Fundamental Concepts for Using Diagnostic Classification Models

Section #2
NCME 2016 Training Session

Lecture Overview

- Nature of attributes
  - What's in a name?
  - Grain sizes explained
  - Q-matrices
- Attribute hierarchies
  - Types of attribute hierarchies
- Reporting of attribute profiles
- Developing cognitive processing models

THE NATURE OF ATTRIBUTES

The Nature of Attributes

- Different labels have been suggested in the literature for the latent variables:
  - Latent characteristics
  - Latent traits
  - Elements of processes
  - Attributes

- Each of these terms carries with it a specific connotation that reflects important beliefs about what analysts hope they can learn from applying DCMs

Meanings of Terms

- Latent characteristic:
  - Highlights that the mental components that are of theoretical interest are unobserved, which is why they are represented with *latent variables* in DCMs
- Latent trait:
  - Highlights that the mental components of interests are believed to be stable across time in contrast to 'latent states' that change over time
- Element:
  - Refers to the fact that the mental components are building blocks of a larger conceptual entity, which is often, but not necessarily, the 'cognitive response process'

Meanings of Terms

- Attribute:
  - Is perhaps most frequently used in the measurement literature on diagnostic assessments
  - Has a history of use in the literature on factor analysis (e.g., McDonald, 1999)
  - Typically synonymous with the terms ‘latent trait’ and ‘latent characteristic’
- We use the term *attribute* for individual latent variables
  - The dominant discourse on DCMs
- *Attribute profile* denotes a particular constellation of latent variable values for a particular respondent

Types of Attributes

Table 4.1

Exemplary Attribute Definitions from Different Domains

Construct: Number Subtraction Domain: Mathematics Source: de la Torre & Douglas (2004)	
- convert a whole number to a fraction - separate a whole number from a fraction - simplify before subtracting - find a common denominator	- borrow from whole number part - column borrow to subtract - subtract numerators - simplify answer
Construct: Reading Comprehension Domain: English Language Learning Source: Buck, Tatsuoka, & Kostin (1997)	
- synthesize scattered information - recognize relevant information - know low-frequency vocabulary - identify the gist of a passage	- apply relevant background knowledge - hold relevant information in WM - use a word-matching strategy - compare two plausible options and choose
Construct: Figural Analysis Domain: Architecture Source: Katz, Martinez, Sheehan, & Tatsuoka (1998)	
- move or rotate objects - read and translate information - activate prior knowledge - identify distracting information	- identify environmental characteristics - process a complex diagram - understand structural technology - apply a learned procedure

Grain Sizes of Attributes

- The degree of definitional specificity of an attribute is often referred to as the *definitional grain size*
- The grain size is driven by the level of specificity with which one desires to make statements about respondents
- The grain size of an attribute is the resolution with which an investigator dissects a cognitive response process and describes its constituent components

Practical Issues with Grain Sizes

- It is possible to decompose individual attributes for more complex tasks further
  - That would increase the number of attributes
- As the number of attributes increases, the number of latent variables in a DCM increases
  - Attribute profiles and item parameters may become impossible to estimate statistically
- It is important to fix the number of attributes to a statistically manageable number for a given diagnostic assessment length and respondent sample size
  - That number is rapidly increasing as technology gets better

Q-matrices

- The specification of which attributes are constitutive of the response process for each item is done numerically in a table with a particular structure called a *Q-matrix* (Tatsuoka, 1983)

- A Q-matrix traditionally contains the items in the rows and the attributes in the columns

- Its entries consist of '1s' and '0s' indicating whether or not an attribute is required to respond to an item
  - A '1' indicates the item measures the attribute

- Identical to factor pattern matrices in CFA
  - Define which parameters are set to zero in the DCM

Example Q-matrix

Table 4.4

Sample Q-matrix for Sample Diagnostic Assessment

	Attribute 1	Attribute 2	Attribute 3	Attribute 4	Attribute 5	Attribute 6
Item 1	0	0	0	1	0	0
Item 2	0	0	0	1	1	0
Item 3	1	0	0	1	0	1
Item 4	1	1	1	1	0	0
Item 5	1	0	0	0	0	1
Item 6	0	1	0	1	1	0
Item 7	1	1	1	0	0	0
Item 8	0	1	0	1	0	1
Item 9	0	1	0	1	1	1
Item 10	0	0	0	0	0	1

This table shows which attributes are required to respond to a particular item. There are 10 items and 6 attributes on this assessment. An entry of '1' indicates that the attribute is required whereas an entry of '0' indicates that it is not required.

