CFA Example Using Forgiveness of Situations (N = 1103)

The Forgiveness of Situations Subscale includes 6 items, 3 of which are reverse-coded, on a 7-point scale:

1. When things go wrong for reasons that can't be controlled, I get stuck in negative thoughts about it. **(R)**
2. With time I can be understanding of bad circumstances in my life.
3. If I am disappointed by uncontrollable circumstances in my life, I continue to think negatively about them. **(R)**
4. I eventually make peace with bad situations in my life.
5. It's really hard for me to accept negative situations that aren't anybody's fault. **(R)**
6. Eventually I let go of negative thoughts about bad circumstances that are beyond anyone's control.

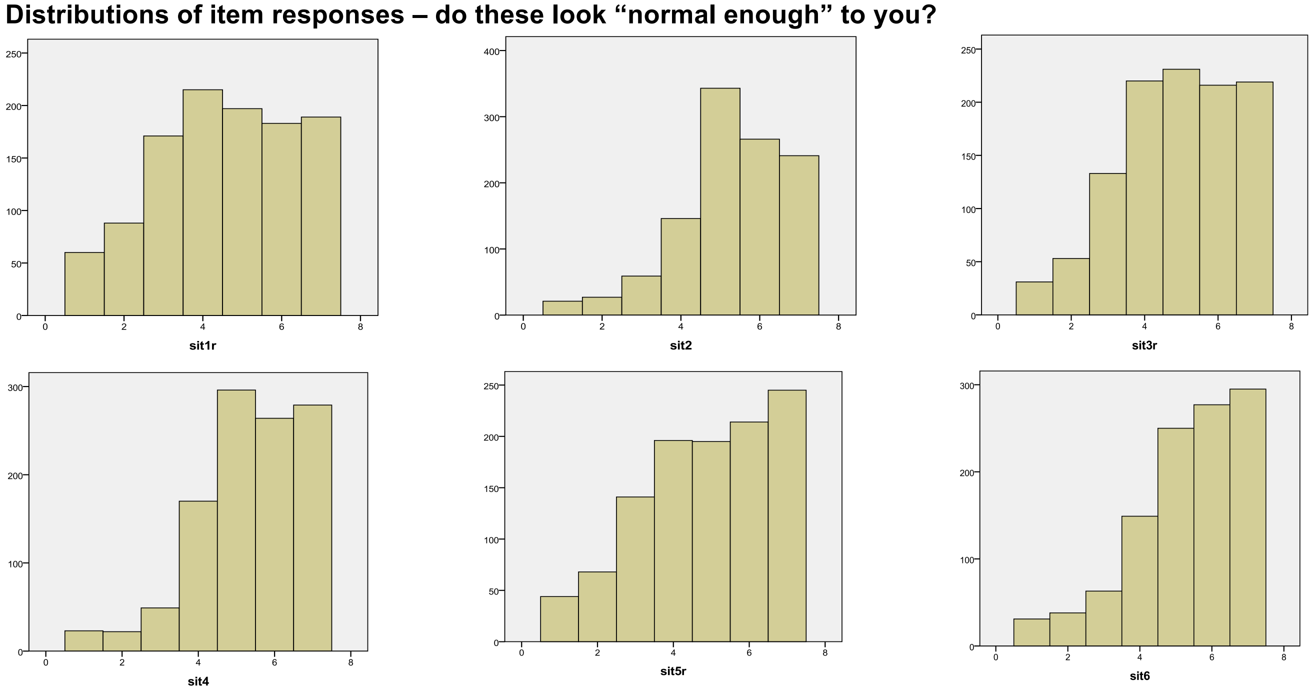
Response Anchors: 1 = Almost Always False of Me, 2=?, 3 = More Often False of Me, 4 = ?,
5 = More Often True of Me, 6 = ?, 7 = Almost Always True of Me

Observed Correlation Matrix	**R1**	**2**	**R3**	**4**	**R5**	**6**
R1	**1.000**					
2	0.240	**1.000**				
R3	0.647	0.317	**1.000**			
4	0.300	0.570	0.369	**1.000**		
R5	0.453	0.255	0.482	0.289	**1.000**	
6	0.297	0.457	0.356	0.448	0.304	**1.000**
Means	4.547	5.289	4.896	5.359	4.860	5.321
Variances	3.049	1.903	2.543	1.967	2.945	2.341

Observed Covariance Matrix	**R1**	**2**	**R3**	**4**	**R5**	**6**
R1	**3.049**					
2	0.577	**1.903**				
R3	1.802	0.697	**2.543**			
4	0.734	1.103	0.824	**1.967**		
R5	1.358	0.604	1.319	0.695	**2.945**	
6	0.795	0.965	0.868	0.962	0.798	**2.341**

To do CFA analysis, you only really need means, variances, and either correlations or covariances among items:
$\text{Covariance}_{y1,y2} = \text{Correlation}_{y1,y2} * SD(Y_1) * SD(Y_2)$ OR $\text{Correlation}_{y1,y2} = \text{Covariance}_{y1,y2} / SD(Y_1) * SD(Y_2)$

Distributions of item responses – do these look "normal enough" to you?



Mplus Code to Read in Data:

```
TITLE:        CFA of Situation Factor
DATA:         FILE IS Study2.dat;        ! Don't need path if in same directory
              FORMAT IS free;            ! Default
              TYPE IS INDIVIDUAL;        ! Default

VARIABLE:     NAMES ARE PersonID Self1 Self2r Self3 Self4r Self5 Self6r
                        Other1r Other2 Other3r Other4 Other5r Other6
                        Sit1r Sit2 Sit3r Sit4 Sit5r Sit6
                        Selfsub Othsub Sitsub HFSsum;                  ! Every variable in DATASET

              USEVARIABLES ARE Sit1r Sit2 Sit3r Sit4 Sit5r Sit6;      ! Every variable in MODEL
              MISSING ARE ALL (99999);                                 ! Identify missing values
              IDVARIABLE IS PersonID;                                  ! Identify person ID variable

ANALYSIS:     TYPE IS GENERAL;           ! Default
              ESTIMATOR IS MLR;          ! Robust ML

SAVEDATA:     SAVE = FSCORES; FILE = FactorScores.dat;  ! To save factor scores

PLOT:         TYPE = PLOT1 PLOT2 PLOT3; ! To get all plots (e.g., factor score distributions)

OUTPUT:       MODINDICES (6.635)  ! Voodoo suggestions to improve the model at p <.01
              STDYX               ! Fully standardized solution
              RESIDUAL            ! Standardized and normalized residuals for local fit
              FSDETERMINACY;      ! Correlation of factor scores with "true" factor scores

MODEL:      (model syntax goes here, to be changed for each model as shown below)
```

Model 1. Fully Z-Scored Factor Model Identification
(Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)

The following code refers to EVERY model parameter for completeness:

```
!Model 1 – Fully Z-Scored Factor Identification Approach

    ! Item factor loadings --> @=fixed, *=free → * REQUIRED for first item if free
      Sit BY Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;

    ! Item intercepts --> [ ] indicates means or intercepts, @=fixed, *=free
      [Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];

    ! Item error variances --> just list item by itself, @=fixed, *=free
      Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;

    ! Factor variance --> just list factor by itself, @=fixed, *=free
      Sit@1;

    ! Factor mean --> [ ] indicates means or intercepts, @=fixed, *=free
      [Sit@0];
```

In reality, all you'd need to write to define this model is:

```
    ! Item factor loadings --> @=fixed, *=free → * REQUIRED for first item if free
      Sit BY Sit1r* Sit2 Sit3r Sit4 Sit5r Sit6;

    ! Factor variance --> just list factor by itself, @=fixed, *=free
      Sit@1;
```

By default, all intercepts are estimated separately and the factor mean is fixed at 0.
By default, all residual variances for the items are estimated separately, too.
By default, factor variances and covariances are estimated freely.

Model 1. Fully Z-Scored Factor Model Identification
(Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)

```
UNSTANDARDIZED MODEL RESULTS
                                                       Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

FACTOR LOADINGS (regression slopes of item response on factor)
SIT      BY
    SIT1R              1.234      0.069     17.906      0.000
    SIT2               0.702      0.074      9.441      0.000
    SIT3R              1.241      0.063     19.846      0.000
    SIT4               0.784      0.069     11.334      0.000
    SIT5R              1.023      0.053     19.179      0.000
    SIT6               0.819      0.069     11.942      0.000

Means (of Factor)
999 = "cannot be computed" - here, because the parameter is fixed to 0 already
    SIT                0.000      0.000    999.000    999.000

Intercepts (of Items) - HERE, ARE ACTUAL ITEM MEANS BECAUSE FACTOR MEAN IS ZERO
    SIT1R              4.547      0.053     86.474      0.000
    SIT2               5.289      0.042    127.347      0.000
    SIT3R              4.896      0.048    101.959      0.000
    SIT4               5.359      0.042    126.895      0.000
    SIT5R              4.860      0.052     94.060      0.000
    SIT6               5.321      0.046    115.493      0.000

Variances (of Factor)
999 = "cannot be computed" - here, because the parameter is fixed to 1 already
    SIT                1.000      0.000    999.000    999.000

 Residual Variances (variance of e's)
    SIT1R              1.526      0.149     10.217      0.000
    SIT2               1.409      0.128     11.014      0.000
    SIT3R              1.004      0.135      7.456      0.000
    SIT4               1.352      0.127     10.672      0.000
    SIT5R              1.899      0.118     16.025      0.000
    SIT6               1.671      0.159     10.517      0.000
```

Making use of the unstandardized model estimates:

<u>Writing out the model—individual predicted values:</u>

$Y_1 = \mu_1 + \lambda_1 F + e_1$

$Y_1 = 4.547 + 1.234F + e_1$

<u>Writing out the model—predicted item variances and covariances:</u>

$Var(Y_1) = (\lambda_1^2) Var(F) + Var(e_1)$

$Var(Y_1) = (1.234^2)*(1) + 1.526 = 3.049$ (= original item variance)

