Psychometric Models:
The Loglinear Cognitive Diagnosis Model

Section #3
NCME 2016 Training Session

Lecture Objectives

- Discuss relevant mathematical prerequisites for understanding diagnostic measurement models
- Introduce the loglinear cognitive diagnosis model – a general measurement model for DCMs
- Show some models the LCDM subsumes

Development of Psychometric Models

- Over the past several years, numerous DCMs have been developed
    - We will focus on DCMs that use latent variables for attributes
    - This lecture focus on the only one you should pay attention to: the Loglinear Cognitive Diagnosis Model
- Each DCM makes assumptions about how mastered attributes combine/interact to produce an item response
    - Compensatory/disjunctive/additive models
    - Non-compensatory/conjunctive/non-additive models
- With so many models, analysts have been unsure which model would best fit their purpose
    - Difficult to imagine all items following same assumptions

General Models for Diagnosis

- Recent developments have produced very general diagnostic models
  - General Diagnostic Model (**GDM**; von Davier, 2005)
  - Loglinear Cognitive Diagnosis Model (**LCDM**; Henson, Templin, & Willse, 2009)
    - Focus of this session
  - Generalized DINA Model (G-DINA; de la Torre, 2011)
    - Is equivalent to the LCDM

- The LCDM provides great modeling flexibility
  - Subsume all other latent variable DCMs
  - Allow both additive and non-additive relationships between attributes/items
  - Sync with other psychometric models allowing for greater understanding of modeling process

Lecture Overview

- Background information
    - ANOVA models and the LCDM

- Logits explained

- The LCDM
    - Parameter structure
    - One-item demonstration

- LCDM general form

- Linking the LCDM to other earlier-developed DCMs

Notation Used Throughout Session

- **Attributes**: $a = 1, \ldots, A$

- **Respondents**: $r = 1, \ldots, R$

- **Attribute Profiles**: $\boldsymbol{\alpha}_r = [\alpha_{r1}, \alpha_{r2}, \ldots, \alpha_{rA}]$
  - Each attribute α_{ra} today is defined as being 0 or 1: $\alpha_{ra} \in \{0,1\}$

- **Latent Classes**: $c = 1, \ldots, C$
  - We have $C = 2^A$ latent classes – one for each possible attribute profile
  - An attribute profile is a specific permutation of all A attributes

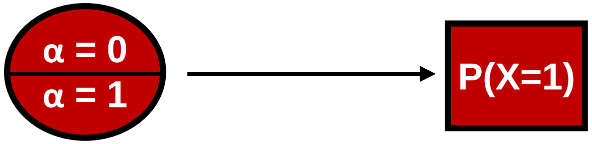
- **Items**: $i = 1, \ldots, I$
  - Restricted to dichotomous item responses (either 0 or 1): $Y_{ri} \in \{0,1\}$

- **Q-matrix**: Elements q_{ia} are indicators an item i measures attribute a
  - q_{ia} is either 0 (does not measure a) or 1 (measures a): $q_{ia} \in \{0,1\}$

BACKGROUND INFORMATION: ANOVA MODELS

Background Information – ANOVA

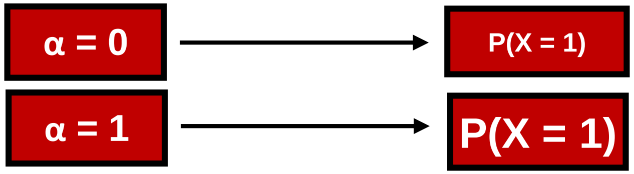
- The LCDM models the probability of a correct response to an item as a function of the latent attributes of a respondent



α = 0
α = 1
P(X=1)

- The latent attributes are categorical, meaning a respondent can have one of countably many possible statuses
  - Each status corresponds to a predicted probability of a correct response

- As such, the LCDM is very similar to an ANOVA model
  - Predicting the a dependent variable as a function of the experimental group of a respondent



α = 0
P(X = 1)
α = 1
P(X = 1)

ANOVA Refresher

- As a refresher on ANOVA, lets imagine that we are interested in the factors that have an effect on *work output* (denoted by Y)

- We design a two-factor study where *work output* may be affected by:
  - Lighting of the workplace
    - High or Low

  - Temperature
    - Cold or Warm

- This experimental design is known as a 2-Way ANOVA

ANOVA Model

- Here is the 2 x 2 Factorial design:

	Low Lighting	High Lighting
Cold Temperature	$\bar{Y}_{Cold,Low}$	$\bar{Y}_{Cold,High}$
Warm Temperature	$\bar{Y}_{Warm,Low}$	$\bar{Y}_{Warm,High}$

- The ANOVA model for a respondent's work output is

$$Y_r = \mu + A_t + B_l + (AB)_{tl} + \varepsilon_r$$

ANOVA Model

- The ANOVA model allows us to test for the presence of
  - A main effect associated with *Temperature* (A_t)
    - Where $A_{Cold} + A_{Warm} = 0$
  - A main effect associated with *Lighting* (B_l)
    - Where $B_{Low} + B_{High} = 0$
  - An interaction effect associated with *Temperature* and *Lighting* $(AB)_{tl}$
    - Where $(AB)_{Cold,Low} + (AB)_{Cold,High} + (AB)_{Warm,Low} + (AB)_{Warm,High} = 0$

$$Y_r = \mu + A_t + B_l + (AB)_{tl} + \varepsilon_r$$

ANOVA with Dummy Coded Variables

- The ANOVA model can also be re-written using two dummy-coded variables

$Warm_r =$

  - 0 for respondents in ***cold temperature*** condition
  - 1 for respondents in ***warm temperature*** condition

$High_r =$

  - 0 for respondents in ***low lighting*** condition
  - 1 for respondents in ***high lighting*** condition

ANOVA with Dummy Coded Variables

- The ANOVA model then becomes:

	$High_r = 0$ Low Lighting	$High_r = 1$ High Lighting
$Warm_r = 0$ Cold Temperature	$\bar{Y}_{Cold,Low}$	$\bar{Y}_{Cold,High}$
$Warm_r = 1$ Warm Temperature	$\bar{Y}_{Warm,Low}$	$\bar{Y}_{Warm,High}$

$$Y_r = \beta_0 + \beta_t Warm_r + \beta_l High_r + \beta_{t*l} Warm_r * High_r + \varepsilon_r$$

ANOVA Effects Explained

$$Y_r = \beta_0 + \beta_t Warm_r + \beta_l High_r + \beta_{t*l} Warm_r * High_r + \varepsilon_r$$

- β_0 is the mean for the cold and low light condition (reference group)
    - The intercept
- β_t is the difference in the average response for warm temperature for a business with low lights $High_r = 0$ (Conditional Main Effect)
- β_l is the difference in the average response for high lights for a business with cold temperature $Warm_r = 0$ (Conditional Main Effect)
- β_{t*l} is additional change in average that is not explained by the shift in temperature and shift and lights, when both occur (2-Way Interaction)

- Respondents from in the same condition have the same predicted value

ANOVA and the LCDM

- The ANOVA model and the LCDM take the same modeling approach
  - Predict a response using dummy coded variables
    - In LCDM dummy coded variables are latent attributes
  - Using a set of main effects and interactions
    - Links attributes to item response
  - Where possible, we may look for ways to reduce the model
    - Removing non-significant interactions and/or main effects

Differences Between LCDM and ANOVA

- The LCDM and the ANOVA model differ in two ways:
  - Instead of a continuous outcome such as work output the LCDM models a function of the probability of a correct response
    - The logit of a correct response (defined next)
  - Instead of observed “factors” as predictors the LCDM uses discrete *latent* variables (the attributes being measured)
- Attributes are given dummy codes (act as latent factors)
  - $\alpha_{ra} = 1$ if respondent *r* has **mastered** attribute *a*
  - $\alpha_{ra} = 0$ if respondent *r* has **not mastered** attribute *a*
- The LCDM treats the attributes as *crossed* experimental factors: all combinations are assumed to exist
  - This assumption can be (and will be) modified

LOGITS EXPLAINED

Model Background

- The LCDM models the log-odds of a correct response conditional on a respondent's attribute pattern $\boldsymbol{\alpha}_r$
  - The log-odds is called a logit