ATTRIBUTE HIERARCHIES

Attribute Hierarchies

- *Attribute hierarchies: s*pecifications of the attribute dependencies in the population of respondents
  - By implication they represent hypotheses about which attribute profiles should be observed in a sample

- Suppose that mastery of Attribute 1 is prerequisite to mastery of Attribute 2:
  - Attribute profiles where the second but not the first attribute is mastered must logically be empty in the population

Types of Attribute Hierarchies

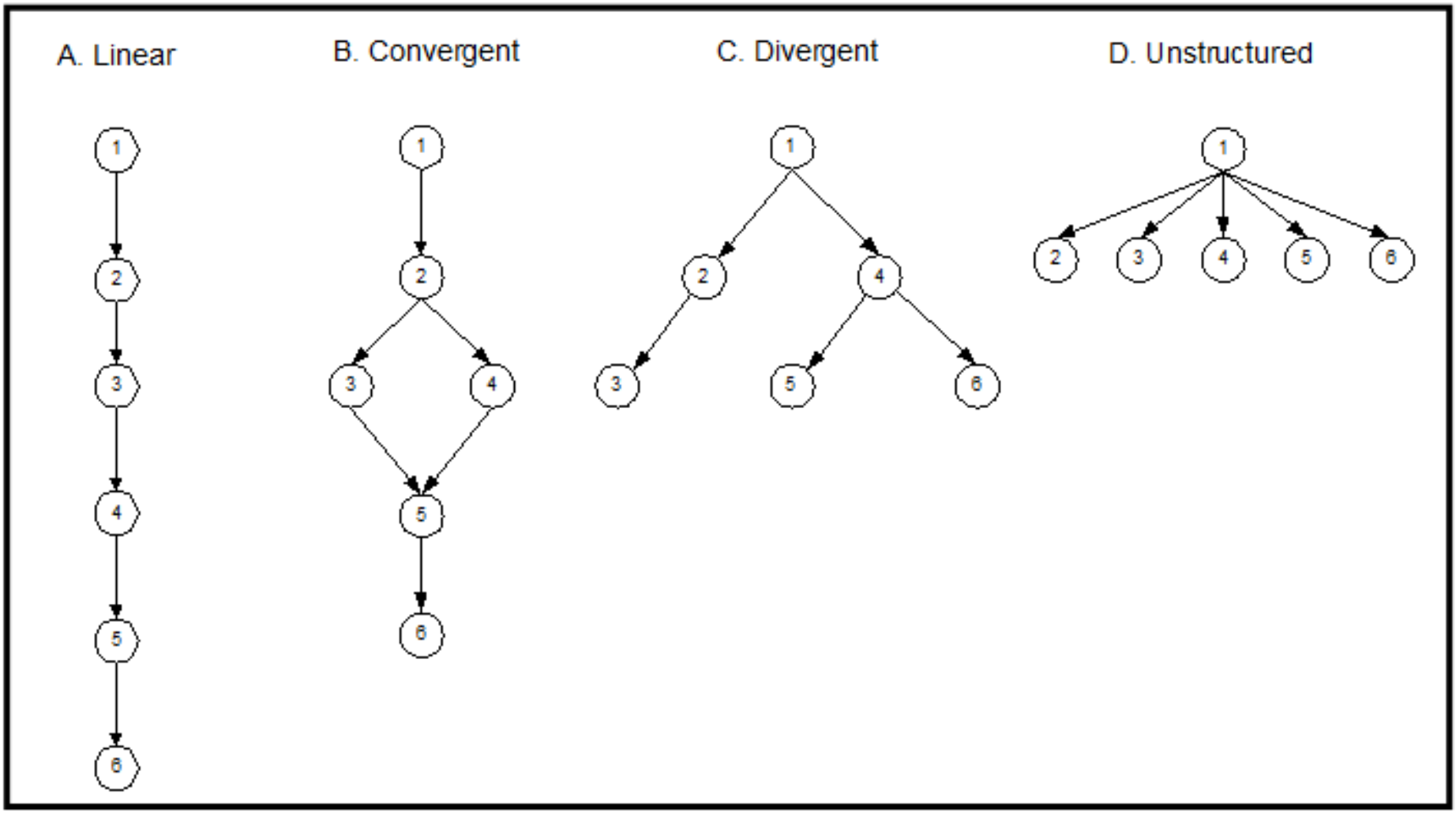


Figure 4.1 Prototypical attribute hierarchies (from Gierl, Leighton & Hunka, 2007)