$Cov(Y_1,Y_2) = \lambda_1*Var(F)*\lambda_2$

$Cov(Y_1,Y_2) = (1.234)*(1)*(.702) = .866$

(actual covariance = .577, so the model over-predicted how related items 1 and 2 should be)

STDYX STANDARDIZED MODEL RESULTS (FULLY STANDARDIZED WITH RESPECT TO X & Y)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS (correlations of item response with factor)				
Square these to get reliability (proportion "true variance") per item				
SIT BY				
SIT1R	0.707	0.035	19.983	0.000
SIT2	0.509	0.053	9.545	0.000
SIT3R	0.778	0.034	22.655	0.000
SIT4	0.559	0.048	11.641	0.000
SIT5R	0.596	0.029	20.528	0.000
SIT6	0.535	0.047	11.392	0.000
Means (of Factor)				
SIT	0.000	0.000	999.000	999.000
Intercepts (of Items) → is intercept / SD(Y) → not usually reported				
SIT1R	2.604	0.057	45.888	0.000
SIT2	3.834	0.111	34.394	0.000
SIT3R	3.070	0.072	42.921	0.000
SIT4	3.821	0.111	34.441	0.000
SIT5R	2.832	0.066	43.095	0.000
SIT6	3.477	0.101	34.573	0.000
Variances (of Factor) → will always be 1 in a standardized solution				
SIT	1.000	0.000	999.000	999.000
Residual Variances (standardized variance of e's)				
SIT1R	0.500	0.050	10.009	0.000
SIT2	0.741	0.054	13.628	0.000
SIT3R	0.395	0.053	7.388	0.000
SIT4	0.687	0.054	12.786	0.000
SIT5R	0.645	0.035	18.619	0.000
SIT6	0.714	0.050	14.187	0.000
R-SQUARE (equals 1-residual variance OR standardized loading squared)				
SIT1R	0.500	0.050	9.991	0.000
SIT2	0.259	0.054	4.772	0.000
SIT3R	0.605	0.053	11.327	0.000
SIT4	0.313	0.054	5.821	0.000
SIT5R	0.355	0.035	10.264	0.000
SIT6	0.286	0.050	5.696	0.000

The standardized solution will look identical across methods of model identification with respect to the factor loadings, error variances, and R-square values for the items. The standardized intercepts will change because they depend on the unstandardized intercepts (but nobody reports them anyway).

Making use of the standardized model estimates:

<u>Writing out the model – predicted item correlations:</u>

$\text{Corr}(Y_1,Y_2) = \lambda_1 * \text{Var}(F) * \lambda_2$

$\text{Corr}(Y_1,Y_2) = (.707)*(1)*(.509) = .360$

(actual correlation = .240, so the model over-predicted how related 1 and 2 should be)

Next up: two equivalent ways of getting the same model, but with different scaling (i.e., different means of identification…)

Now let's see the model parameters when using the marker item for model identification instead…
Model 2. Marker Item Loading = 1, Factor Mean = 0 (Factor Variance, All Intercepts Estimated)

```
! Model 2 -- Marker Item Loading with Factor Mean = 0 – MOST COMMON APPROACH AND DEFAULT IN MPLUS
    Sit BY Sit1r@1 Sit2* Sit3r* Sit4* Sit5r* Sit6*;      ! Loadings (#1 fixed=1)
    [Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];            ! Intercepts (all free)
     Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;             ! Residual variances (all free)
    Sit*;                                                ! Factor variance (free)
    [Sit@0];                                             ! Factor mean (fixed=0)
```

```
UNSTANDARDIZED MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

FACTOR LOADINGS (regression slopes of item response on factor)
Here, loading for SIT1R is not tested because it is fixed=1
 SIT      BY
    SIT1R              1.000      0.000    999.000    999.000
    SIT2               0.569      0.083      6.830      0.000
    SIT3R              1.005      0.035     28.555      0.000
    SIT4               0.636      0.082      7.741      0.000
    SIT5R              0.829      0.053     15.698      0.000
    SIT6               0.664      0.081      8.143      0.000

 Means (of Factor)
    SIT                0.000      0.000    999.000    999.000

 Intercepts (of Items) – EXPECTED Y WHEN FACTOR = 0, or for mean of factor in sample
    SIT1R              4.547      0.053     86.474      0.000
    SIT2               5.289      0.042    127.347      0.000
    SIT3R              4.896      0.048    101.960      0.000
    SIT4               5.359      0.042    126.896      0.000
    SIT5R              4.860      0.052     94.060      0.000
    SIT6               5.321      0.046    115.492      0.000

 Variances (of Factor)
    SIT                1.523      0.170      8.954      0.000
 Residual Variances (variances of e's)
    SIT1R              1.526      0.149     10.217      0.000
    SIT2               1.409      0.128     11.014      0.000
    SIT3R              1.004      0.135      7.456      0.000
    SIT4               1.352      0.127     10.673      0.000
    SIT5R              1.899      0.118     16.026      0.000
    SIT6               1.671      0.159     10.517      0.000
```

Yet another equivalent alternative method for scaling the factor…
Model 3. Marker Item Loading = 1 and Intercept = 0 (Factor Variance and Mean Estimated)

```
! Model 3 -- Marker Item Loading and Intercept
    Sit BY Sit1r@1 Sit2* Sit3r* Sit4* Sit5r* Sit6*;      ! Loadings (1 fixed=1)
    [Sit1r@0 Sit2* Sit3r* Sit4* Sit5r* Sit6*];           ! Intercepts (1 fixed=0)
     Sit1r*  Sit2* Sit3r* Sit4* Sit5r* Sit6*;            ! Residual variances (all free)
    Sit*;                                                ! Factor variance (free)
    [Sit*];                                              ! Factor mean (free)
```

```
Means (of Factor) → Note is mean of marker item 1
    SIT                4.547      0.053     86.474      0.000

Intercepts (of Items) – EXPECTED Y WHEN FACTOR = 0
HERE, WHICH IS WHEN ITEM 1 = 0 → beyond scale of item, so values are very low
    SIT1R              0.000      0.000    999.000    999.000
    SIT2               2.701      0.383      7.046      0.000
    SIT3R              0.325      0.171      1.899      0.058
    SIT4               2.469      0.380      6.504      0.000
    SIT5R              1.092      0.246      4.431      0.000
    SIT6               2.304      0.369      6.250      0.000
```

Calculating model degrees of freedom:

Total df = [v(v+1) / 2] + v = 27
Spent by model = 18
Leftover df = 9

Model fit information for a single-factor model (same regardless of factor scaling method):

```
Number of Free Parameters                   18  → is # of estimated parameters ("free" to be not 0)

Loglikelihood - use for testing differences in model fit across nested models

  H0 Value                         -11536.404  → this is for your specified model
  H0 Scaling Correction Factor         1.4158  → indicates how far off from normal=1
       for MLR
  H1 Value                         -11322.435  → this is for a saturated (perfect) model
  H1 Scaling Correction Factor         1.4073  → indicates how far off from normal=1
       for MLR

Information Criteria → "smaller is better" - use for nested or non-nested model comparisons

   Akaike (AIC)                     23108.808  → AIC = (-2*LL_H0) + (2*estimated parameters)
   Bayesian (BIC)                   23198.912  → BIC = (-2*LL_H0) + (LN N*estimated parameters)
   Sample-Size Adjusted BIC         23141.739  → BIC replacing N with (N + 2) / 24
     (n* = (n + 2) / 24)


Chi-Square Test of Model Fit (Significance is bad here) → for your specified model

    Value                             307.799
    Degrees of Freedom                      9 → leftover after estimating our one-factor model
    P-Value                            0.0000
    Scaling Correction Factor          1.3903 → indicates how far off from normal=1
        for MLR                                 > 1 = leptokurtic distribution (too-fat tails)
                                                < 1 = platykurtotic distribution (too-thin tails)

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
    for chi-square difference testing in the regular way.  MLM, MLR and WLSM
    chi-square difference testing is described on the Mplus website.  MLMV, WLSMV,
    and ULSMV difference testing is done using the DIFFTEST option.
```

Where does this χ^2 value for "model fit" come from? A rescaled −2LL model comparison of this one-factor model (H0) against the saturated model (H1) that perfectly reproduces the data covariances:

Step 1: Original $-2\Delta LL = -2*(LL_{fewer} - LL_{more}) = -2(-11{,}536.404 + 11{,}322.435) = 427.938$

Step 2: Scaling correction $= [\,(\#parms_{fewer}*scale_{fewer}) - (\#parms_{more}*scale_{more})\,] / (\#parms_{fewer} - \#parms_{more})$
$= [\,(18 * 1.4158) - (27 * 1.4073)\,] / (18 - 27) = -12.501 / -9 = 1.3903$

Step 3: Rescaled $-2\Delta LL = -2\Delta LL$ / scaling correction = 427.938 / 1.903 = **307.803** → ~matches model χ^2
Step 4: Difference in df = $\#parms_{more} - \#parms_{fewer}$ = 27 – 18 = **9**

How to fit the saturated (Unstructured) Baseline Model: Item means, variances, and covariances in original data

```
! Saturated Model
    ! Item means --> [ ] indicates means or intercepts, @=fixed, *=free
        [Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];
    ! Item variances --> just list item by itself, @=fixed, *=free
        Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;
    ! Item covariances --> just list all by all, @=fixed, *=free
        Sit1r  Sit2  Sit3r  Sit4  Sit5r  Sit6  WITH
       Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;
```

Model fit information for the saturated model

```
Number of Free Parameters                          27 → all possible means, variances, covariances

Loglikelihood
          H0 Value                        -11322.435
          H0 Scaling Correction Factor        1.4073
            for MLR
          H1 Value                        -11322.435
          H1 Scaling Correction Factor        1.4073
            for MLR

Information Criteria
          Akaike (AIC)                     22698.870
          Bayesian (BIC)                   22834.027
          Sample-Size Adjusted BIC         22748.268
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                                0.000*
          Degrees of Freedom                       0
          P-Value                             0.0000
          Scaling Correction Factor           1.0000
            for MLR
```

Note that H0 and H1 are now the same! Our H0 model IS the H1 saturated model.

