption is all that is needed than the assumption of compound symmetry.
- Hence, compound symmetry need not hold for the scores themselves, but only for the differences between pairs of scores.
- This condition is referred to as circularity or sphericity.

The Sphericity Assumption

- There are tests for violations of sphericity of compound symmetry.
- The most widely used of these, a likelihood-ratio test statistic W developed by Mauchly (1940), is included in a number of computer programs.
- This statistic should not be significant for the analysis to proceed.

Dealing with Violations of Sphericity

- There are four approaches we can take.
  1. Box (1954a) suggested using the values:

     $df_{num} = e(a-1)$ and $df_{denum} = e(a-1)(n-1)$

where e measures the extent to which sphericity is violated.

Use Geisser and Greenhouse (1958) or Huynh and Feldt (1976), of which the latter has the greater power.

Dealing with Violations of Sphericity

2. The smallest value of e = 1/(a-1) can be used and, hence,

$$df_{num} = 1 \quad \text{and} \quad df_{denum} = n\text{-}1.$$

- This is known as the conservative F test suggested by Geisser and Greenhouse (1958) (i.e., Lower-bound in SPSS).

Dealing with Violations of Sphericity

3. We may use the multivariate approach.

4. We may forget about the omnibus test and use tests of contrasts, which are immune to violations of sphericity.

Incidental Effects

Incidental Effects

- Factors such as the position in testing sequence or the type of material are examples of the nuisance variables.
- When such a variable becomes an explicit factor in the design, we will refer to it as either a nuisance factor or an incidental factor.

Incidental Effects

- The biases that arise when the treatments are confounded with incidental aspects of the study, such as the order of testing or the materials, can be avoided by breaking up any consistent relationship between them.

- There are two ways to do this:
  - In randomization, the relationship between the treatments and the incidental aspects of the study is chosen randomly.
  - In counterbalancing, it is constructed in a way that systematically balances the incidental effects across the study (e.g., Latin square, see p. 381).

Randomization

- The randomization procedures are the easiest to apply, but it cannot assure that the incidental factor is completely balanced across treatment and may have large error term.

Counterbalancing and the Latin Square

- The arrangement of the conditions in Table 17.1 is known as a Latin square.
- The key feature of the Latin square arrangement is that every latter appears exactly once in each row and each column.

Analyzing a Counterbalanced Design

The Omnibus Analysis

- See the analysis using the numerical example in Table 17.2.

- Two within-subject analyses were performed, one for the treatment conditions (factor A) and the other for the order in which the conditions were administered (factor P).

- The error sum of square is
  - $SS_{residual} = SS_{total} - SS_A - SS_S$

with the degrees of freedom
  - $df_{residual} = df_{total} - df_A - df_S$

The Importance of Interactions in a Latin Square

- The particular configuration of conditions in a Latin square makes it impossible to extract information about any interaction that may be present.

- See Table 17.3 for the steps to test an effect after removing the influence of an incidental factor from the individual scores. See Table 17.4 for an example.

Final Thought

- The repeated measures ANOVA partitions variability due to a subject.
- Removing such variability aids in the power of the test.

- The repeated measures analysis described in this class was an initial first pass at the approach.
  - We will see assumptions of such an approach are very strong.
- Newer methods will relax some of these assumptions.

Next Time…

- Final exam discussion (12/6).