Additional Matrices Implied by Attribute Hierarchies

- There are additional attribute matrices implied by attribute hierarchies:
  - Adjacency Matrix
    - Lists hierarchically dependent attributes
  - Reachability Matrix
    - Lists which attributes can be reached by others
    - Comes from Adjacency Matrix

- Used in Attribute Hierarchy/Rule Space techniques
  - We use these matrices to reduce the number of parameters in the structural model (see Chapter 8)

- Adjacency Matrix is a network/graph theoretic entity

REPORTING OF ATTRIBUTE PROFILES

Example Profile Report (Page 1)

DiagnOsis scoring report

Student Name: Margo **LanguEdge Reading Comprehension Test 1**

Review Your Answers

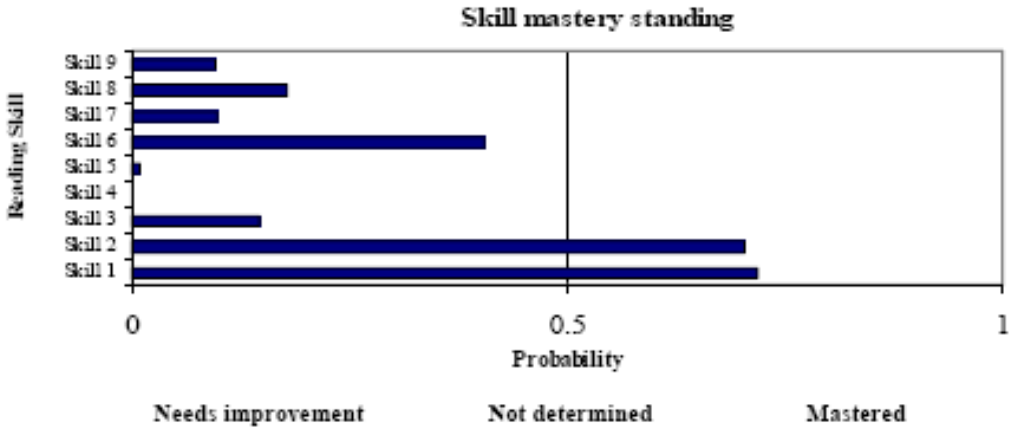
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
Your Answer	√	√	√	2	√	1	4	√	√	√	3	2	2	√	√	√	√	2	√	√	√	3	2	3	5	1	√	1	4	4	√	√	√	o	√	3	1,4,6 2,3
Correct Answer	2	3	2	3	3	3	1	1	1	4	4	3	2,4,6	2	3	2	1	3	2	3	4	2	4	1	1,5,6	4	2	2	3	3	1	2	2	2	4	1	1,5,6 3,7
Difficulty	e	m	e	h	m	m	h	h	m	m	h	m	m	e	m	m	m	e	e	e	e	h	m	m	m	m	e	h	m	e	e	e	e	m	h	m	h

Scoring
Correct answer to questions with 4 choices = Plus 1 point
Wrong or omitted answer = No point
Q13 & 25: 3 correct = 2 points, 2 correct=1 point
Q37: 5 correct=3 points, 4 correct=2 points, 3 correct = 1 point

Key
√ Correct
o Omitted
+ Plus partial points
e = Easy, m = Medium, h = Hard
(Difficulty is based on 1372 students' performance on this test)

Score
You earned *20* out of maximum *41* points.
10 points from *12* easy questions
7 points from *17* medium questions
3 points from *8* hard questions
You omitted 1 question.

Improve Your Skills



How to Interpret Skill Mastery

- Nine primary reading skills are assessed in this reading comprehension test. Please review skill descriptions and example questions attached to this scoring report.
- The graph on the left side shows your probable mastery standing of each skill.
- The grey region indicates that your probable mastery standing cannot be determined.
- There may be some measurement error associated with the classification.
- This diagnostic information can be more useful when used in combination with your teacher's and your own evaluation of your reading skills.