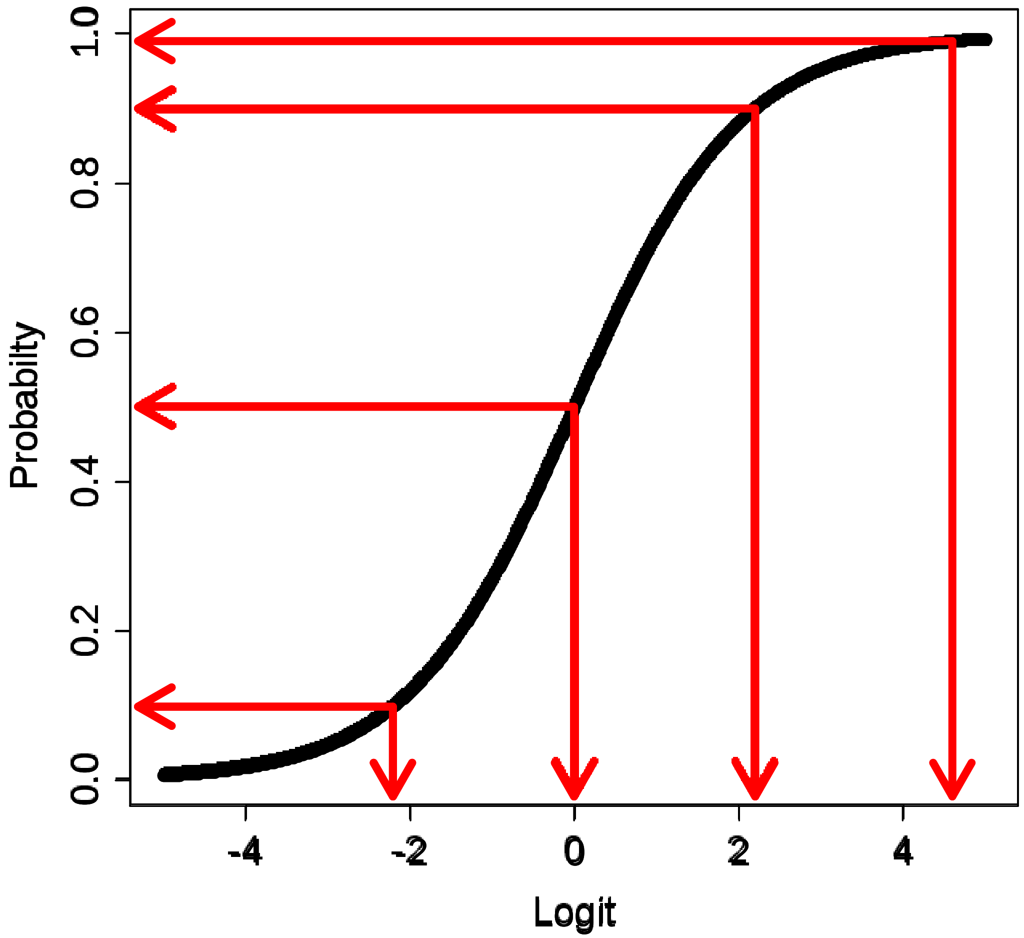
$$Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \log\left(\frac{P(Y_{ri} = 1|\boldsymbol{\alpha}_r)}{1 - P(Y_{ri} = 1|\boldsymbol{\alpha}_r)}\right)$$

  - Here $\log(\cdot)$ is the natural log

- The logit is used because the responses are binary
  - Items are either answered correctly (1) or incorrectly (0)

- The linear model with an identity link and Gaussian error is inappropriate for categorical data
  - Can lead to impossible predictions (i.e., probabilities greater than 1 or less than 0)

More on Logits



Probability	Logit
0.5	0.0
0.9	2.2
0.1	-2.2
0.99	4.6

From Logits to Probabilities

- Whereas logits are useful as the are unbounded continuous variables, categorical data analyses rely on estimated probabilities

- The inverse logit function coverts the unbounded logit to a probability
  - This is also the form of an IRT model (and logistic regression)

$$P(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \frac{\exp\big(Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r)\big)}{1 + \exp\big(Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r)\big)}$$

  - Here, $\exp(\cdot) = 2.718282$: the inverse function of the natural log (Euler's number)
  - Sometimes this is written:

$$P(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \frac{\exp\big(Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r)\big)}{1 + \exp\big(Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r)\big)} = \big[1 + \exp\big(-Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r)\big)\big]^{-1}$$

THE LCDM

Building the LCDM

- To demonstrate the LCDM, consider the item 2+3-1=? from our basic math example
  - Measures addition (attribute 1: α_{r1}) and subtraction (attribute 2: α_{r2})

- Only attributes defined by the Q-matrix are modeled for an item

- The LCDM provides the logit of a correct response as a function of the latent attributes mastered by a respondent:

$$Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \lambda_{i,0} + \lambda_{i,1,(1)}\alpha_{r1} + \lambda_{i,1,(2)}\alpha_{r2} + \lambda_{i,2,(1,2)}\alpha_{r1}\alpha_{r2}$$

LCDM Explained

$$\boxed{Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r)} = \boxed{\lambda_{i,0}} + \lambda_{i,1,(1)}\alpha_{r1} + \lambda_{i,1,(2)}\alpha_{r2} + \lambda_{i,2,(1,2)}\alpha_{r1}\alpha_{r2}$$

- $Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r)$ is the logit of a correct response to item *i* by respondent *r*

- $\lambda_{i,0}$ is the intercept
  - The logit for non-masters of addition and subtraction
  - The reference group is respondents who have not mastered ***either*** attribute ($\alpha_{r1} = 0$ and $\alpha_{r2} = 0$)

LCDM Explained

$$Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \lambda_{i,0} + \boxed{\lambda_{i,1,(1)}}\alpha_{r1} + \boxed{\lambda_{i,1,(2)}}\alpha_{r2} + \boxed{\lambda_{i,2,(1,2)}}\alpha_{r1}\alpha_{r2}$$

- $\lambda_{i,1,(1)}$= **<u>conditional</u> main effect** for addition (attribute 1)
  - The increase in the logit for mastering addition (for someone who has *<u>not mastered</u>* subtraction)
- $\lambda_{i,1,(2)}$= **<u>conditional</u> main effect** for subtraction (attribute 2)
  - The increase in the logit for mastering subtraction (for someone who has *<u>not mastered</u>* addition)
- $\lambda_{i,2,(1,2)}$= is the **2-way interaction** between addition and subtraction (attributes 1 and 2)
  - Change in the logit for mastering ***both*** addition & subtraction

Understanding LCDM Notation

- The LCDM item parameters have several subscripts:

$$\lambda_{\boxed{i,}\boxed{e,}\boxed{(a_1,\ldots)}}$$

- Subscript #1 – i: the item to which parameters belong

- Subscript #2 – e: the level of the effect
  - 0 is the intercept
  - 1 is the main effect
  - 2 is the two-way interaction
  - 3 is the three-way interaction

- Subscript #3 –$(a_1, \ldots)$: the attributes to which the effect applies
  - Same number of attributes listed as number in Subscript #2

LCDM: A NUMERICAL EXAMPLE

LCDM with Example Numbers

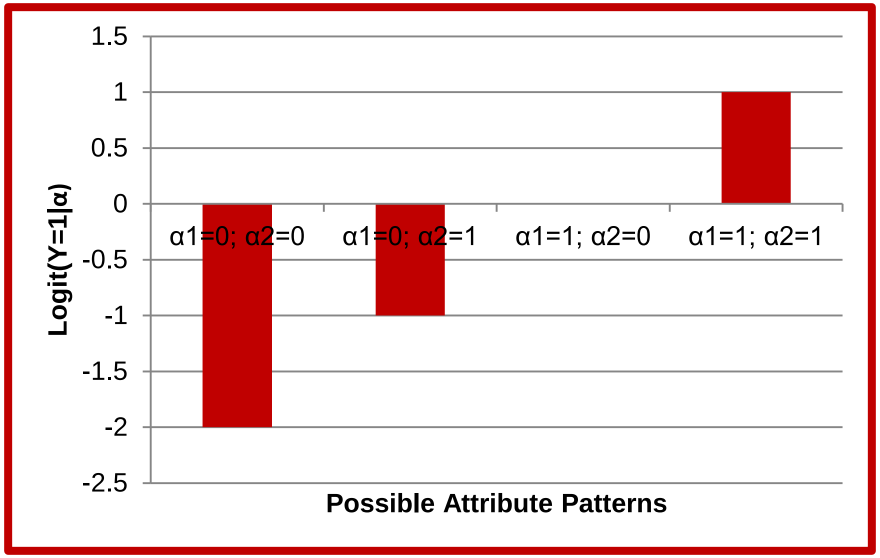
- Imagine we obtained the following estimates for the item $2 + 3 - 1 = ?$:

Parameter	Estimate	Effect Name
$\lambda_{i,0}$	-2	Intercept
$\lambda_{i,1,(1)}$	2	Addition Conditional Main Effect
$\lambda_{i,1,(2)}$	1	Subtraction Conditional Main Effect
$\lambda_{i,2,(1,2)}$	0	Addition/Subtraction Interaction

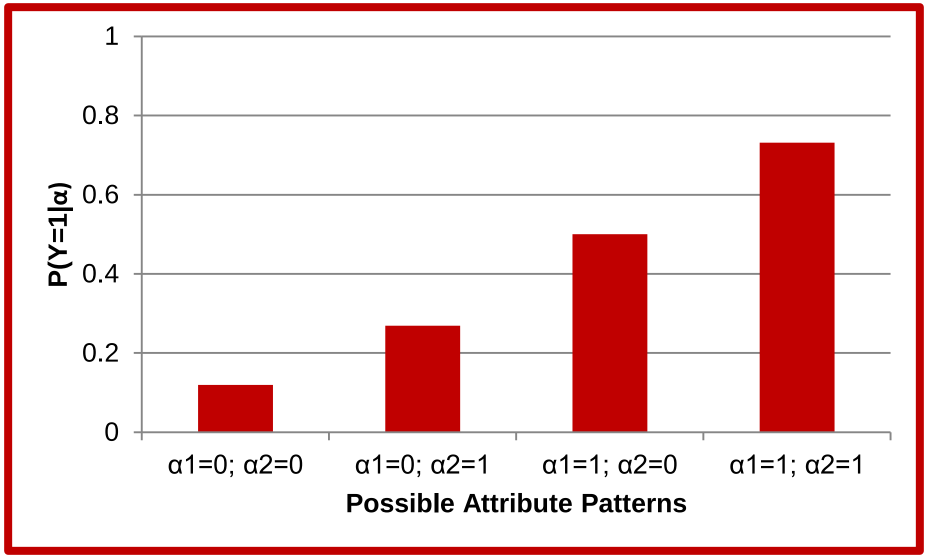
LCDM Predicted Logits and Probabilities

α_{r1}	α_{r2}	LCDM Logit Function	Logit	Probability
0	0	$\lambda_{i,0} + \lambda_{i,1,(1)} \times (0) + \lambda_{i,1,(2)} \times (0) + \lambda_{i,2,(1,2)} \times (0) \times (0)$	-2	0.12
0	1	$\lambda_{i,0} + \lambda_{i,1,(1)} \times (0) + \lambda_{i,1,(2)} \times (1) + \lambda_{i,2,(1,2)} \times (0) \times (1)$	-1	0.27
1	0	$\lambda_{i,0} + \lambda_{i,1,(1)} \times (1) + \lambda_{i,1,(2)} \times (0) + \lambda_{i,2,(1,2)} \times (1) \times (0)$	0	0.50
1	1	$\lambda_{i,0} + \lambda_{i,1,(1)} \times (1) + \lambda_{i,1,(2)} \times (1) + \lambda_{i,2,(1,2)} \times (1) \times (1)$	1	0.73

Logit Response Function



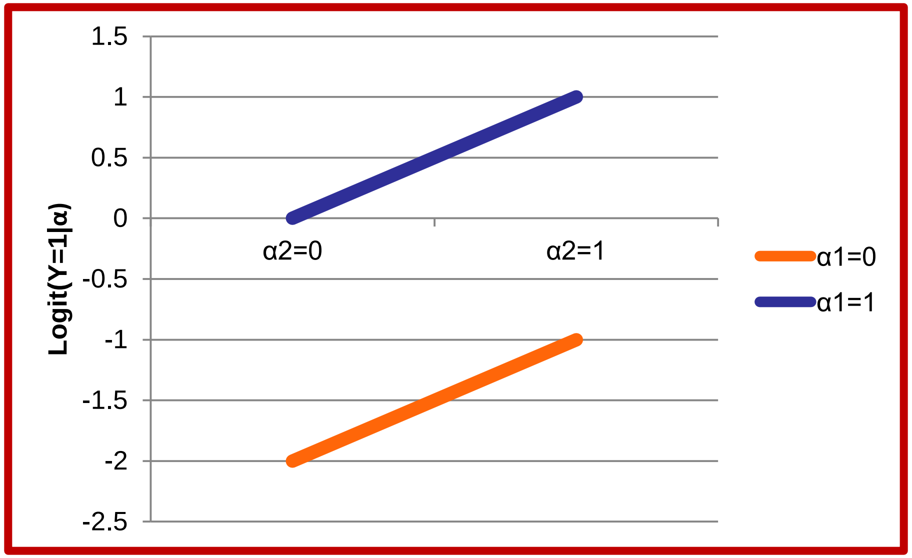
Probability Response Function (Item Characteristic Bar Chart)



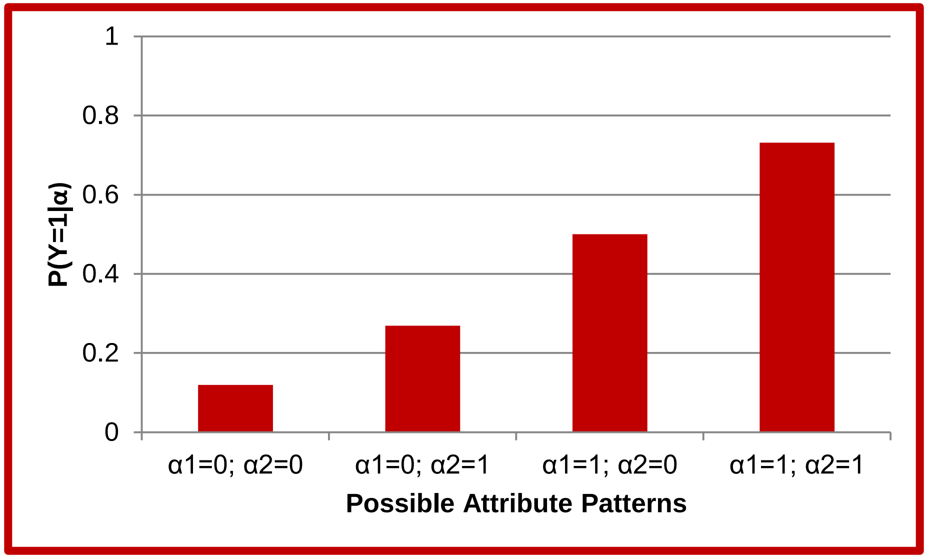
LCDM Interaction Plots

- The LCDM interaction term can be investigated via plots
- **No interaction**: parallel lines for the logit
  - Compensatory RUM (Hartz, 2002)

Logit Response Function



Probability Response Function (Item Characteristic Bar Chart)

