

Testing Multiple-Group Measurement Invariance using ML in Item Factor Models in Mplus version 7.4

Example data: 634 older adults (age 80-100) self-reporting on 7 items assessing the Instrumental Activities of Daily Living (IADL) as follows. We are examining differences between men ($N=214$) and women ($N=420$). Each item has **two** response options: 0 = needs help, 1 = does not need help. The items are: 1. Housework (cleaning and laundry), 2. Bedmaking, 3. Cooking, 4. Everyday shopping, 5. Getting to places outside of walking distance, 6. Handling banking and other business, and 7. Using the telephone.

Multiple Group IFA Model Syntax and Truncated Output:

```
TITLE: Assess binary IADL items using ML
DATA: FILE = ADL2.dat;
VARIABLE: NAMES = case female cia1-cia7;
          USEVARIABLES = dial-dia7;
          CATEGORICAL = dial-dia7;
          CLASSES = group(2);           ! 2 fake latent classes
          KNOWNCLASS = group(female=0 1); ! Latent classes = gender
          IDVARIABLE = case;
          MISSING = .;

ANALYSIS: ESTIMATOR = ML; LINK = LOGIT;   ! Full info estimator
          TYPE = MIXTURE; ALGORITHM = INTEGRATION;

PLOT: TYPE = PLOT1 PLOT2 PLOT3;

OUTPUT: STDYX RESIDUAL; ! No voo-doo for IFA in ML

! MEN REFERENCE GROUP CONFIGURAL MODEL (will stay the same)
MODEL:
%OVERALL% ! Needed for fake latent class model
! Factor loadings all estimated, just labeled
IADL BY dial-dia7* (L1-L7);
! Item thresholds all free, just labeled
[dial$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

!!! CONFIGURAL MODEL FOR WOMEN ALTERNATIVE GROUP 2
%group#2% ! Needed for fake latent class model
! Factor loadings all estimated
IADL BY dial-dia7*;
! Item thresholds all free
[dial$1-dia7$1*];
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                29

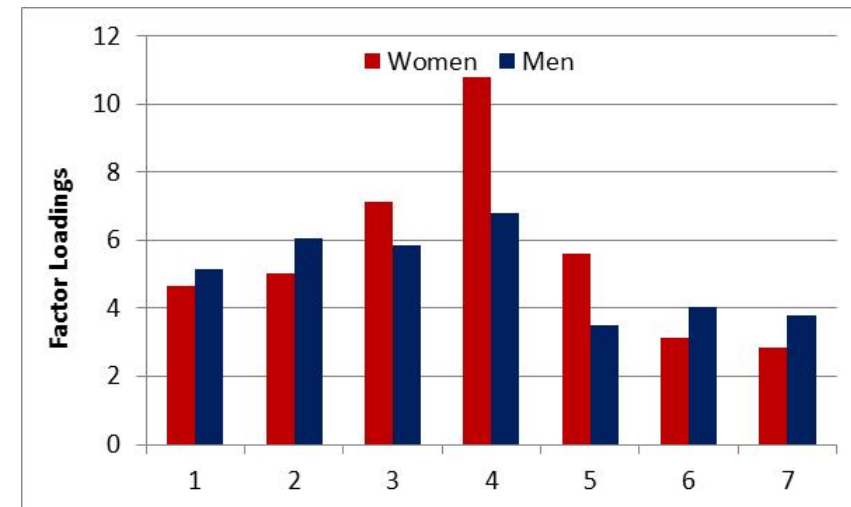
Loglikelihood
  H0 Value                               -1796.106

Information Criteria
  Akaike (AIC)                           3650.212
  Bayesian (BIC)                          3779.367
  Sample-Size Adjusted BIC                3687.295
  (n* = (n + 2) / 24)
```

This will be our baseline configural model.

29 parameters estimated = $2 \times [7 \text{ loadings} + 7 \text{ thresholds}] = 28$

1 extra parameter is estimated as the logit of the proportion of the sample in group#1 (the female=0 group).