Now back to the rest of the one-factor model fit statistics:

```
RMSEA (Root Mean Square Error Of Approximation)(want close to 0 = saturated model)

    Estimate                           0.173
    90 Percent C.I.             0.157  0.190
    Probability RMSEA <= .05           0.000 → so RMSEA does NOT overlap .05 (is signif > .05)


CFI/TLI (want close to 1 = saturated model)

    CFI                                0.732
    TLI                                0.553

SRMR (Standardized Root Mean Square Residual)(want close to 0 = saturated model)

   Value                               0.086

Chi-Square Test of Model Fit for the Baseline Model → for the "no covariances" model

     Value                          1128.693
     Degrees of Freedom                   15
     P-Value                          0.0000
```

Where does this χ^2 value for "fit of the baseline model" come from? A rescaled −2LL model comparison of the independence model with NO covariances to the saturated model:

Step 1: Original $-2\Delta LL = -2*(LL_{fewer} - LL_{more}) = -2(-12{,}312.952 + 11{,}322.435) = 1{,}981.034$

Step 2: Scaling correction = $[(\#parms_{fewer}*scale_{fewer}) - (\#parms_{more}*scale_{more})] / (\#parms_{fewer} - \#parms_{more})$
$= [(12 * 0.9725) - (27 * 1.4073)] / (12 - 27) = -26.372 / -15 = 1.7551$

Step 3: Rescaled $-2\Delta LL = -2\Delta LL$ / scaling correction = 1,981.034 / 1.7551 = **1,128.704** → ~matches baseline χ^2
Step 4: Difference in df = $\#parms_{more} - \#parms_{fewer} = 27 - 12 =$ **15**

What's the point? This baseline model fit test tells us whether there are any covariances at all (i.e., whether it even makes sense to try to fit latent factors to predict them).

How to fit the Independence (Null) Baseline Model: Item means and variances, but NO covariances

```
! Independence Model
    ! Item means --> [ ] indicates means or intercepts, @=fixed, *=free
        [Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*];
    ! Item variances --> just list item by itself, @=fixed, *=free
        Sit1r* Sit2* Sit3r* Sit4* Sit5r* Sit6*;
```

Model fit information for the independence model

```
Number of Free Parameters                         12

Loglikelihood
          H0 Value                        -12312.952
          H0 Scaling Correction Factor        0.9725
            for MLR
          H1 Value                        -11322.435
          H1 Scaling Correction Factor        1.4073
            for MLR

Information Criteria
          Akaike (AIC)                     24649.904
          Bayesian (BIC)                   24709.974
          Sample-Size Adjusted BIC         24671.859
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                             1128.692*
          Degrees of Freedom                      15
          P-Value                             0.0000
          Scaling Correction Factor           1.7552
            for MLR

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
    for chi-square difference testing in the regular way.  MLM, MLR and WLSM
    chi-square difference testing is described on the Mplus website.  MLMV, WLSMV,
    and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                             0.259
          90 Percent C.I.                      0.247  0.272
          Probability RMSEA <= .05             0.000

CFI/TLI
          CFI                                  0.000
          TLI                                  0.000

Chi-Square Test of Model Fit for the Baseline Model
          Value                             1128.693
          Degrees of Freedom                      15
          P-Value                             0.0000

SRMR (Standardized Root Mean Square Residual)
          Value                                0.300
```

Note that the model fit is the same as the “baseline” model fit given before.

Although not 0, this is the worst possible RMSEA while still allowing separate means and variances per item in these data. RMSEA is a parsimony-corrected absolute fit index (so, its fit is relative to the saturated model).

CFI and TLI are 0 because they are “incremental fit” indices relative to the independence model (which this is).

SRMR is also an absolute fit index (relative to saturated model), so this is the worst it gets for these data, too.

So global fit for the one-factor model is not so good… (RMSEA = .173, CFI = .732)
What do the voo-doo modification indices suggest we do to fix it?

```
MODEL MODIFICATION INDICES
Minimum M.I. value for printing the modification index    6.635
EPC = EXPECTED PARAMETER CHANGE

                                    M.I.     E.P.C.   Std E.P.C.   StdYX E.P.C.

WITH Statements (SUGGESTED ERROR COVARIANCES for unknown multidimensionality)

SIT2     WITH SIT1R             49.618    -0.464     -0.464       -0.316
SIT3R    WITH SIT1R            143.624     1.023      1.023        0.827
SIT3R    WITH SIT2              34.877    -0.357     -0.357       -0.300
SIT4     WITH SIT1R             36.280    -0.403     -0.403       -0.280
SIT4     WITH SIT2             161.318     0.702      0.702        0.509
SIT4     WITH SIT3R             29.202    -0.336     -0.336       -0.288
SIT6     WITH SIT1R             24.079    -0.358     -0.358       -0.224
SIT6     WITH SIT2              63.893     0.486      0.486        0.317
SIT6     WITH SIT3R             22.386    -0.319     -0.319       -0.246
SIT6     WITH SIT4              46.541     0.415      0.415        0.276
```

Another approach—how about we examine local fit and see where the problems seem to be?
The means and variances of the items will be perfectly reproduced, so that's not an issue…
misfit results from the difference between the observed and model-predicted covariances.

Mplus gives us the "residual" (defined as observed – predicted) or "leftover" covariance matrix, but it is scale dependent and thus not so helpful. We can calculate the residual correlation matrix (see spreadsheet):

Residual Correlation Matrix	R1	2	R3	4	R5	6
R1						
2	-0.120					
R3	0.097	-0.079				
4	-0.095	**0.285**	-0.066			
R5	0.032	-0.048	0.018	-0.044		
6	-0.081	**0.185**	-0.060	**0.149**	-0.015	

Mplus also gives us "normalized" residuals, which can be thought of as z-scores for how large the residual leftover covariance is in absolute terms. Because the denominator decreases with sample size, however, these values may be inflated in large samples, so look for *relatively* large values.

"Normalized" Residuals for Inter-Item Covariances = (observed – predicted) / SD(observed)

```
Normalized Residuals for Covariances/Correlations/Residual Correlations
                SIT1R         SIT2          SIT3R         SIT4          SIT5R         SIT6
              ________      ________      ________      ________      ________      ________
 SIT1R          0.000
 SIT2          -3.503         0.000
 SIT3R          2.977        -2.253         0.000
 SIT4          -2.928         6.560        -1.959         0.000
 SIT5R          0.960        -1.434         0.548        -1.372         0.000
 SIT6          -2.345         4.721        -1.756         3.925        -0.444         0.000
```

NEGATIVE NORMALIZED RESIDUAL → Less related than you predicted (don't want to be together)
POSITIVE NORMALIZED RESIDUAL → More related than you predicted (want to be more together)

Why might the normalized residuals (leftover correlations) for the positive-worded items be larger than for the negatively-worded items?

These results suggest that wording valence is playing a larger role in the pattern of covariance across items than what the one-factor model predicts. Rather than adding voo-doo covariances among the residuals for specific items, how about a two-factor model based on wording instead?

Model 4. Fully Z-Scored, 2-Factor Model

```
! Model 4 -- Fully Z-Scored 2-Factor Model
    SitP BY Sit2*  Sit4*  Sit6*;             ! SitP loadings (all free)
    SitN BY Sit1r* Sit3r* Sit5r*;            ! SitN loadings (all free)
    [Sit2*  Sit4*  Sit6*];                   ! SitP intercepts (all free)
    [Sit1r* Sit3r* Sit5r*];                  ! SitN intercepts (all free)
    Sit2*   Sit4*  Sit6*;                    ! SitP residual variances (all free)
    Sit1r*  Sit3r* Sit5r*;                   ! SitN residual variances (all free)
    SitP@1; SitN@1;                          ! Factor variances (fixed=1)
    SitP WITH SitN*;                         ! Factor covariance (free)
    [SitP@0 SitN@0];                         ! Factor means (fixed=0)
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       19

Loglikelihood
    H0 Value                            -11340.140
    H0 Scaling Correction Factor            1.4017
         for MLR
    H1 Value                            -11322.435
    H1 Scaling Correction Factor            1.4073
         for MLR

Information Criteria
    Akaike (AIC)                         22718.281
    Bayesian (BIC)                       22813.391
    Sample-Size Adjusted BIC             22753.042
      (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
    Value                                  24.924*
    Degrees of Freedom                          8
    P-Value                                0.0016
    Scaling Correction Factor              1.4207
          for MLR
```

Is the 2-factor model better than the 1-factor model? How do we know?