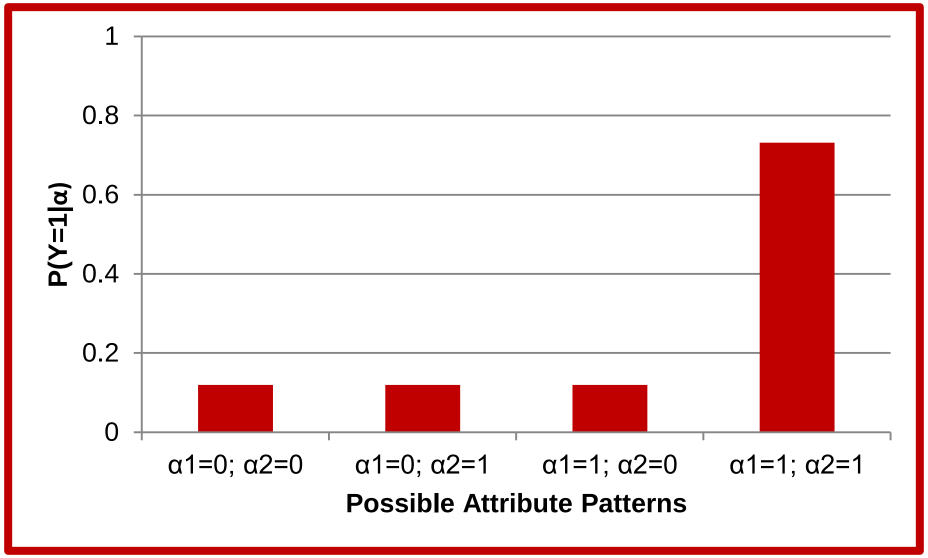


Strong Positive Interactions

- **Positive interaction**: over-additive logit model
  - Conjunctive model (i.e., all-or-none)
  - DINA model (Haertel, 1989; Junker & Sijtsma, 1999)

Logit Response Function

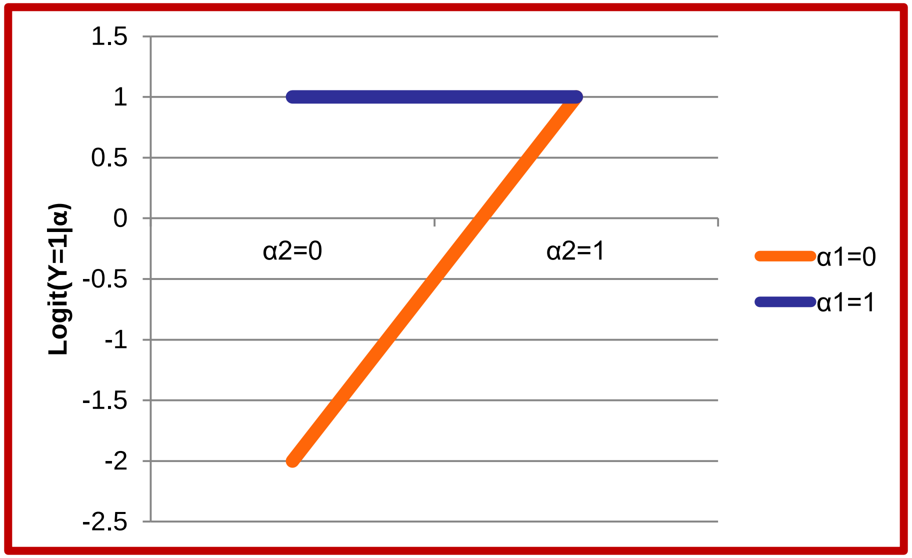
Probability Response Function (Item Characteristic Bar Chart)



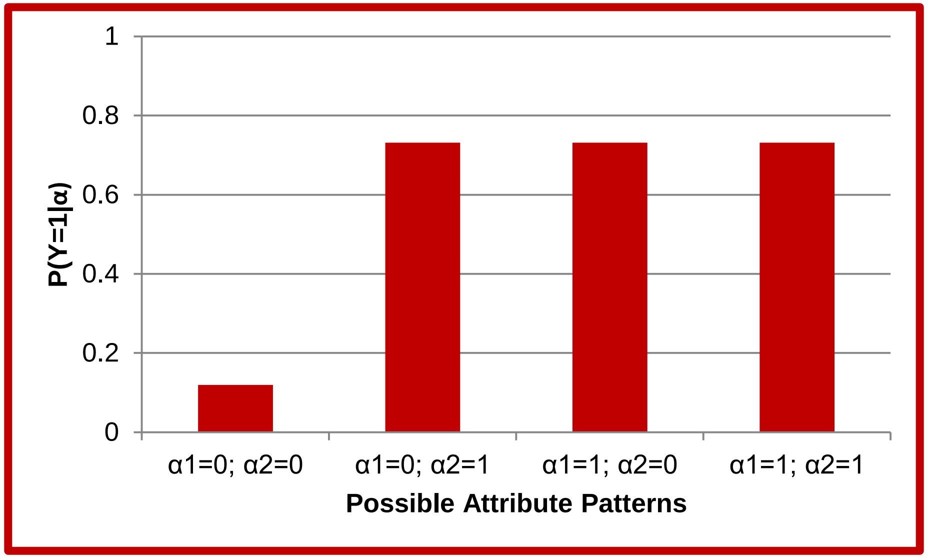
Strong Negative Interactions

- **Negative interaction**: under-additive logit model
  - Disjunctive model (i.e., one-or-more)
  - DINO model (Templin & Henson, 2006)

Logit Response Function



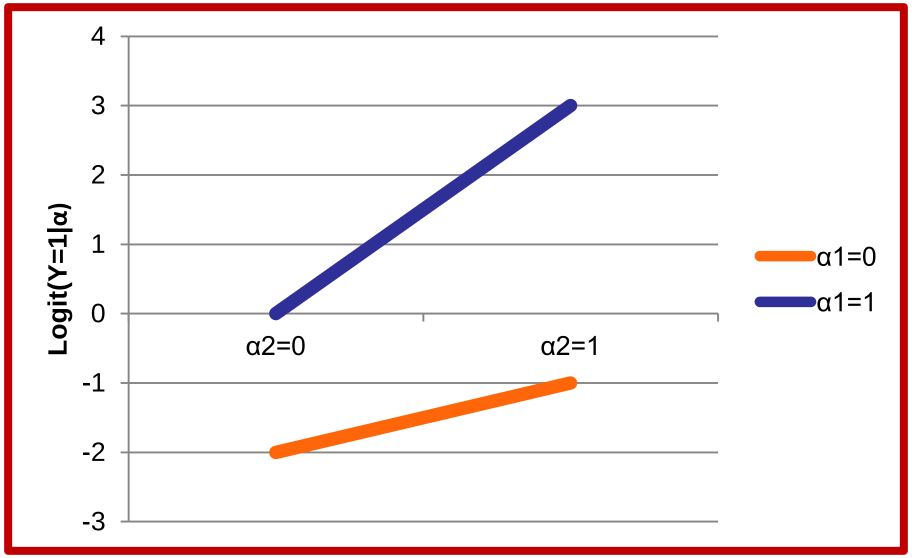
Probability Response Function (Item Characteristic Bar Chart)



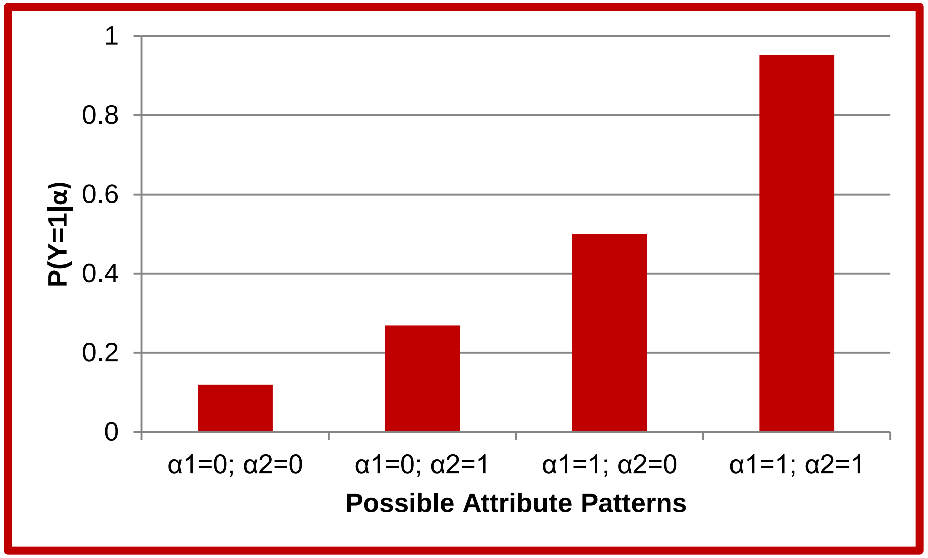
Less Extreme Interactions

- Extreme interactions are unlikely in practice
- Below: positive interaction with positive main effects

