ANALYTICAL COMPARISONS AMONG TREATMENT MEANS (CHAPTER 4)

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Today's Class

- The need for analytic comparisons.
- Planned comparisons.
- Comparisons among treatment means.
- Evaluating contrasts with t-tests.
- Orthogonal contrasts.
- Mean comparison in SPSS.

Today's Example Data Set

Still Vigilant…

Vigilance Task While Sleep Deprived

- There are a = 4 conditions, namely, 4, 12, 20, and 28 hours without sleep.
- There are n = 4 subjects randomly assigned to each of the different treatment conditions.
- The vigilance task score represents the number of failures to spot objects on a radar screen during a 30-minute test period.

Data (computation p. 51)

Hours without sleep			
4 hr	12 hr	20 hr	28 hr
a_1	a_2	a_3	A_4
37	36	43	76
22	45	75	66
22	47	66	43
25	23	46	62

SPSS Results (From Lab Thursday)

Descriptives

Errors

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum
1.00	4	26.5000	7.14143	3.57071	15.1364	37.8636	22.00	37.00
2.00	4	37.7500	10.93542	5.46771	20.3493	55.1507	23.00	47.00
3.00	4	57.5000	15.50269	7.75134	32.8318	82.1682	43.00	75.00
4.00	4	61.7500	13.81726	6.90863	39.7637	83.7363	43.00	76.00
Total	16	45.8750	18.47476	4.61869	36.0305	55.7195	22.00	76.00

ANOVA

Errors

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3314.250	3	1104.750	7.343	.005
Within Groups	1805.500	12	150.458		
Total	5119.750	15			

The Need for Analytic Comparisons

The Need for Analytic Comparisons

- This chapter focuses on the analysis of experiments in which the independent variable consists of qualitative differences among the treatment conditions, where the interest is in isolating and assessing meaningful comparisons between specific treatment conditions.

- An analytical comparison refers to a meaningful comparison between two or more treatment conditions that are components of a larger experimental design (i.e., planned or post hoc comparisons).

The Composite Nature of SS_A

- For an equal n observations per group:

$$SS_A = n\sum_{j=1}^{a}\left(\bar{Y}_j - \bar{Y}_T\right)^2$$

The Composite Nature of SS_A

- Because:

$$\bar{Y}_T = \frac{\sum_{j=1}^{a} \bar{Y}_T}{a}$$

- We can re-express SS_A as:

$$SS_A = \frac{n}{a} \sum_{pairs} \left(\bar{Y}_i - \bar{Y}_k \right)^2$$

Example Decomposition

	A	B	C	D
1	Comparison	Mean i	Mean j	(M_i-M_j)^2
2	1 v. 2	26.5	37.75	126.5625
3	1 v. 3	26.5	57.5	961
4	1 v. 4	26.5	61.75	1242.5625
5	2 v. 3	37.75	57.5	390.0625
6	2 v. 4	37.75	61.75	576
7	3 v. 4	57.5	61.75	18.0625
8	n/a = 1			
9	Sum*(n/a)			3314.25

The Omnibus F Test

- The overall variation among the treatment means reflected in SS_A may be better understood by examining these contributing parts, namely, the comparisons between pairs of means.
- An F ratio based on more than two treatment levels is called the omnibus or overall F test.
- Identifying the sources that contribute to the significant overall F should be performed to understand differences among the treatment means.

Planned Comparisons

Planned Comparisons

- The analytical comparisons conducted directly on a set of data without reference to the result of the omnibus F test is possible. These are called the planned comparisons.

An Example of Planned Comparisons

- Instead of testing the overall hypothesis (e.g., H_0: $m_1 = m_2 = m_3 = m_4 = m_5$, where m_i represents the amount of learning for the jth group), we may perform the four comparisons. The four null hypotheses are as follows:

1.

$$H_0 : \mu_1 = \mu_2 \quad (4)$$

3.

$$H_0 : \mu_1 = \mu_4 \quad (5)$$

5.

$$H_0 : \mu_2 = \mu_3 \quad (6)$$

7.

$$H_0 : \mu_4 = \mu_5 \quad (7)$$

Comparisons Among Treatment Means

Comparisons Among Treatment Means

- Comparisons that can be reduced to miniature experiment, each of which consists of $a = 2$ (i.e., df = 1), are called also contrasts or single-df comparisons.