**Model 1. Configural Invariance Model
(Everything separate across groups *except* for parameters needed to be constrained for identification)**

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)					UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Latent Class 1 (0) - refers to female=0 from KNOWNCLASS					Latent Class 2 (1) - refers to female=1 from KNOWNCLASS				
IADL	BY -- FACTOR LOADINGS				IADL	BY -- FACTOR LOADINGS			
DIA1	5.138	1.321	3.890	0.000	DIA1	4.664	0.731	6.377	0.000
DIA2	6.066	1.830	3.315	0.001	DIA2	5.014	0.998	5.026	0.000
DIA3	5.828	1.566	3.722	0.000	DIA3	7.108	1.670	4.257	0.000
DIA4	6.792	2.086	3.256	0.001	DIA4	10.774	8.194	1.315	0.189 !!!
DIA5	3.520	0.717	4.907	0.000	DIA5	5.611	1.145	4.902	0.000
DIA6	4.026	0.890	4.525	0.000	DIA6	3.143	0.422	7.440	0.000
DIA7	3.789	1.211	3.129	0.002	DIA7	2.857	0.610	4.681	0.000
Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION					Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	0.000	0.000	999.000	999.000	IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED LOGIT OF Y=0 IF THETA=0					Thresholds				
DIA1\$1	-1.372	0.622	-2.204	0.028	DIA1\$1	-2.188	0.423	-5.177	0.000
DIA2\$1	-5.632	1.614	-3.489	0.000	DIA2\$1	-5.694	1.020	-5.583	0.000
DIA3\$1	-3.005	0.919	-3.270	0.001	DIA3\$1	-6.715	1.494	-4.496	0.000
DIA4\$1	-4.378	1.337	-3.274	0.001	DIA4\$1	-3.791	2.941	-1.289	0.197
DIA5\$1	-2.722	0.552	-4.928	0.000	DIA5\$1	-1.798	0.491	-3.659	0.000
DIA6\$1	-3.311	0.707	-4.684	0.000	DIA6\$1	-2.184	0.313	-6.976	0.000
DIA7\$1	-6.398	1.701	-3.762	0.000	DIA7\$1	-5.577	0.886	-6.298	0.000
Variances: VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION					Variances: VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION				
IADL	1.000	0.000	999.000	999.000	IADL	1.000	0.000	999.000	999.000
Although Mplus does give IRT a-parameters and b-parameters for binary items, it rescales them to assume a theta mean=0 and variance=1 in both groups. Thus, they will not be invariant even when the loadings and thresholds are invariant. For this reason, they are not shown here.					Categorical Latent Variables				
					Means = logit of proportion of men (group#1) = .34				
					GROUP#1 -0.670 0.084 -7.985 0.000				

Model 2a. Metric Invariance Model (IFA loadings held equal across groups – Mplus IRT discriminations still vary via factor variances)

```

! MEN REFERENCE GROUP CONFIGURAL MODEL (will stay the same)
MODEL:
%OVERALL% ! Needed for fake latent class model
! Factor loadings all estimated, just labeled
IADL BY dial-dia7* (L1-L7);
! Item thresholds all free, just labeled
[dial$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

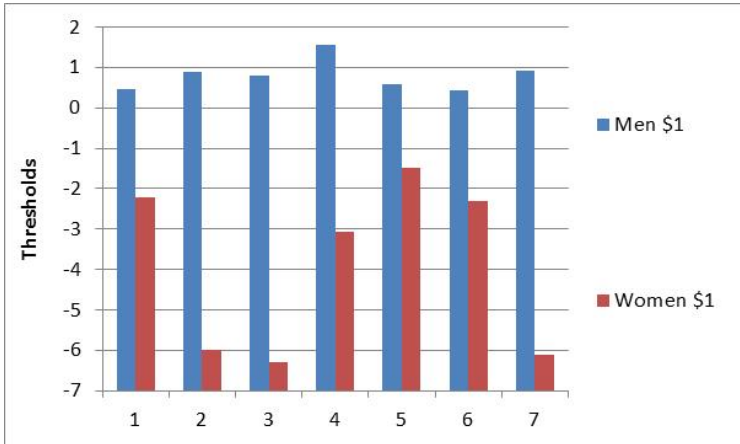
!!! 2 METRIC MODEL FOR WOMEN ALTERNATIVE GROUP 2
%group#2% ! Needed for fake latent class model
! Factor loadings NOW CONSTRAINED EQUAL TO MEN
IADL BY dial-dia7* (L1-L7);
! Item thresholds all free
[dial$1-dia7$1*];
! Factor mean=0 for identification
! Factor variance NOW ESTIMATED
[IADL@0]; IADL*;

MODEL FIT INFORMATION
Number of Free Parameters                23

Loglikelihood
H0 Value                                -1799.928

Information Criteria
Akaike (AIC)                            3645.855
Bayesian (BIC)                          3748.288
Sample-Size Adjusted BIC                 3675.266
(n* = (n + 2) / 24)
    
```