Logit Response Function



Probability Response Function (Item Characteristic Bar Chart)



GENERAL FORM OF THE LCDM

More General Versions of the LCDM

- The LCDM is based on the General Diagnostic Model by von Davier (GDM; 2005)
  - The GDM allows for both categorical and continuous latent variables

- For items measuring more than two attributes, higher level interactions are possible
  - Difficult to estimate in practice

- The LCDM appears in the psychometric literature in a more general form
  - See Henson, Templin, & Willse (2009)

General Form of the LCDM

- The LCDM specifies the probability of a correct response as a function of a set of attributes and a Q-matrix:

$$P(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \frac{\exp\left(\boldsymbol{\lambda}_i^T \boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r)\right)}{1 + \exp\left(\boldsymbol{\lambda}_i^T \boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r)\right)}$$

Unpacking the General Form of the LCDM: Components α_r and q_i

- The key to understanding the general form of the LCDM is to understand that it is a general equation that makes any possible number of attributes be measured by an item

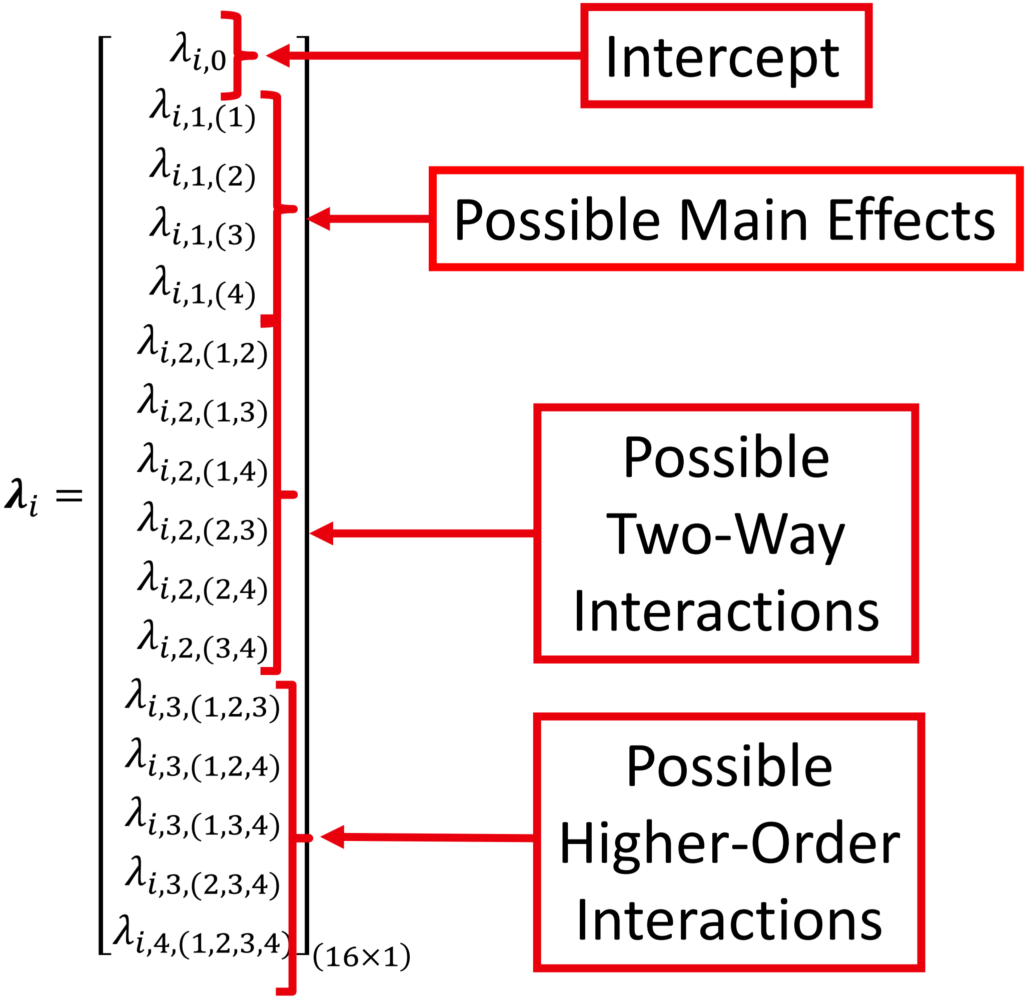
- To put this into context, we will continue with our basic mathematics example
  - Overall four attributes measured: Addition (α_{r1}), Subtraction (α_{r2}), Multiplication (α_{r3}), Division (α_{r4})
    - Attribute profile vector for respondent r: $\boldsymbol{\alpha}_r = [\alpha_{r1} \quad \alpha_{r2} \quad \alpha_{r3} \quad \alpha_{r4}]$ (size 1 x 4)

  - Item i: $2 + 3 - 1 = ?$
    - Measures Addition (α_{r1}) and Subtraction (α_{r2})
    - Q-matrix row vector for item i: $\boldsymbol{q}_i = [1 \quad 1 \quad 0 \quad 0]$

Unpacking the General Form of the LCDM: Parameter Vector λ_i

- From the general LCDM notation, $\boldsymbol{\lambda}_i$ is a vector of all possible item parameters for item i
    - All possible: if all A Q-matrix entries in $\boldsymbol{q}_i$ were equal to 1 (so size is $2^A \times 1$)
    - Not all parameters will be estimated if some $q_{ia} = 0$

- For four-attribute example,



$$\boldsymbol{\lambda}_i = \begin{bmatrix} \lambda_{i,0} \\ \lambda_{i,1,(1)} \\ \lambda_{i,1,(2)} \\ \lambda_{i,1,(3)} \\ \lambda_{i,1,(4)} \\ \lambda_{i,2,(1,2)} \\ \lambda_{i,2,(1,3)} \\ \lambda_{i,2,(1,4)} \\ \lambda_{i,2,(2,3)} \\ \lambda_{i,2,(2,4)} \\ \lambda_{i,2,(3,4)} \\ \lambda_{i,3,(1,2,3)} \\ \lambda_{i,3,(1,2,4)} \\ \lambda_{i,3,(1,3,4)} \\ \lambda_{i,3,(2,3,4)} \\ \lambda_{i,4,(1,2,3,4)} \end{bmatrix}_{(16\times 1)}$$

Intercept

Possible Main Effects

Possible Two-Way Interactions

Possible Higher-Order Interactions

Unpacking the General Form of the LCDM: Helper Function $h(q_i, \alpha_r)$

- From the general LCDM notation $\boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r)$ is a vector-valued function
  - Vector valued = result is a vector

$\boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r)$	$\boldsymbol{\lambda}_i$
$\boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r) = \begin{bmatrix} 1 \\ (q_{i1}\alpha_{r1}) \\ (q_{i2}\alpha_{r2}) \\ (q_{i3}\alpha_{r3}) \\ (q_{i4}\alpha_{r4}) \\ (q_{i1}\alpha_{r1})(q_{i2}\alpha_{r2}) \\ (q_{i1}\alpha_{r1})(q_{i3}\alpha_{r3}) \\ (q_{i1}\alpha_{r1})(q_{i4}\alpha_{r4}) \\ (q_{i2}\alpha_{r2})(q_{i3}\alpha_{r3}) \\ (q_{i2}\alpha_{r2})(q_{i4}\alpha_{r4}) \\ (q_{i3}\alpha_{r3})(q_{i4}\alpha_{r4}) \\ (q_{i1}\alpha_{r1})(q_{i2}\alpha_{r2})(q_{i3}\alpha_{r3}) \\ (q_{i1}\alpha_{r1})(q_{i2}\alpha_{r2})(q_{i4}\alpha_{r4}) \\ (q_{i1}\alpha_{r1})(q_{i3}\alpha_{r3})(q_{i4}\alpha_{r4}) \\ (q_{i2}\alpha_{r2})(q_{i3}\alpha_{r3})(q_{i4}\alpha_{r4}) \\ (q_{i1}\alpha_{r1})(q_{i2}\alpha_{r2})(q_{i3}\alpha_{r3})(q_{i4}\alpha_{r4}) \end{bmatrix}_{(16\times1)}$	$\boldsymbol{\lambda}_i = \begin{bmatrix} \lambda_{i,0} \\ \lambda_{i,1,(1)} \\ \lambda_{i,1,(2)} \\ \lambda_{i,1,(3)} \\ \lambda_{i,1,(4)} \\ \lambda_{i,2,(1,2)} \\ \lambda_{i,2,(1,3)} \\ \lambda_{i,2,(1,4)} \\ \lambda_{i,2,(2,3)} \\ \lambda_{i,2,(2,4)} \\ \lambda_{i,2,(3,4)} \\ \lambda_{i,3,(1,2,3)} \\ \lambda_{i,3,(1,2,4)} \\ \lambda_{i,3,(1,3,4)} \\ \lambda_{i,3,(2,3,4)} \\ \lambda_{i,4,(1,2,3,4)} \end{bmatrix}_{(16\times1)}$