Example Profile Report (Page 2)

DiagnOsis scoring report

Primary Skill Descriptions and Example Questions — **Margo**

	Skill Descriptions	Example Questions
	Skill 1: Deduce word meaning from context Deducing the meaning of a word or a phrase by searching and analyzing text and by using contextual clues in the text.	33, 14, 32, 4, 3, 11
	Skill 2: Determine word meaning out of context Determine word meaning out of context with recourse to background knowledge	9, 27, 10, 29, 19, 21, 7
⚑	**Skill 3: Comprehend text through syntactic and semantic links** Comprehend relations betweens parts of text through lexical and grammatical cohesion devices within and across successive sentences without logical problems	3, 26, 12, 36, 4, 2, 22, 33, 24
⚑	**Skill 4: Comprehension of text-explicit information** Read quickly across sentences within a paragraph and comprehend literal meaning of explicitly stated information.	22, 18, 30, 17, 8, 24, 36, 20, 12, 25, 14
⚑	**Skill 5: Comprehend text-implicit information at global level** Read selectively a paragraph or across paragraphs to recognize salient ideas paraphrased based on implicit information in text.	6, 34, 26, 4, 5, 35
?	**Skill 6: Infer major arguments or a writer's purpose** Skim through paragraphs and make propositional inferences about arguments or a writer's purpose with recourse to implicitly stated information or prior knowledge	31, 16, 23, 15, 28, 2, 11, 7, 32
⚑	**Skill 7: Comprehend negatively stated information** Read carefully or expeditiously to locate relevant information in text and to determine which information is true or not true.	22, 7, 28, 5
⚑	**Skill 8: Summarize major ideas from minor details** Analyze and evaluate relative importance of information in the text by distinguishing major ideas from supporting details.	13, 5, 17, 25, 20
⚑	**Skill 9: Determine contrasting ideas through diagrammatic display** Recognize major contrasts and arguments in the text whose rhetorical structure contains the relationships such as compare/contrast, cause/effect or alternative arguments and map them into mental framework	37, 23, 35

- **Not all example questions are equally informative in assessing related skills. Questions are listed in the order from most informative to least informative.**
- ⚑ **indicates that these skills are weak areas you need to improve. '?' indicates that your mastery is not determined.**

DEVELOPING COGNITIVE PROCESSING MODELS

Attributes Exist Because of Theory

- Current applications of DCMs typically involve attributes that are defined via a theory of response processing supported by research in applied cognitive psychology and educational measurement

- In order to use DCMs to represent and, ideally, *validate cognitive processing models* one first needs to have developed a plausible model from theory and empirical investigations

- Cognitive response processes can be decomposed more easily into their constituent attributes for tasks that are narrower in scope and are can be solved with fewer alternative response strategies that rely on combinations of different attributes

Methods Used for Cognitive Models

- Verbal reports and protocol studies
  - Both require that the items under consideration are presented to a sample of respondents from the population who are probed about the way they respond to them
- Eye-tracking research
  - Evidence for the cognitive processes that respondents engage in comes from the physiological manifestations of these processes
- Expert panels
  - Ask experts to describe the cognitive processes behind item responses based on prior research and experience with assessments in the domain

Limitations of the Methodologies

- Researchers from disciplines outside of educational measurement are often disconcerted with the use of certain terms and procedures in the educational measurement literature on diagnostic assessment

- In cognitive psychology, the dominant focus is on understanding the basic mental architecture of human beings and its consequences on how human beings process information to solve particular tasks

- In differential psychology, the dominant focus is on explaining intra- and inter-individual differences on these mental components and capacities and their structural relationships in a population

APPLYING CONCEPTS:
THE DIAGNOSING TEACHERS' MULTIPLICATIVE REASONING PROJECT

Introduction

- Diagnosing Teachers' Multiplicative Reasoning* (DTMR)
  - NSF funded grant (DRL-0903411)

- Goal was to create a test that will assess fine-grained components of teachers' reasoning multiplicatively with rational numbers

- The test was used to
  - Tailor professional development to teachers' needs
  - Quantitatively study teachers' fine-grained abilities to reason multiplicatively
    - Quantify findings based on extensive qualitative research base
    - Generalize to larger populations

Big Picture

- Most psychometric models are designed to measure a unidimensional continuous trait or ability

- Examples of continuous traits
  - Student's "math" ability at the 8th grade level
  - In-service teachers' mathematical knowledge for teaching number and operations
    - The content area of focus for this study

- As a result, many tests are designed to measure a unidimensional ability

- This project took a different approach
  - A multidimensional diagnostic approach
  - Using a new class of psychometric models

Diagnosing Multiplicative Reasoning

- Instead of measuring an overall ability to reason multiplicatively with fractions, we can break that continuous trait down into more fine-grained cognitive facilities or attributes:
  - Ability to identify appropriate referent units for numbers
  - Ability to partition quantities and iterate unit fractions
  - Ability to identify appropriate arithmetic operations
  - Ability to make multiplicative comparisons

- We treat these attributes as categorical
  - Dichotomous (have two categories)
  - Mastery of an attribute (= 1) or non-mastery of an attribute (= 0)

Example Item

- This item is analogous to Item 22 on the DTMR test
  - » Measures Referent Unit (Attribute 1) and Partitioning and Iterating (Attribute 2)

Ms. Roland gave her students the following problem to solve:

Candice has 4/5 of a meter of cloth. She uses 1/8 of a meter for a project.
How much cloth does she have left after the project?