Rescaled likelihood ratio test
(−2LL rescaled difference test):

1. **−2ΔLL = −2* difference in LL:**
   −2*(−11,536.404 + 11,340.140) = 392.528

2. **difference scaling correction:**
   (parms_1*scale_1) − (parms_2*scale_2) / (parms_1 – parms_2)
   (18*1.4158) – (19*1.4017) / (18 – 19) = 1.1479

3. **rescaled difference = −2ΔLL / scaling correction:**
   392.528 / 1.1479 = 341.953

4. **compare rescaled difference to χ^2 with df = Δdf :**
   critical χ^2 for df =1 is 3.84, so because 341.953 is > 3.84, the model fit significantly improved

```
*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
    for chi-square difference testing in the regular way.  MLM, MLR and WLSM
    chi-square difference testing is described on the Mplus website.  MLMV, WLSMV,
    and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
    Estimate                                0.044
    90 Percent C.I.                  0.025  0.064
    Probability RMSEA <= .05                0.667

CFI/TLI
    CFI                                     0.985
    TLI                                     0.972

Chi-Square Test of Model Fit for the Baseline Model
    Value                                1128.693
    Degrees of Freedom                         15
    P-Value                                0.0000

SRMR (Standardized Root Mean Square Residual)
    Value                                   0.029
```

UNSTANDARDIZED RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP BY				
SIT2	1.007	0.052	19.487	0.000
SIT4	1.064	0.050	21.195	0.000
SIT6	0.956	0.053	18.203	0.000
SITN BY				
SIT1R	1.325	0.048	27.698	0.000
SIT3R	1.349	0.044	30.514	0.000
SIT5R	1.009	0.055	18.358	0.000
SITP WITH SITN = factor covariance (= correlation if variances=1)				
	0.564	**0.041**	**13.776**	**0.000**
Means				
SITP	0.000	0.000	999.000	999.000
SITN	0.000	0.000	999.000	999.000
Intercepts				
SIT1R	4.547	0.053	86.474	0.000
SIT2	5.289	0.042	127.347	0.000
SIT3R	4.896	0.048	101.959	0.000
SIT4	5.359	0.042	126.896	0.000
SIT5R	4.860	0.052	94.060	0.000
SIT6	5.321	0.046	115.492	0.000
Variances				
SITP	1.000	0.000	999.000	999.000
SITN	1.000	0.000	999.000	999.000
Residual Variances				
SIT1R	1.294	0.103	12.547	0.000
SIT2	0.888	0.097	9.173	0.000
SIT3R	0.724	0.092	7.857	0.000
SIT4	0.835	0.093	9.003	0.000
SIT5R	1.926	0.119	16.128	0.000
SIT6	1.428	0.134	10.684	0.000

Omega = (Sum of loadings)^2 / (Sum of loadings)^2 + Sum of error variances + 2* Sum of error covariances

Omega for Positive Factor = .744
(1.007+1.064+0.956)^2 /
(1.007+1.064+0.956)^2 +
(0.888+0.835+1.428) + 2*0

(alpha was .746)

Omega for Negative Factor = .775
(1.325+1.349+1.009)^2 /
(1.325+1.349+1.009)^2 +
(1.294+0.724+1.926) + 2*0

(alpha was .780)

STDYX STANDARDIZED RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SITP BY				
SIT2	0.730	0.032	22.794	0.000
SIT4	0.759	0.029	25.995	0.000
SIT6	0.625	0.035	17.949	0.000
SITN BY				
SIT1R	0.759	0.022	34.072	0.000
SIT3R	0.846	0.021	39.657	0.000
SIT5R	0.588	0.030	19.651	0.000
SITP WITH				
SITN	0.564	0.041	13.776	0.000
Residual Variances				
SIT1R	0.425	0.034	12.567	0.000
SIT2	0.467	0.047	9.976	0.000
SIT3R	0.285	0.036	7.895	0.000
SIT4	0.425	0.044	9.589	0.000
SIT5R	0.654	0.035	18.576	0.000
SIT6	0.610	0.043	14.029	0.000
R-SQUARE				
SIT1R	0.575	0.034	17.036	0.000
SIT2	0.533	0.047	11.397	0.000
SIT3R	0.715	0.036	19.829	0.000
SIT4	0.575	0.044	12.998	0.000
SIT5R	0.346	0.035	9.826	0.000
SIT6	0.390	0.043	8.974	0.000

Wouldn't it be nice if Mplus would compute Omegas for you? It can, if you (a) label the parameters it needs to do the math, and (b) create new terms for the Omega estimates via MODEL CONSTRAINT:

Model 4. Fully Z-Scored, 2-Factor Model again, now with parameter labels

```
! Model 4 -- Fully Z-Scored 2-Factor Model
    SitP BY Sit2*  Sit4*  Sit6*  (L1-L3);      ! SitP loadings (all free)
    SitN BY Sit1r* Sit3r* Sit5r* (L4-L6);      ! SitN loadings (all free)
    [Sit2*  Sit4*  Sit6*];                     ! SitP intercepts (all free)
    [Sit1r* Sit3r* Sit5r*];                    ! SitN intercepts (all free)
    Sit2*   Sit4*  Sit6*  (E1-E3);             ! SitP residual variances (all free)
    Sit1r*  Sit3r* Sit5r* (E4-E6);             ! SitN residual variances (all free)
    SitP@1; SitN@1;                            ! Factor variances (fixed=1)
    SitP WITH SitN*;                           ! Factor covariance (free)
    [SitP@0 SitN@0];                           ! Factor means (fixed=0)

MODEL CONSTRAINT:  ! Calculate omega model-based reliability per factor
    NEW(OmegaP OmegaN);
    OmegaP = ((L1+L2+L3)**2) / (((L1+L2+L3)**2) + (E1+E2+E3));
    OmegaN = ((L4+L5+L6)**2) / (((L4+L5+L6)**2) + (E4+E5+E6));
```

Output now provided in unstandardized solution:

```
New/Additional Parameters
    OMEGAP             0.744      0.020     37.956      0.000
    OMEGAN             0.775      0.014     56.803      0.000
```

Any more local fit problems? Let's see…

Residuals of covariance matrix (so unstandardized estimate of how far off each covariance is):

	SIT1R	SIT2	SIT3R	SIT4	SIT5R	SIT6
SIT1R	0.000					
SIT2	-0.176	0.000				
SIT3R	0.016	-0.069	0.000			
SIT4	-0.062	0.031	0.015	0.000		
SIT5R	0.021	0.030	-0.042	0.089	0.000	
SIT6	0.080	0.003	0.140	-0.055	0.254	0.000

"Normalized" residuals (z-like statistic for how far off each covariance is):

	SIT1R	SIT2	SIT3R	SIT4	SIT5R	SIT6
SIT1R	0.000					
SIT2	**-2.125**	0.000				
SIT3R	0.172	-0.896	0.000			
SIT4	-0.768	0.370	0.192	0.000		
SIT5R	0.212	0.382	-0.464	1.128	0.000	
SIT6	0.869	0.031	1.658	-0.676	**2.847**	0.000

Any suggested voo-doo? (only available when not using MODEL CONSTRAINT, though)

```
MODEL MODIFICATION INDICES
Minimum M.I. value for printing the modification index     6.635

                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

BY Statements – these are cross-loadings

SITN     BY SIT2             9.775    -0.224     -0.224       -0.162
SITN     BY SIT6            10.828     0.245      0.245        0.160

WITH Statements – these are error covariances

SIT4     WITH SIT2          10.830     0.332      0.332        0.386
SIT6     WITH SIT4           9.773    -0.273     -0.273       -0.250
```

Because we have no real theoretical or defendable reason to fit any of these suggested parameters, we will not add any new parameters. This will be about as good as it gets.

Let's examine the estimated distribution of the factor scores for each factor:

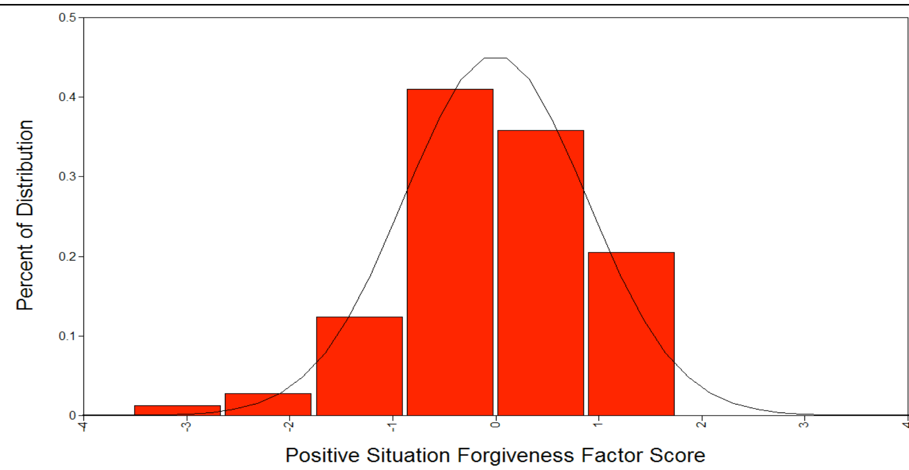
```
SUMMARY OF FACTOR SCORES
     FACTOR SCORE INFORMATION (COMPLETE-DATA PATTERN)
           FACTOR DETERMINACIES
           SITP        0.882
           SITN        0.908

SAMPLE STATISTICS FOR ESTIMATED FACTOR SCORES
     SAMPLE STATISTICS
           Means
              SITP          SITP_SE       SITN          SITN_SE
              ________      ________      ________      ________
 1              0.000         0.472         0.000         0.418
           Covariances
              SITP          SITP_SE       SITN          SITN_SE
              ________      ________      ________      ________
 SITP           0.777
 SITP_SE        0.000         0.000
 SITN           0.533         0.000         0.825
 SITN_SE        0.000         0.000         0.000         0.000
           Correlations
              SITP          SITP_SE       SITN          SITN_SE
              ________      ________      ________      ________
 SITP           1.000
 SITP_SE      999.000         1.000
 SITN           0.665       999.000         1.000
 SITN_SE      999.000       999.000       999.000         1.000
```

The factor determinacy, the correlation between the estimated and true factor scores, is .882 for the positive factor and .908 for the negative factor.