More on the Helper Function $h(q_i, \alpha_r)$

- For a specific item i with a specific Q-matrix row vector $\boldsymbol{q}_i$, the role of the helper function $\boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r)$ becomes more transparent

$\boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r)$	$\boldsymbol{\lambda}_i$
$\boldsymbol{h}(\boldsymbol{q}_i, \boldsymbol{\alpha}_r) = \begin{bmatrix} 1 \\ (q_{i1}\alpha_{r1}) = \alpha_{r1} \\ (q_{i2}\alpha_{r2}) = \alpha_{r2} \\ (q_{i3}\alpha_{r3}) = 0 \\ (q_{i4}\alpha_{r4}) = 0 \\ (q_{i1}\alpha_{r1})(q_{i2}\alpha_{r2}) = \alpha_{r1}\alpha_{r2} \\ (q_{i1}\alpha_{r1})(q_{i3}\alpha_{r3}) = 0 \\ (q_{i1}\alpha_{r1})(q_{i4}\alpha_{r4}) = 0 \\ (q_{i2}\alpha_{r2})(q_{i3}\alpha_{r3}) = 0 \\ (q_{i2}\alpha_{r2})(q_{i4}\alpha_{r4}) = 0 \\ (q_{i3}\alpha_{r3})(q_{i4}\alpha_{r4}) = 0 \\ (q_{i1}\alpha_{r1})(q_{i2}\alpha_{r2})(q_{i3}\alpha_{r3}) = 0 \\ (q_{i1}\alpha_{r1})(q_{i2}\alpha_{r2})(q_{i4}\alpha_{r4}) = 0 \\ (q_{i1}\alpha_{r1})(q_{i3}\alpha_{r3})(q_{i4}\alpha_{r4}) = 0 \\ (q_{i2}\alpha_{r2})(q_{i3}\alpha_{r3})(q_{i4}\alpha_{r4}) = 0 \\ (q_{i1}\alpha_{r1})(q_{i2}\alpha_{r2})(q_{i3}\alpha_{r3})(q_{i4}\alpha_{r4}) = 0 \end{bmatrix}_{(16\times1)}$	$\boldsymbol{\lambda}_i = \begin{bmatrix} \lambda_{i,0} \\ \lambda_{i,1,(1)} \\ \lambda_{i,1,(2)} \\ \lambda_{i,1,(3)} \\ \lambda_{i,1,(4)} \\ \lambda_{i,2,(1,2)} \\ \lambda_{i,2,(1,3)} \\ \lambda_{i,2,(1,4)} \\ \lambda_{i,2,(2,3)} \\ \lambda_{i,2,(2,4)} \\ \lambda_{i,2,(3,4)} \\ \lambda_{i,3,(1,2,3)} \\ \lambda_{i,3,(1,2,4)} \\ \lambda_{i,3,(1,3,4)} \\ \lambda_{i,3,(2,3,4)} \\ \lambda_{i,4,(1,2,3,4)} \end{bmatrix}_{(16\times1)}$

Putting It All Together: The Matrix Product $\lambda_i^T h(q_i,\alpha_r)$

- The term in the exponent is the logit we have been using all along:

$$Logit(Y_{ri}=1|\boldsymbol{\alpha}_r)$$

- In general:

$$\boldsymbol{\lambda}_i^T \boldsymbol{h}(\boldsymbol{q}_i,\boldsymbol{\alpha}_r) = \lambda_{i,0} + \sum_{a=1}^{A}\lambda_{i,1,(a)}(q_{ia}\alpha_{ra}) + \sum_{a=1}^{A-1}\sum_{b=a+1}^{A}\lambda_{i,2,(a,b)}(q_{ia}\alpha_{ra})(q_{ib}\alpha_{rb}) + \cdots$$

Intercept

Main Effects

Two-Way Interactions

Higher Interactions

- For our example item:

$$(\boldsymbol{\lambda}_i^T)_{(1\times16)}\boldsymbol{h}(\boldsymbol{q}_i,\boldsymbol{\alpha}_r)_{(16\times1)} = \lambda_{i,0} + \lambda_{i,1,(1)}\alpha_{r1} + \lambda_{i,1,(2)}\alpha_{r2} + \lambda_{i,2,(1,2)}\alpha_{r1}\alpha_{r2}$$

- Result is a scalar (1×1)

SUBSUMED MODELS

Previously Popular DCMs

- Because the advent of the GDM and LCDM has been fairly recent, other earlier DCMs are still in use

- Such DCMs are much more restrictive than the LCDM
  - Not discussed at length here
  - It is anticipated that field will adapt to more general forms

- Each of these models can be fit using the LCDM
  - Fixing certain model parameters

- Shown for reference purposes
  - See Henson, Templin, & Willse (2009) for more detail

Other DCMs with the LCDM

- The Big 6 - DCMs with latent variables:
  - **DINA** (Deterministic Inputs, Noisy ‘AND’ Gate)
    - Haertel (1989); Junker and Sijtsma (1999)
  - **NIDA** (Noisy Inputs, Deterministic ‘AND’ Gate)
    - Maris (1995)
  - **RUM** (Reparameterized Unified Model)
    - Hartz (2002)
  - **DINO** (Deterministic Inputs, Noisy ‘OR’ Gate)
    - Templin & Henson (2006)
  - **NIDO** (Noisy Inputs, Deterministic ‘OR’ Gate)
    - Templin (2006)
  - **C-RUM** (Compensatory Reparameterized Unified Model)
    - Hartz (2002)

Other DCMs with the LCDM

LCDM Parameters	Non-compensatory Models			Compensatory Models		
	DINA	NIDA	NC-RUM	DINO	NIDO	C-RUM
Main Effects	Zero	Positive	Positive	Positive	Positive	Positive
Interactions	Positive	Positive	Positive	Negative	Zero	Zero
Parameter Restrictions	Across Attributes	Across Items	---	Across Attributes	Across Items	---

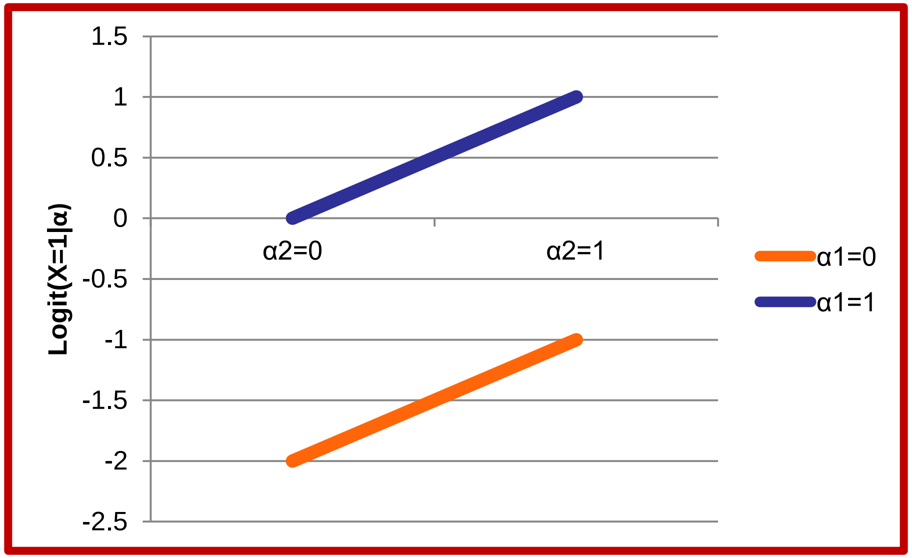
Adapted from: Rupp, Templin, and Henson (2010)