- Note that we may extend the comparisons to situations where the subsets of three or more means are tested. (i.e., df ^{1} 1).

Linear Contrasts

- From the null hypothesis of a comparison we may represent the difference between two means with ψ.
- For example $$H_0\text{: } \mu_1 = \frac{\mu_2 + \mu_3}{2}$$

Can be rewritten: $$H_0\text{: } \mu_1 - \frac{\mu_2 + \mu_3}{2} = 0.$$

Yielding: $$\psi = (+1)(\mu_1) + \left(-\frac{1}{2}\right)(\mu_2) + \left(-\frac{1}{2}\right)(\mu_3).$$

The New Null Hypothesis

- From the last slide, our contrast is:

$$\psi = (+1)(\mu_1) + \left(-\frac{1}{2}\right)(\mu_2) + \left(-\frac{1}{2}\right)(\mu_3).$$

where the numbers multiplied by each mean are called coefficients.

- Note that, now, H_0: $\psi = 0$.

Contrasts, In General

- The general formula is:

$$\psi = \sum_{j=1}^{a} c_j \mu_{j.}$$

where the c_j's represent the coefficients appropriate for a particular comparison.

- Note: there is a constraint that the c's sum to zero.

Constructing Coefficients

- A pairwise comparison is a comparison between two single groups (e.g., $\psi = \mu_j - \mu_k$).
- A complex comparison is a comparison between an average of two or more groups and either a single group or an average of two or more groups.
- For an example (a = 5),

$$\psi = \frac{\mu_2 + \mu_4}{2} - \frac{\mu_1 + \mu_3 + \mu_5}{3}.$$

where c_j's are called a standard set.

- If c_js are expressed as a set of integer numbers (i.e., {-2, +3, -2, +3, -2}), we call the c_j's the relative weights of the groups.

The Sums of Squares for a Comparison

- If

$$\hat{\psi} = \sum_{j=1}^{a} c_j \bar{Y}_j \; .$$

- then the sum of sauares for the contrast is:

$$SS_{\psi} = \frac{n\,\hat{\psi}^2}{\sum_{j=1}^{a} c_j^2} \; .$$

Evaluating Comparisons

- If the comparisons are planned comparisons, we may not even bother to perform the omnibus F test.
- Note that the two comparisons in Table 4.4 account for the total between groups sum of squares (i.e., a complete set of orthogonal comparisons).
- The F ratio is formed as

$$F = \frac{MS_{\psi}}{MS_{S/A}},$$

- where $df_{num} = 1$ and $df_{denom} = a(n-1)$.

Unequal Sample Sizes

- Let n_i indicate the number of subjects in group a_i. The sum of square is

$$SS_\psi = \frac{\hat{\psi}^2}{\sum_{j=1}^{a} (c_j^2/n_j)} .$$

- The MS_ψ and F can be obtained accordingly.

Evaluating Constrasts with a t-test

The t-test

- The one-sample t test is

$$t = \frac{\bar{Y} - \mu_0}{s_M}.$$

- where $s_M = s/\sqrt{n}$.

Two-Sample t-test

- The independent samples t test for the equality of the population means, under the assumptions of the ANOVA procedure are valid, can be written as

$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{\hat{\sigma}_{[\bar{Y}]_1 - [\bar{Y}]_2}} . \qquad (18)$$

$$\hat{\sigma}_{[\bar{Y}]_1 - [\bar{Y}]_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) MS_{S/A}} .$$

t-tests v. ANOVA

- It can be noted that $t^2(df) = F(1,df)$

Orthogonal Contrasts

Orthogonal Contrasts

- In general, we can divide any sum of squares into as many independent sums of squares as there are df.
- The valuable property of orthogonal comparisons is that they reflect independent or nonoverlapping pieces of information.
- The outcome of one comparison given no indication whatsoever about the outcome of another orthogonal comparison.
- Orthogonality means independence of information.

What is Orthogonal?