Positive factor score SE = 0.472
Negative factor score SE = 0.418

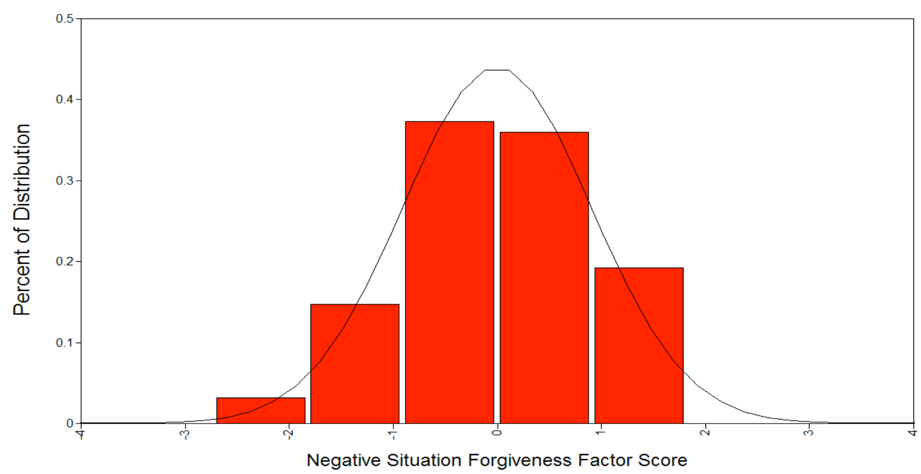
Although the correlation between the factors was originally .56, the correlation between the estimated factor scores is .67 instead due to shrinkage.



The positive factor scores have an estimated mean of 0 with a variance of 0.78 instead of 1.00.

The SE for each person's factor score is .472. Treating factor scores as observed variables is like saying SE = 0.

Positive factor score = Score ± 2*.472 = Score ± .944

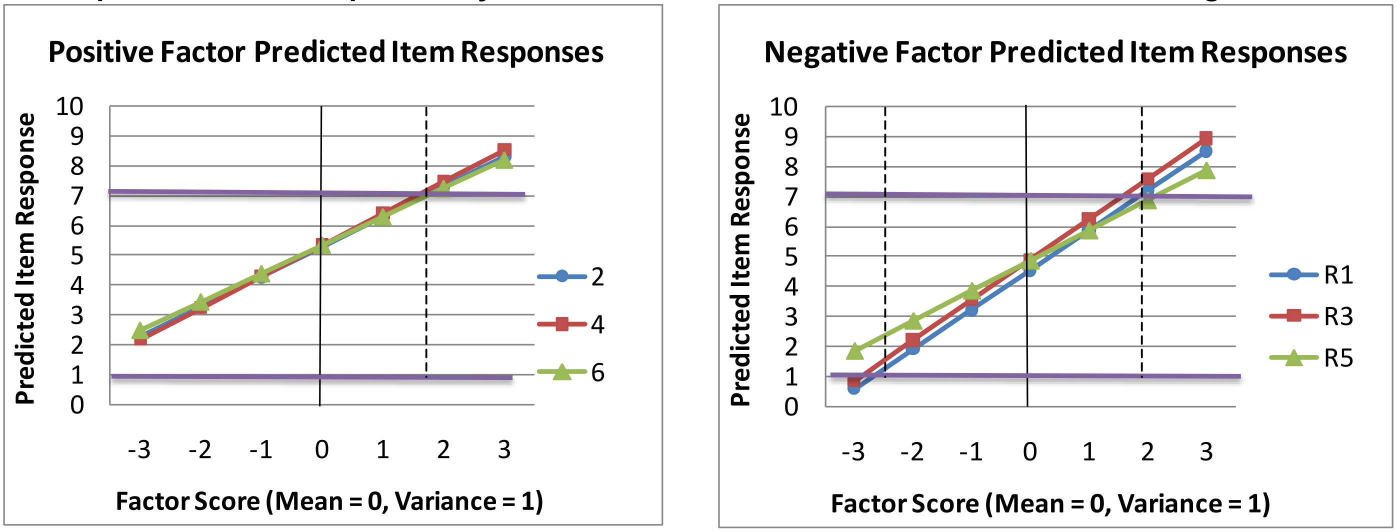


The negative factor scores have an estimated mean of 0 with a variance of 0.825 instead of 1.00.

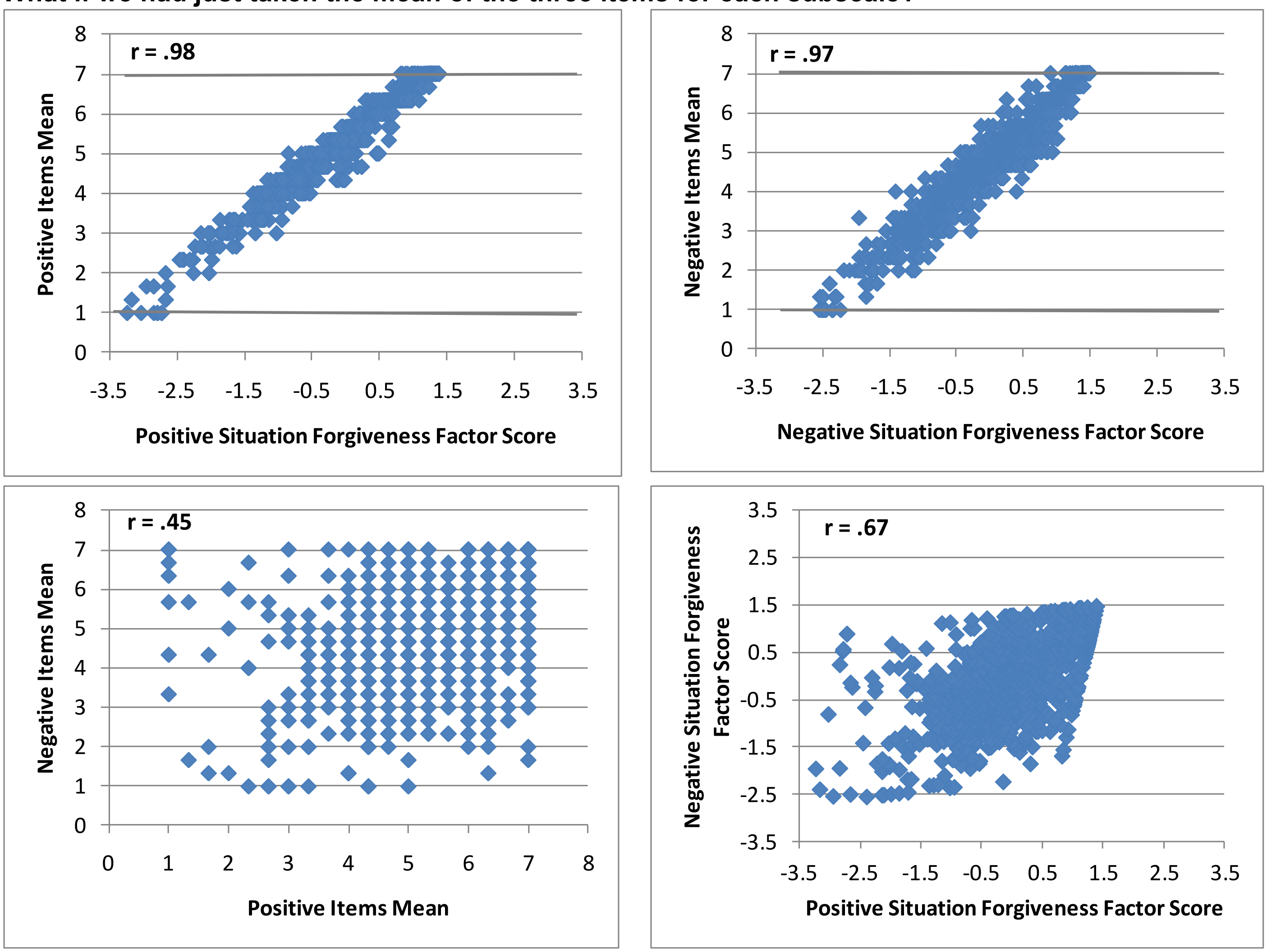
The SE for each person's factor score is .418, so ± .836.

The negative factor scores retain more variance (and have a smaller SE) because there is more information in them, due to higher factor loadings (greater reliability) of their items.

Model-predicted item responses by factor scores with dashed lines for floor and ceiling effects:



What if we had just taken the mean of the three items for each subscale?



There are problems with either of these observed variable approaches: The **mean of the items** appears to have less variability (i.e., fewer possible scores) and assumes that all items should be weighted equally and have no error. The **estimated factor scores** do not have the same properties as estimated for the factor in the model (i.e., less variance for each factor, higher correlation among the factors).

What to do instead of either of these? Stay tuned for how to use plausible values.

Another example: Formal Tests of CTT Assumptions

We will test the CTT assumption of tau-equivalence (equal factor loadings), one factor at a time.
If those hold, we can then test the assumption of parallel items (equal error variances, too).

First, tau-equivalence of the negative factor only:

```
! Model 5 -- Tau-Equivalent Negative Items Only 2-Factor Model
    SitP BY Sit2*  Sit4*  Sit6*;                  ! SitP loadings (all free)
    SitN BY Sit1r* Sit3r* Sit5r* (NegLoad);       ! SitN loadings (all held equal)
    [Sit2*  Sit4*  Sit6*];                        ! SitP intercepts (all free)
    [Sit1r* Sit3r* Sit5r*];                       ! SitN intercepts (all free)
    Sit2*  Sit4*  Sit6*;                          ! SitP residual variances (all free)
    Sit1r* Sit3r* Sit5r*;                         ! SitN residual variances (all free)
    SitP@1; SitN@1;                               ! Factor variances (fixed=1)
    SitP WITH SitN*;                              ! Factor covariance (free)
    [SitP@0 SitN@0];                              ! Factor means (fixed=0)
```

```
UNSTANDARDIZED MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
SITP     BY
    SIT2               1.007      0.052     19.491      0.000
    SIT4               1.063      0.050     21.202      0.000
    SIT6               0.957      0.052     18.257      0.000

 SITN     BY
    SIT1R              1.254      0.032     38.957      0.000
    SIT3R              1.254      0.032     38.957      0.000
    SIT5R              1.254      0.032     38.957      0.000

 SITP     WITH
    SITN               0.575      0.041     13.855      0.000

 Residual Variances
    SIT1R              1.335      0.083     16.150      0.000
    SIT2               0.889      0.096      9.217      0.000
    SIT3R              0.857      0.069     12.337      0.000
    SIT4               0.837      0.092      9.045      0.000
    SIT5R              1.806      0.115     15.716      0.000
    SIT6               1.425      0.134     10.630      0.000

STANDARDIZED STYDX MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
SITP     BY
    SIT2               0.730      0.032     22.840      0.000
    SIT4               0.758      0.029     26.037      0.000
    SIT6               0.626      0.035     17.958      0.000

 SITN     BY
    SIT1R              0.735      0.016     46.189      0.000
    SIT3R              0.805      0.016     50.774      0.000
    SIT5R              0.682      0.015     45.076      0.000

 Residual Variances
    SIT1R              0.459      0.023     19.604      0.000
    SIT2               0.467      0.047     10.017      0.000
    SIT3R              0.353      0.025     13.835      0.000
    SIT4               0.425      0.044      9.633      0.000
    SIT5R              0.535      0.021     25.887      0.000
    SIT6               0.609      0.044     13.969      0.000
```

Why are the standardized factor loadings for the negative factor not held equal like the unstandardized loadings are?