Compensatory RUM (Hartz, 2002)

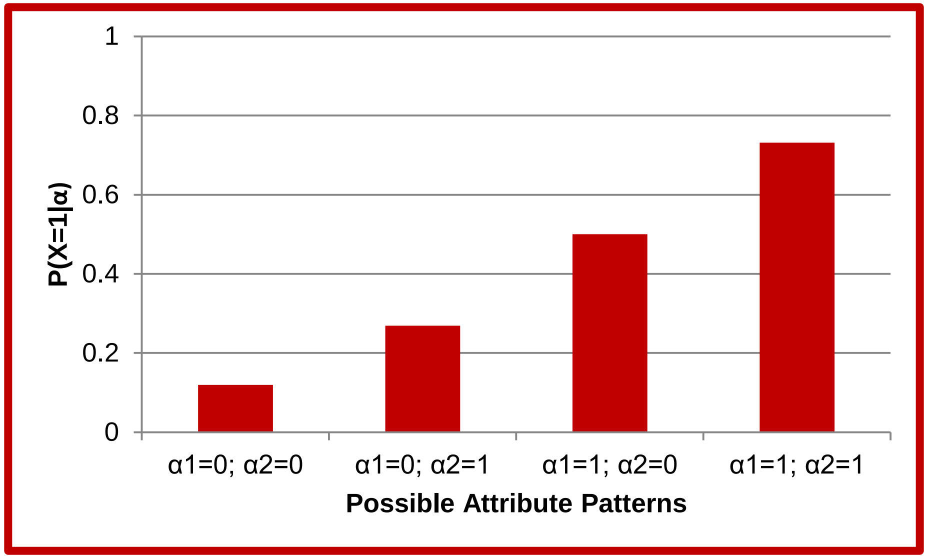
- No interactions in model
- **No interaction**: parallel lines for the logit

$$Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \lambda_{i,0} + \lambda_{i,1,(1)}\alpha_{r1} + \lambda_{i,1,(2)}\alpha_{r2}$$

Logit Response Function



Item Characteristic Bar Chart



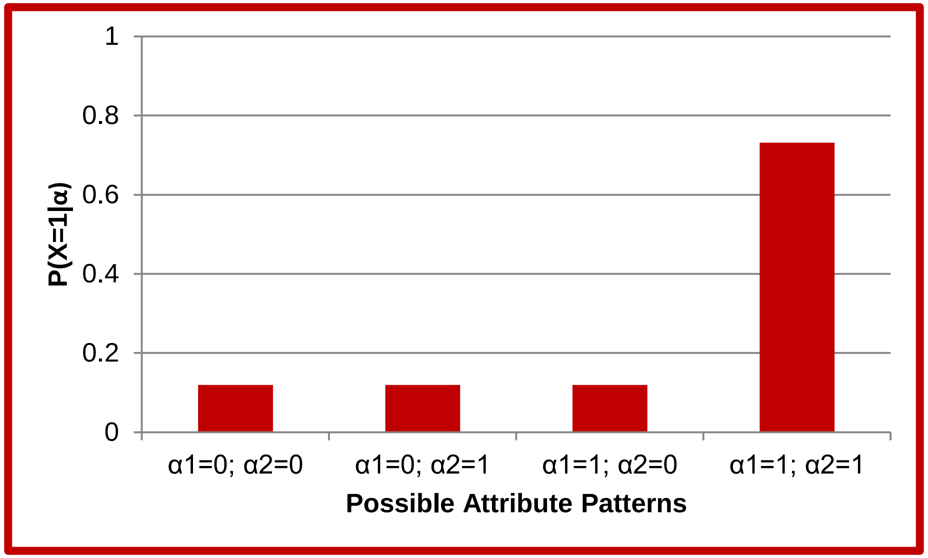
DINA Model (Haertel, 1989; Junker & Sijstma, 1999)

- **Positive interaction**: over-additive logit model
  - Highest interaction parameter is non-zero
  - All main effects (and lower interactions) zero

$$Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \lambda_{i,0} + \lambda_{i,2,(1,2)}\alpha_{r1}\alpha_{r2}$$

Logit Response Function

Item Characteristic Bar Chart

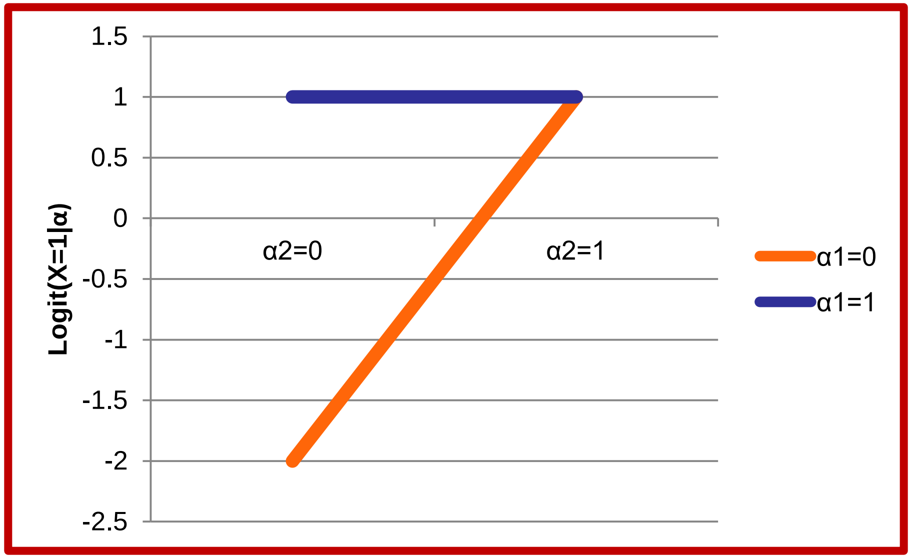


DINO Model (Templin & Henson, 2006)

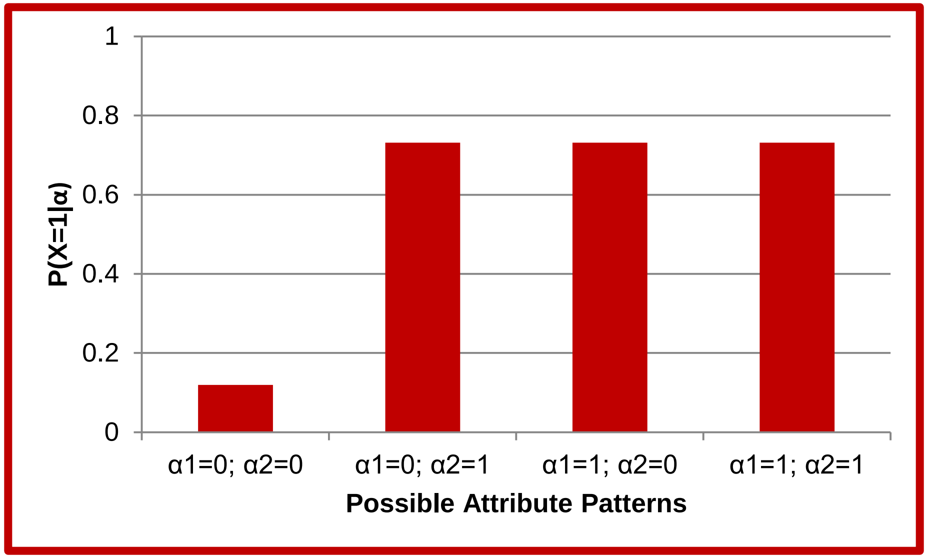
- **Negative interaction**: under-additive logit model
  - All main effects equal
  - Interaction terms are -1 sum of corresponding lower effects

$$Logit(Y_{ri} = 1|\boldsymbol{\alpha}_r) = \lambda_{i,0} + \lambda_{i,1}\alpha_{r1} + \lambda_{i,1}\alpha_{r2} - \lambda_{i,1}\alpha_{r1}\alpha_{r2}$$