- A numerical test of the orthogonality of any two comparisons is provided by the following relationship between the two sets of coefficients:

$$\sum_{j=1}^{a} c_{1j}\, c_{2j} = 0,$$

- where c_{1j} and c_{2j} are corresponding coefficients in the two comparisons.

Orthogonality and the ANOVA

- The number of orthogonal comparisons for a given set of data is a-1, that is the df for the SS_A (e.g., Helmert contrasts).
- Authors of statistical sourcebook are not in agreement on the issue of placing orthogonality restrictions on the nature of the planned comparisons.
- Researchers must exercise judgment in the planning stages to guarantee that the important questions studies in an investigation can be answered unambiguously by the proposed experimental design.
- Nonorthogonal comparisons, however, require special care to avoid logical ambiguities.

Composite Contrasts Derived from Theory

Contrast Coefficients that Match a Pattern

- The contrast coefficients that reflect theoretical pattern can be constructed using the steps:
  1. Use each predicted mean as a starting coefficient.
  2. Subtract the average of these means from the predicted means so that they sum to zero.
  3. Optionally, simplify the coefficients to be integers.

- The testing of the observed pattern of means with the theoretically-derived expectation as well as the assessment of the fit of the outcome to the predicted pattern can be performed.

Back to the Example

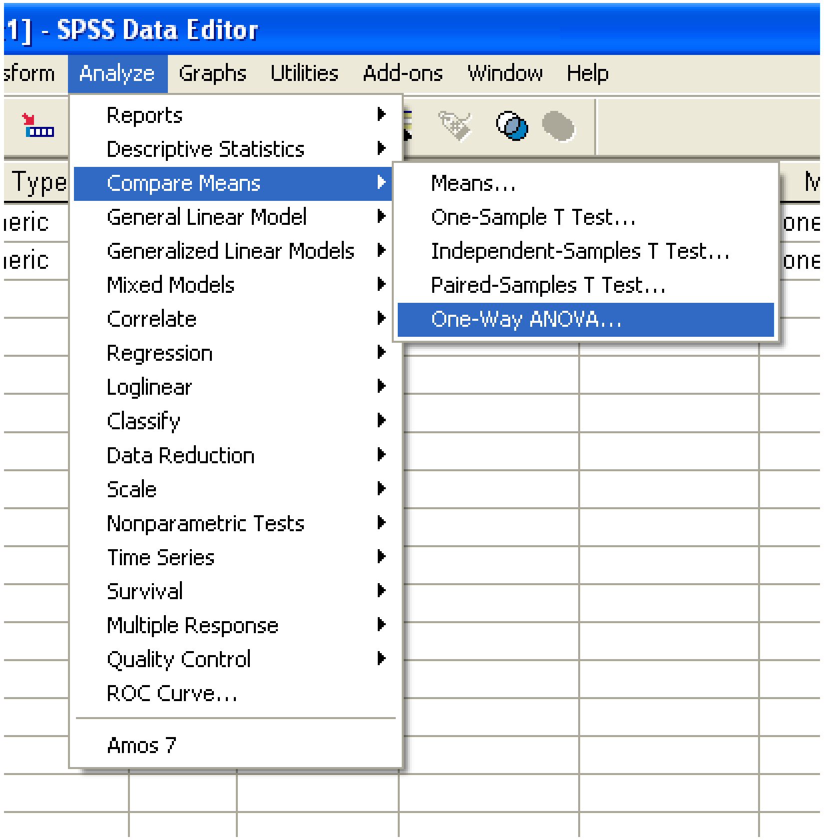
Which vigilance task means are significantly different?

Comparisons in SPSS

- To make mean comparisons in SPSS, a basic method is to use t-tests.
- Because we will have multiple mean comparisons, we need to adjust our overall Type-I error rate so that we have an overall level of error that is acceptable.
- To do this, we must first go and run our analysis as we did in lab…