<table>
<tr>
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Fit of previous 2-factor model:

```
Number of Free Parameters                        19

Loglikelihood
    H0 Value                               -11340.140
    H0 Scaling Correction Factor               1.4017
         for MLR
    H1 Value                               -11322.435
    H1 Scaling Correction Factor               1.4073
         for MLR

RMSEA (Root Mean Square Error Of Approximation)
    Estimate                                    0.044
    90 Percent C.I.                   0.025     0.064
    Probability RMSEA <= .05                    0.667

CFI/TLI
     CFI                                         0.985
     TLI                                         0.972
```

</td>
<td>

Fit of tau-equivalent negative items 2-factor model:

```
Number of Free Parameters                        17

Loglikelihood
     H0 Value                              -11357.612
     H0 Scaling Correction Factor              1.4474
            for MLR
     H1 Value                              -11322.435
     H1 Scaling Correction Factor              1.4073
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
      Estimate                                   0.062
      90 Percent C.I.                   0.046    0.079
      Probability RMSEA <= .05                   0.102

CFI/TLI
     CFI                                        0.962
     TLI                                        0.943
```

</td>
</tr>
</table>

Does the assumption of tau-equivalence hold for the negative items? How do we know?

Second, tau-equivalence of the factor loadings for the positive factor only:

```
! Model 6 -- Tau-Equivalent Positive Items Only 2-Factor Model
    SitP BY Sit2*  Sit4*  Sit6* (PosLoad);     ! SitP loadings (all held equal)
    SitN BY Sit1r* Sit3r* Sit5r*;              ! SitN loadings (all free)
    [Sit2*  Sit4*  Sit6*];                     ! SitP intercepts (all free)
    [Sit1r* Sit3r* Sit5r*];                    ! SitN intercepts (all free)
    Sit2*  Sit4*  Sit6*;                       ! SitP residual variances (all free)
    Sit1r* Sit3r* Sit5r*;                      ! SitN residual variances (all free)
    SitP@1; SitN@1;                            ! Factor variances (fixed=1)
    SitP WITH SitN*;                           ! Factor covariance (free)
    [SitP@0 SitN@0];                           ! Factor means (fixed=0)
```

```
Number of Free Parameters                        17

Loglikelihood
    H0 Value                               -11341.773
    H0 Scaling Correction Factor               1.4187
          for MLR
    H1 Value                               -11322.435
    H1 Scaling Correction Factor               1.4073
          for MLR

RMSEA (Root Mean Square Error Of Approximation)
    Estimate                                    0.040
    90 Percent C.I.                   0.023     0.058
    Probability RMSEA <= .05                    0.797

CFI/TLI
    CFI                                         0.984
    TLI                                         0.976
```

Does the assumption of tau-equivalence hold for the positive items? How do we know?

```
UNSTANDARDIZED MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
SITP     BY
    SIT2               1.014      0.036     28.389      0.000
    SIT4               1.014      0.036     28.389      0.000
    SIT6               1.014      0.036     28.389      0.000

 SITN     BY
    SIT1R              1.325      0.048     27.727      0.000
    SIT3R              1.349      0.044     30.531      0.000
    SIT5R              1.010      0.055     18.370      0.000

 SITP     WITH
    SITN               0.567      0.040     14.131      0.000

 Residual Variances
    SIT1R              1.295      0.103     12.580      0.000
    SIT2               0.881      0.083     10.587      0.000
    SIT3R              0.725      0.092      7.873      0.000
    SIT4               0.886      0.075     11.767      0.000
    SIT5R              1.925      0.119     16.117      0.000
    SIT6               1.384      0.118     11.737      0.000

STANDARDIZED STDYX MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
SITP     BY
    SIT2               0.734      0.023     32.593      0.000
    SIT4               0.733      0.021     35.611      0.000
    SIT6               0.653      0.022     29.743      0.000

 SITN     BY
    SIT1R              0.759      0.022     34.139      0.000
    SIT3R              0.846      0.021     39.706      0.000
    SIT5R              0.588      0.030     19.663      0.000

 SITP     WITH
    SITN               0.567      0.040     14.131      0.000

 Residual Variances
    SIT1R              0.425      0.034     12.598      0.000
    SIT2               0.461      0.033     13.965      0.000
    SIT3R              0.285      0.036      7.910      0.000
    SIT4               0.463      0.030     15.350      0.000
    SIT5R              0.654      0.035     18.562      0.000
    SIT6               0.574      0.029     20.019      0.000
```

Given that tau-equivalence held for the positive factor, we can also test the assumption of parallel items as equal residual variances (in addition to equal factor loadings):

```
! Model 7 -- Parallel Items on Positive Only 2-Factor Model
    SitP BY Sit2*  Sit4*  Sit6*  (PosLoad);    ! SitP loadings (all held equal)
    SitN BY Sit1r* Sit3r* Sit5r*;              ! SitN loadings (all free)
    [Sit2*  Sit4*  Sit6*];                     ! SitP intercepts (all free)
    [Sit1r* Sit3r* Sit5r*];                    ! SitN intercepts (all free)
    Sit2*  Sit4*  Sit6*         (PosError);    ! SitP residual variances (all held equal)
    Sit1r* Sit3r* Sit5r*;                      ! SitN residual variances (all free)
    SitP@1; SitN@1;                            ! Factor variances (fixed=1)
    SitP WITH SitN*;                           ! Factor covariance (free)
    [SitP@0 SitN@0];                           ! Factor means (fixed=0)
```

```
Number of Free Parameters                       15

Loglikelihood
          H0 Value                      -11361.960
          H0 Scaling Correction Factor      1.3443
            for MLR
          H1 Value                      -11322.435
          H1 Scaling Correction Factor      1.4073
            for MLR


RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.056
          90 Percent C.I.             0.041  0.072
          Probability RMSEA <= .05           0.244

CFI/TLI
          CFI                                0.963
          TLI                                0.954
```

Does the assumption of parallel items hold for the positive items? How do we know?

UNSTANDARDIZED MODEL RESULTS

```
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

SITP     BY
    SIT2               1.005      0.035     28.455      0.000
    SIT4               1.005      0.035     28.455      0.000
    SIT6               1.005      0.035     28.455      0.000

 SITN     BY
    SIT1R              1.325      0.048     27.816      0.000
    SIT3R              1.347      0.044     30.623      0.000
    SIT5R              1.011      0.055     18.408      0.000

 SITP     WITH
    SITN               0.581      0.040     14.581      0.000

 Residual Variances
    SIT1R              1.294      0.102     12.645      0.000
    SIT2               1.060      0.061     17.452      0.000
    SIT3R              0.728      0.091      7.992      0.000
    SIT4               1.060      0.061     17.452      0.000
    SIT5R              1.922      0.119     16.095      0.000
    SIT6               1.060      0.061     17.452      0.000
```

STANDARDIZED STDYX MODEL RESULTS

```
 SITP     BY
    SIT2               0.698      0.019     37.365      0.000
    SIT4               0.698      0.019     37.365      0.000
    SIT6               0.698      0.019     37.365      0.000

 SITN     BY
    SIT1R              0.759      0.022     34.339      0.000
    SIT3R              0.845      0.021     40.011      0.000
    SIT5R              0.589      0.030     19.713      0.000

 SITP     WITH
    SITN               0.581      0.040     14.581      0.000

 Residual Variances
    SIT1R              0.424      0.034     12.652      0.000
    SIT2               0.512      0.026     19.616      0.000
    SIT3R              0.286      0.036      8.024      0.000
    SIT4               0.512      0.026     19.616      0.000
    SIT5R              0.653      0.035     18.520      0.000
    SIT6               0.512      0.026     19.616      0.000
```

Example write-up describing these analyses…

(Note: You may borrow the phrasing contained in this example to describe various aspects of your analyses, but your own results sections will not mimic this example exactly—they should be customized to describe the how and the why of what you did, specifically).

(Descriptive information for the sample and items would have already been given in the method section…)

The reliability and dimensionality of six items each assessing forgiveness of situations was assessed in a sample of 1,103 persons with a confirmatory factor analysis using robust maximum likelihood estimation (MLR) in Mplus v. 7.4 (Muthén & Muthén, 1998-2015). All models were identified by setting any latent factor means to 0 and latent factor variances to 1, such that all item intercepts, item factor loadings, and item residual variances were then estimated. The six items utilized a seven-point response scale, and three items were reverse-coded prior to analysis such that higher values then indicated greater levels of forgiveness of situations for all items. Model fit statistics reported in Table 1 include the obtained model χ^2, its scaling factor (in which values different than 1.000 indicate deviations from normality), its degrees of freedom, and its *p*-value (in which non-significance is desirable for good fit), CFI, or Comparative Fit Index (in which values higher than .95 are desirable for good fit), and the RMSEA, or Root Mean Square Error of Approximation, point estimate and 90% confidence interval (in which values lower than .06 are desirable for good fit). As reported in Table 2, nested model comparisons were conducted using the rescaled −2ΔLL with degrees of freedom equal to the rescaled difference in the number of parameters between models (i.e., a rescaled likelihood ratio test). The specific models examined are described in detail below.