Logit Response Function



Item Characteristic Bar Chart



EXAMPLE RESULTS: DTMR PROJECT

LCDM Example Item Response Function

- Referent unit (α_1) and partitioning and iterating (α_2) are measured
  - Q-matrix entries:

	RU	PI	APP	MC
Item 22	1	1	0	0

  - LCDM item response function:

$$logit(X_{ei} = 1|\boldsymbol{\alpha}_e) = \underbrace{\boxed{\lambda_{i,0}}}_{\text{Intercept}} + \underbrace{\boxed{\lambda_{i,1,(1)}}}_{\substack{\text{Main Effect} \\ \text{(RU)}}}\alpha_{e1} + \underbrace{\boxed{\lambda_{i,1,(2)}}}_{\substack{\text{Main Effect} \\ \text{(PI)}}}\alpha_{e2} + \underbrace{\boxed{\lambda_{i,2,(1,2)}}}_{\substack{\text{Interaction} \\ \text{(Between RU and PI)}}}\alpha_{e1}\alpha_{e2}$$

From the DTMR Paper

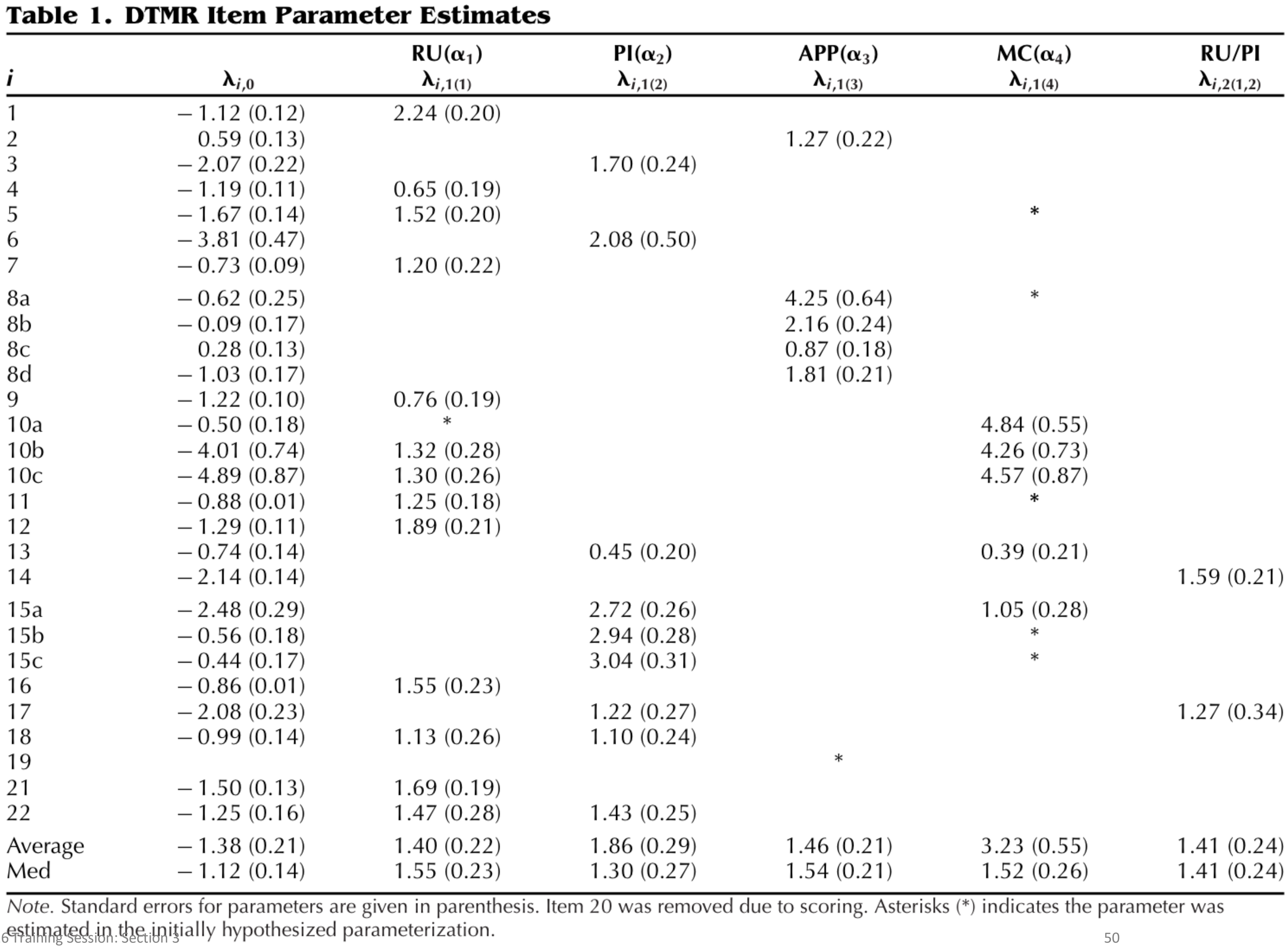


Table 1. DTMR Item Parameter Estimates

i	$\lambda_{i,0}$	RU(α_1) $\lambda_{i,1(1)}$	PI(α_2) $\lambda_{i,1(2)}$	APP(α_3) $\lambda_{i,1(3)}$	MC(α_4) $\lambda_{i,1(4)}$	RU/PI $\lambda_{i,2(1,2)}$
1	− 1.12 (0.12)	2.24 (0.20)				
2	0.59 (0.13)			1.27 (0.22)		
3	− 2.07 (0.22)		1.70 (0.24)			
4	− 1.19 (0.11)	0.65 (0.19)				
5	− 1.67 (0.14)	1.52 (0.20)			*	
6	− 3.81 (0.47)		2.08 (0.50)			
7	− 0.73 (0.09)	1.20 (0.22)				
8a	− 0.62 (0.25)			4.25 (0.64)	*	
8b	− 0.09 (0.17)			2.16 (0.24)		
8c	0.28 (0.13)			0.87 (0.18)		
8d	− 1.03 (0.17)			1.81 (0.21)		
9	− 1.22 (0.10)	0.76 (0.19)				
10a	− 0.50 (0.18)	*			4.84 (0.55)	
10b	− 4.01 (0.74)	1.32 (0.28)			4.26 (0.73)	
10c	− 4.89 (0.87)	1.30 (0.26)			4.57 (0.87)	
11	− 0.88 (0.01)	1.25 (0.18)			*	
12	− 1.29 (0.11)	1.89 (0.21)				
13	− 0.74 (0.14)		0.45 (0.20)		0.39 (0.21)	
14	− 2.14 (0.14)					1.59 (0.21)
15a	− 2.48 (0.29)		2.72 (0.26)		1.05 (0.28)	
15b	− 0.56 (0.18)		2.94 (0.28)		*	
15c	− 0.44 (0.17)		3.04 (0.31)		*	
16	− 0.86 (0.01)	1.55 (0.23)				
17	− 2.08 (0.23)		1.22 (0.27)			1.27 (0.34)
18	− 0.99 (0.14)	1.13 (0.26)	1.10 (0.24)			
19				*		
21	− 1.50 (0.13)	1.69 (0.19)				
22	− 1.25 (0.16)	1.47 (0.28)	1.43 (0.25)			
Average	− 1.38 (0.21)	1.40 (0.22)	1.86 (0.29)	1.46 (0.21)	3.23 (0.55)	1.41 (0.24)
Med	− 1.12 (0.14)	1.55 (0.23)	1.30 (0.27)	1.54 (0.21)	1.52 (0.26)	1.41 (0.24)

Note. Standard errors for parameters are given in parenthesis. Item 20 was removed due to scoring. Asterisks (*) indicates the parameter was estimated in the initially hypothesized parameterization.

CONCLUDING REMARKS

Wrapping Up – Lecture Take-Home Points

- The LCDM uses an ANOVA-like approach to map latent attributes onto item responses
  - Uses main effects and interactions for each attribute
  - Uses a logit link function

- Multiple diagnostic models are subsumed by the LCDM