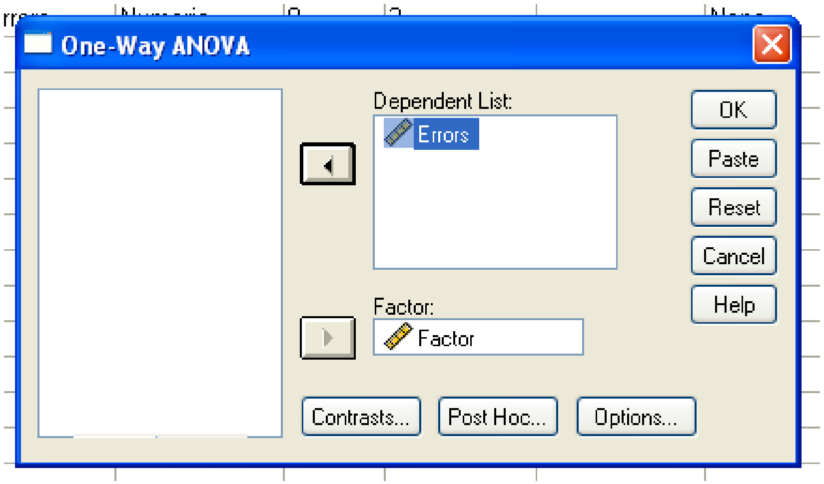
Running the ANOVA

- Go to Analyze…Compare Means…One-Way ANOVA



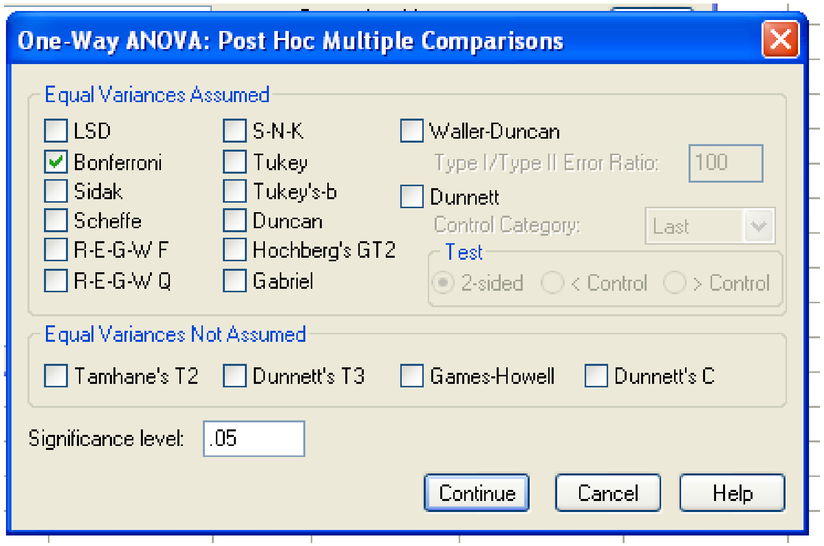
Choose the Variables

- Next put the factor variable in the Factor box…
- Then put the dependent variable in the Dependent List Box.
- THEN: Click on the Post Hoc…button.



Select the Bonferroni Method Test

- Check the box next to Bonferroni…
- Be sure to set the significance level for your overall experiment (0.05 is the default).



Results

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Errors
Bonferroni

(I) Factor	(J) Factor	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-11.25000	8.67347	1.000	-38.5947	16.0947
	3.00	-31.00000*	8.67347	.023	-58.3447	-3.6553
	4.00	-35.25000*	8.67347	.009	-62.5947	-7.9053
2.00	1.00	11.25000	8.67347	1.000	-16.0947	38.5947
	3.00	-19.75000	8.67347	.251	-47.0947	7.5947
	4.00	-24.00000	8.67347	.102	-51.3447	3.3447
3.00	1.00	31.00000*	8.67347	.023	3.6553	58.3447
	2.00	19.75000	8.67347	.251	-7.5947	47.0947
	4.00	-4.25000	8.67347	1.000	-31.5947	23.0947
4.00	1.00	35.25000*	8.67347	.009	7.9053	62.5947
	2.00	24.00000	8.67347	.102	-3.3447	51.3447
	3.00	4.25000	8.67347	1.000	-23.0947	31.5947

*. The mean difference is significant at the .05 level.

Final Thought

- The ANOVA procedure yields an omnibus F test that tells you that at least one group mean is different from the rest.
- This class talked about ways in which you could find out which mean that happened to be.

- Contrasts are specific hypothesis tests that examine how each mean may differ from all the other means.

Next Class

- Chapter 5: Analysis of Trends