Although a one-factor model was initially posited to account for the pattern of covariance across these six items, it resulted in poor fit, as shown in Table 1. Although each item had a significant factor loading (with standardized loadings ranging from .509 to .778), a single latent factor did not adequately describe the pattern of relationship across these six items as initially hypothesized. Sources of local misfit were identified using the normalized residual covariance matrix, available via the RESIDUAL output option in Mplus, in which individual values were calculated as: (observed covariance – expected covariance) / SD(observed covariance). Relatively large positive residual covariances were observed among items 2, 4, and 6 (the positively-worded items), indicating that these items were more related than was predicted by the single-factor model. Modification indices, available via the MODINDICES output option in Mplus, corroborated this pattern, further suggesting additional remaining relationships among the negatively-worded items as well.

The necessity of separate latent factors for the positively-worded and negatively-worded items was tested by specifying a two-factor model in which the positively-worded items 2, 4, and 6 indicated a *forgiveness* factor, and in which negatively-worded items 1, 3, and 5 indicated a *not unforgiveness* factor, and in which the two factors were allowed to correlate. The two-factor model fit was acceptable by every criterion except the significant χ^2, likely due to the large sample. In addition, the two-factor model fit significantly better than the one-factor model, as reported in Table 2, indicating that the estimated correlation between the two factors of .564 was significantly less than 1.000. Thus, the six items appeared to measure two separate but related constructs. Further examination of local fit via normalized residual covariances and modification indices yielded no interpretable remaining relationships, and thus this two-factor model was retained.

Table 3 provides the estimates and their standard errors for the item factor loadings, intercepts, and residual variances from both the unstandardized and standardized solutions. All factor loadings and the factor covariance were statistically significant. As shown in Table 3, standardized loadings for the forgiveness factor items ranged from .625 to .759 (with R^2 values for the amount of item variance accounted for by the factor ranging from .390 to .575), and standardized loadings for the not unforgiveness factor ranged from .588 to .846 (with R^2 values of .346 to .715), suggesting the factor loadings were practically significant as well. Omega model-based reliability was calculated for each factor as described in Brown (2006) as the squared sum of the factor loadings divided by the squared sum of the factor loadings plus the sum of the error variances plus twice the sum of the error covariances (although no error covariances were included here). Omega was .744 for the forgiveness factor and .775 for the not unforgiveness factor, suggesting marginal reliability for both of the three-item scales.

The resulting distribution of the factors was examined by requesting empirical Bayes estimates of the individual scores for each factor, as shown in Figure 1. Factor determinacy estimates, available via the FSDETERMINACY output option in Mplus, were .882 and .908, respectively, for the forgiveness and not unforgiveness factors (with standard errors for the factor scores of .472 and .418), indicating that the estimated factor scores were strongly related to their model-based counterparts. In addition, Figure 2 shows the predicted response for each item as a linear function of the latent factor based on the estimated model parameters. As shown, the predicted item response goes above the highest response option just before a latent factor score of 2 (i.e., 2 SDs above the mean), resulting in a ceiling effect for both sets of factor scores, as also shown in Figure 1. In addition, for the not unforgiveness factor, the predicted item response goes below the lowest response option just before a latent factor score of -3 (i.e., 3 SDs below the mean), resulting in a floor effect for the not unforgiveness factor, as also shown in Figure 1.

The extent to which the items within each factor could be seen as exchangeable was then examined via an additional set of nested model comparisons, as reported in Table 1 (for fit) and Table 2 (for comparisons of fit). First, the assumption of tau-equivalence (i.e., true-score equivalence, equal discrimination across items) was examined by constraining the factor loadings to be equal within a factor. For the not unforgiveness factor, the tau-equivalent model fit was acceptable but was significantly worse than the original two-factor model fit (i.e., in which all loadings were estimated freely). For the forgiveness factor, however, the tau-equivalent model fit was acceptable and was not significantly worse than the original two-factor model fit. Thus, the assumption of tau-equivalence held for the forgiveness factor items only. Finally, the assumption of parallel items (i.e., equal factor loadings and equal residual variances, or equal reliability across items) was examined for the forgiveness factor items only, and the resulting model fit was acceptable but was significantly worse than the tau-equivalent forgiveness factor model fit. Thus, the assumption of parallel items did not hold for the forgiveness factor items. In summary, while the not unforgiveness factor items were not exchangeable, the forgiveness factor items were exchangeable with respect to their factor loadings only (i.e., equal discrimination, but not equal residual variances or reliability).

Tables would be built as seen in the excel workbook:

Table 1 → “Model Fit Table 1” worksheet
Table 2 → “MLR Comparisons Table 2” worksheet
Table 3 → “Model Estimates Table 3” worksheet

Figures would be built as seen in this example:

Figure 1 → Can be built in Mplus
Figure 2 → Can be built using “Factor Model Predictions” worksheet

References:

Muthén, L. K., & Muthén, B.O. (1998-2015). *Mplus User’s Guide*. Seventh Edition. Los Angeles, CA: Muthén & Muthén.

CFA Example Using Forgiveness of Situations (N = 1103) using SAS MIXED

SAS Code to Read in Mplus Data:

```
* Import data from Mplus, becomes var1-var23 without names at top;
PROC IMPORT OUT=work.Situation DATAFILE= "&example.\Study2.dat" DBMS=TAB REPLACE;
     GETNAMES=NO; DATAROW=1; RUN;

* Rename variables;
DATA Situation; SET Situation;
       ARRAY old(23) var1-var23;
       ARRAY new(23) PersonID Self1 Self2r Self3 Self4r Self5 Self6r
                     Other1r Other2 Other3r Other4 Other5r Other6
                     Sit1r Sit2 Sit3r Sit4 Sit5r Sit6
                     Selfsub Othsub Sitsub HFSsum;
       DO i=1 TO 23; new(i)=old(i); IF new(i)=99999 THEN new(i)=.; END;
       DROP i var1-var23; RUN;

* Stack situation items;
DATA SituationStacked; SET Situation;
       ARRAY aitem(6)  Sit1r Sit2 Sit3r Sit4 Sit5r Sit6;
       DO i=1 TO 6; itemnum=i; response=aitem(i); OUTPUT; END; DROP i; RUN;
```

Independence (Null) Baseline Model: Item means and variances, but NO covariances

```
TITLE "Independence (Null) CFA Model in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
       CLASS PersonID itemnum;
       MODEL response = itemnum / SOLUTION NOINT NOTEST;
       REPEATED itemnum / TYPE=TOEPH(1) SUBJECT=PersonID R; RUN;
```

TYPE=TOEPH(1) predicts a diagonal matrix (would be the same as TYPE=UN(1).

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493					
2		1.9028				
3			2.5431			
4				1.9672		
5					2.9451	
6						2.3412

The **R** matrix shows the unconditional variances per item—repeated in the next piece of output as Var(item). Note that this independence model predicts no covariances between items.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
Var(1)	PersonID	3.0493	0.1298	23.48	<.0001
Var(2)	PersonID	1.9028	0.08102	23.48	<.0001
Var(3)	PersonID	2.5431	0.1083	23.48	<.0001
Var(4)	PersonID	1.9672	0.08377	23.48	<.0001
Var(5)	PersonID	2.9451	0.1254	23.48	<.0001
Var(6)	PersonID	2.3412	0.09969	23.48	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
24625.9	12	24649.9	24650.0	24672.6	24710.0	24722.0

Model fit is given as −2LL rather than LL (but otherwise is the same as given from Mplus).

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > |t|
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects show the unconditional means per item.

Saturated (Unstructured) Baseline Model: Item means, variances, and covariances in original data

```
TITLE "Saturated (Unstructured) CFA Model in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
       CLASS PersonID itemnum;
       MODEL response = itemnum / SOLUTION NOINT NOTEST;
```

TYPE=UN(6) predicts a fully-estimated matrix without any constraints whatsoever.

```
      REPEATED itemnum / TYPE=UN(6) SUBJECT=PersonID R RCORR; RUN;
```

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493	0.5772	1.8022	0.7339	1.3583	0.7946
2	0.5772	1.9028	0.6974	1.1029	0.6043	0.9652
3	1.8022	0.6974	2.5431	0.8244	1.3191	0.8676
4	0.7339	1.1029	0.8244	1.9672	0.6947	0.9618
5	1.3583	0.6043	1.3191	0.6947	2.9451	0.7982
6	0.7946	0.9652	0.8676	0.9618	0.7982	2.3412

The **R** matrix shows the unconditional variances and covariances for the items.

RCORR is the unconditional correlation matrix.

Note THIS IS THE DATA— the only discrepancies you'd see relative to descriptive statistics would be from missing data, as these are ML estimates (that assume MAR rather than MCAR as in listwise deletion).