She had students use the number line so that they could draw the lengths. Which of the following diagrams shows the solution? Assume all intervals are subdivided equally.

a)

b)

c)

d)

e)

Groups According to Attribute Mastery

- The groups are based on patterns of mastery according to the set of attributes
- A classification of each individual skill results in a classification into one of these 16 patterns

2^A possible patterns or groups:

Pattern	RU	PI	APP	MC
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	1
5	0	1	0	0
6	0	1	0	1
7	0	1	1	0
8	0	1	1	1
9	1	0	0	0
10	1	0	0	1
11	1	0	1	0
12	1	0	1	1
13	1	1	0	0
14	1	1	0	1
15	1	1	1	0
16	1	1	1	1

Designing Diagnostic Tests

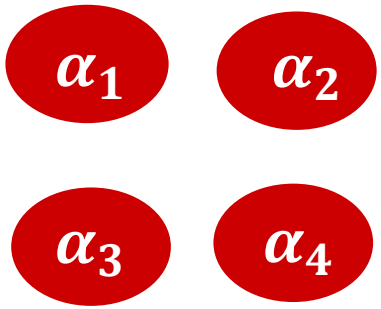
- Diagnostic tests are written so that each item measures one or more of the attributes
- The attributes measured by each item are recorded in a Q-matrix
  - Describes whether an item measures an attribute (q = 1) or not (q = 0)
  - Mapping is established by content experts
    - Confirmed by item response interviews
- First several items on DTMR test:

	RU	PI	APP	MC
Item 1	1	0	0	0
Item 2	0	0	1	0
Item 3	1	0	0	0
Item 4	1	0	0	1

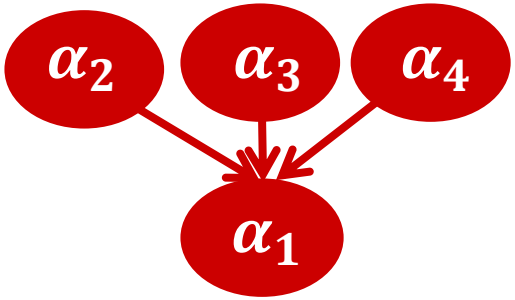
Attribute Hierarchies

- We tested the following hierarchies using the Hierarchical Diagnostic Classification Model* (HDCM)
  - All hierarchies fit significantly worse ($p<.001$) than the no hierarchy

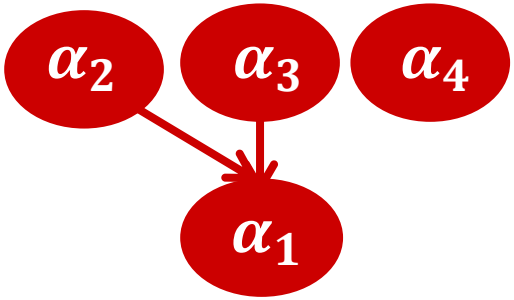
No Hierarchy



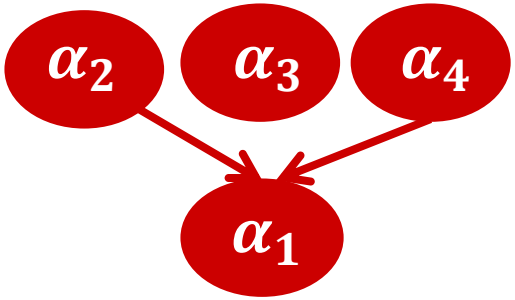
Hierarchy 1



Hierarchy 2



Hierarchy 3



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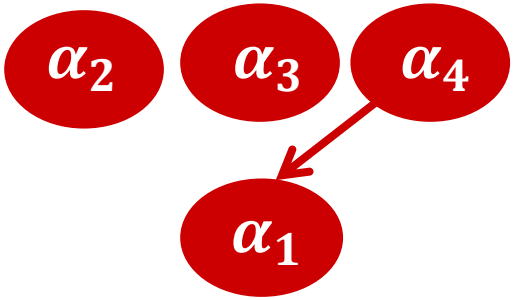
Hierarchy 7



Wrapping Up

- In this lecture we defined:
  - Attributes
  - Q-matrices
  - Attribute hierarchies
  - How cognitive theories are built
  - Limitations of the approaches