Estimated R Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.2396	0.6472	0.2997	0.4533	0.2974
2	0.2396	1.0000	0.3170	0.5700	0.2553	0.4573
3	0.6472	0.3170	1.0000	0.3686	0.4820	0.3555
4	0.2997	0.5700	0.3686	1.0000	0.2886	0.4482
5	0.4533	0.2553	0.4820	0.2886	1.0000	0.3040
6	0.2974	0.4573	0.3555	0.4482	0.3040	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	PersonID	3.0493	0.1298	23.48	<.0001
UN(2,1)	PersonID	0.5772	0.07458	7.74	<.0001
UN(2,2)	PersonID	1.9028	0.08102	23.48	<.0001
UN(3,1)	PersonID	1.8022	0.09988	18.04	<.0001
UN(3,2)	PersonID	0.6974	0.06948	10.04	<.0001
UN(3,3)	PersonID	2.5431	0.1083	23.48	<.0001
UN(4,1)	PersonID	0.7339	0.07699	9.53	<.0001
UN(4,2)	PersonID	1.1029	0.06705	16.45	<.0001
UN(4,3)	PersonID	0.8244	0.07178	11.49	<.0001
UN(4,4)	PersonID	1.9672	0.08377	23.48	<.0001
UN(5,1)	PersonID	1.3583	0.09907	13.71	<.0001
UN(5,2)	PersonID	0.6043	0.07356	8.21	<.0001
UN(5,3)	PersonID	1.3191	0.09148	14.42	<.0001
UN(5,4)	PersonID	0.6947	0.07543	9.21	<.0001
UN(5,5)	PersonID	2.9451	0.1254	23.48	<.0001
UN(6,1)	PersonID	0.7946	0.08393	9.47	<.0001
UN(6,2)	PersonID	0.9652	0.06988	13.81	<.0001
UN(6,3)	PersonID	0.8676	0.07798	11.13	<.0001
UN(6,4)	PersonID	0.9618	0.07081	13.58	<.0001
UN(6,5)	PersonID	0.7982	0.08264	9.66	<.0001
UN(6,6)	PersonID	2.3412	0.09969	23.48	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
22644.9	27	22698.9	22699.1	22750.0	22834.0	22861.0

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > |t|
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects again show the unconditional means per item.

Model 1. Fully Z-Scored Factor Model Identification (Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)

```
TITLE "Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
      CLASS PersonID itemnum;
      MODEL response = itemnum / SOLUTION NOINT NOTEST;
      REPEATED itemnum / TYPE=FA(1) SUBJECT=PersonID R RCORR;
RUN;
```

TYPE=FA(1) creates the covariance matrix that would be predicted by a single-factor model.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493	0.8670	1.5313	0.9682	1.2626	1.0108
2	0.8670	1.9028	0.8716	0.5511	0.7187	0.5753
3	1.5313	0.8716	2.5431	0.9733	1.2692	1.0161
4	0.9682	0.5511	0.9733	1.9672	0.8025	0.6424
5	1.2626	0.7187	1.2692	0.8025	2.9451	0.8378

The **R** matrix shows the predicted variances and covariances for the items.

RCORR is the single-factor predicted correlation matrix.

6	1.0108	0.5753	1.0161	0.6424	0.8378	2.3412

Estimated R Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3600	0.5499	0.3953	0.4213	0.3783
2	0.3600	1.0000	0.3962	0.2848	0.3036	0.2726
3	0.5499	0.3962	1.0000	0.4351	0.4638	0.4164
4	0.3953	0.2848	0.4351	1.0000	0.3334	0.2994
5	0.4213	0.3036	0.4638	0.3334	1.0000	0.3191
6	0.3783	0.2726	0.4164	0.2994	0.3191	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
FA(1)	PersonID	1.5259	0.09440	16.16	<.0001
FA(2)	PersonID	1.4093	0.07096	19.86	<.0001
FA(3)	PersonID	1.0038	0.07755	12.94	<.0001
FA(4)	PersonID	1.3518	0.07071	19.12	<.0001
FA(5)	PersonID	1.8986	0.09312	20.39	<.0001
FA(6)	PersonID	1.6706	0.08330	20.05	<.0001
FA(1,1)	PersonID	1.2342	0.05332	23.15	<.0001
FA(2,1)	PersonID	0.7025	0.04720	14.88	<.0001
FA(3,1)	PersonID	1.2407	0.04783	25.94	<.0001
FA(4,1)	PersonID	0.7845	0.04679	16.76	<.0001
FA(5,1)	PersonID	1.0230	0.05202	19.67	<.0001
FA(6,1)	PersonID	0.8190	0.05019	16.32	<.0001

The FA(item) terms are the item residual variances. The FA(item, factor) terms are the item factor loadings.

So the total variance per item is given by: loading^2(1) + error variance, as shown in the **R** matrix above.

Item 1 = 1.2342^2 + 1.5259 = 3.0493

The covariance between items is given by their loadings multiplied together.

Item 1 and 2 cov = 1.2342*0.7025 = 0.8670

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23072.8	18	23108.8	23108.9	23142.9	23198.9	23216.9

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > |t|
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects now show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

Tau-Equivalent Items Single-Factor Model with Marker Item Factor Model Identification (Factor Variance = ?, Factor Mean = 0, All Loadings Equal at 1)

```
TITLE "Tau-Equivalent Items Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
       CLASS PersonID itemnum;
       MODEL response = itemnum / SOLUTION NOINT NOTEST;
       RANDOM INTERCEPT / TYPE=UN SUBJECT=PersonID  G V VCORR;
       REPEATED itemnum / TYPE=TOEPH(1) SUBJECT=PersonID R; RUN;
```

A random intercept creates a constant source of covariance across all items.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.0017					
2		1.1357				
3			1.4550			
4				1.0866		
5					2.0552	
6						1.4565

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	1	0.9127

Estimated V Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.9143	0.9127	0.9127	0.9127	0.9127	0.9127
2	0.9127	2.0483	0.9127	0.9127	0.9127	0.9127
3	0.9127	0.9127	2.3677	0.9127	0.9127	0.9127
4	0.9127	0.9127	0.9127	1.9993	0.9127	0.9127
5	0.9127	0.9127	0.9127	0.9127	2.9679	0.9127
6	0.9127	0.9127	0.9127	0.9127	0.9127	2.3691

Estimated V Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3735	0.3474	0.3781	0.3103	0.3473
2	0.3735	1.0000	0.4144	0.4510	0.3702	0.4143
3	0.3474	0.4144	1.0000	0.4195	0.3443	0.3853
4	0.3781	0.4510	0.4195	1.0000	0.3747	0.4194
5	0.3103	0.3702	0.3443	0.3747	1.0000	0.3442
6	0.3473	0.4143	0.3853	0.4194	0.3442	1.0000

The **R** matrix shows the item residual variances.

The **G** matrix shows the variance due to the factor for all items.

V is the predicted covariance matrix from putting **G** and **R** back together, and **VCORR** is the predicted correlation matrix.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	PersonID	0.9127	0.04938	18.48	<.0001
Var(1)	PersonID	2.0017	0.09613	20.82	<.0001
Var(2)	PersonID	1.1357	0.05929	19.15	<.0001
Var(3)	PersonID	1.4550	0.07304	19.92	<.0001
Var(4)	PersonID	1.0866	0.05703	19.05	<.0001
Var(5)	PersonID	2.0552	0.09729	21.13	<.0001
Var(6)	PersonID	1.4565	0.07161	20.34	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23131.1	13	23157.1	23157.1	23181.7	23222.2	23235.2

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > |t|
itemnum	1	4.5467	0.05140	5510	88.45	<.0001
itemnum	2	5.2892	0.04309	5510	122.74	<.0001
itemnum	3	4.8957	0.04633	5510	105.67	<.0001
itemnum	4	5.3590	0.04257	5510	125.87	<.0001
itemnum	5	4.8604	0.05187	5510	93.70	<.0001
itemnum	6	5.3209	0.04635	5510	114.81	<.0001

The fixed effects still show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

Parallel Items Single-Factor Model with Marker Item Factor Model Identification (Factor Variance = ?, Factor Mean = 0, All Loadings = 1 and All Error Variances Equal)

```
TITLE "Parallel Items Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
       CLASS PersonID itemnum;
       MODEL response = itemnum / SOLUTION NOINT NOTEST;
       RANDOM INTERCEPT / TYPE=UN SUBJECT=PersonID G V VCORR;
       REPEATED itemnum / TYPE=VC SUBJECT=PersonID R; RUN;
```

A random intercept creates a constant source of covariance across all items. A Type=VC R matrix means equal residual variance across items.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.5180					
2		1.5180				
3			1.5180			
4				1.5180		
5					1.5180	
6						1.5180

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	1	0.9401

Estimated V Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.4581	0.9401	0.9401	0.9401	0.9401	0.9401
2	0.9401	2.4581	0.9401	0.9401	0.9401	0.9401
3	0.9401	0.9401	2.4581	0.9401	0.9401	0.9401
4	0.9401	0.9401	0.9401	2.4581	0.9401	0.9401
5	0.9401	0.9401	0.9401	0.9401	2.4581	0.9401
6	0.9401	0.9401	0.9401	0.9401	0.9401	2.4581

Estimated V Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3825	0.3825	0.3825	0.3825	0.3825
2	0.3825	1.0000	0.3825	0.3825	0.3825	0.3825
3	0.3825	0.3825	1.0000	0.3825	0.3825	0.3825
4	0.3825	0.3825	0.3825	1.0000	0.3825	0.3825
5	0.3825	0.3825	0.3825	0.3825	1.0000	0.3825
6	0.3825	0.3825	0.3825	0.3825	0.3825	1.0000

The **R** matrix shows the item residual variances.

The **G** matrix shows the variance due to the factor for all items.

V is the predicted covariance matrix from putting **G** and **R** back together, and **VCORR** is the predicted correlation matrix.

This type of predicted covariance matrix has a special name: **compound symmetry**.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	PersonID	0.9401	0.05103	18.42	<.0001
itemnum	PersonID	1.5180	0.02891	52.51	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23254.0	8	23270.0	23270.1	23285.2	23310.1	23318.1

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > |t|
itemnum	1	4.5467	0.04721	5510	96.31	<.0001
itemnum	2	5.2892	0.04721	5510	112.04	<.0001
itemnum	3	4.8957	0.04721	5510	103.71	<.0001
itemnum	4	5.3590	0.04721	5510	113.52	<.0001
itemnum	5	4.8604	0.04721	5510	102.96	<.0001
itemnum	6	5.3209	0.04721	5510	112.71	<.0001

The fixed effects still show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

Unfortunately, multiple factor models in MIXED appear to be EFA models instead of CFA models, so no examples of two-factor models are given.