Does the full metric invariance model (2a) fit significantly worse than the configural model (1)? Nope, $-2\Delta LL(df=6) = 7.64, p = .27$



	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Latent Class 1 (0)				
IADL BY				
DIA1	4.691	0.714	6.570	0.000
DIA2	5.209	0.938	5.550	0.000
DIA3	6.517	1.258	5.179	0.000
DIA4	8.631	2.433	3.548	0.000
DIA5	4.611	0.669	6.893	0.000
DIA6	3.337	0.465	7.181	0.000
DIA7	3.195	0.629	5.082	0.000
Means: Factor mean fixed=0 for identification				
IADL	0.000	0.000	999.000	999.000
Thresholds				
DIA1\$1	-1.267	0.473	-2.681	0.007
DIA2\$1	-4.872	0.876	-5.563	0.000
DIA3\$1	-3.308	0.811	-4.078	0.000
DIA4\$1	-5.408	1.559	-3.468	0.001
DIA5\$1	-3.387	0.580	-5.838	0.000
DIA6\$1	-2.828	0.441	-6.409	0.000
DIA7\$1	-5.626	0.924	-6.091	0.000
Variances: Factor variance fixed=1 for identification				
IADL	1.000	0.000	999.000	999.000
Latent Class 2 (1)				
IADL BY				
DIA1	4.691	0.714	6.570	0.000
DIA2	5.209	0.938	5.550	0.000
DIA3	6.517	1.258	5.179	0.000
DIA4	8.631	2.433	3.548	0.000
DIA5	4.611	0.669	6.893	0.000
DIA6	3.337	0.465	7.181	0.000
DIA7	3.195	0.629	5.082	0.000
Means: Factor mean fixed=0 for identification				
IADL	0.000	0.000	999.000	999.000
Thresholds				
DIA1\$1	-2.220	0.402	-5.520	0.000
DIA2\$1	-5.983	0.913	-6.555	0.000
DIA3\$1	-6.310	1.089	-5.794	0.000
DIA4\$1	-3.079	0.995	-3.095	0.002
DIA5\$1	-1.495	0.358	-4.179	0.000
DIA6\$1	-2.317	0.315	-7.348	0.000
DIA7\$1	-6.127	0.882	-6.949	0.000
Variances: Factor variance now estimated				
IADL	1.056	0.240	4.409	0.000

Model 3a. Full Threshold Invariance Model (all IFA thresholds held equal across groups – Mplus IRT difficulties can still vary)

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! MEN REFERENCE GROUP CONFIGURAL MODEL (stays the same)
MODEL:
%OVERALL% ! Needed for fake latent class model
! Factor loadings all estimated, just labeled
IADL BY dial-dia7* (L1-L7);
! Item thresholds all free, just labeled
[dial$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

!!! 3 SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP 2
%group#2% ! Needed for fake latent class model
! Factor loadings NOW CONSTRAINED EQUAL TO MEN
IADL BY dial-dia7* (L1-L7);
! Item thresholds NOW CONSTRAINED EQUAL TO MEN
[dial$1-dia7$1*] (T1-T7);
! Factor mean=0 NOW ESTIMATED
! Factor variance NOW ESTIMATED
[IADL*]; IADL*;

MODEL FIT INFORMATION
Number of Free Parameters          17

Loglikelihood
H0 Value                          -1860.402
Information Criteria

Akaike (AIC)                      3754.804
Bayesian (BIC)                    3830.515
Sample-Size Adjusted BIC          3776.542
    
```

Does the full scalar invariance model (3a) fit significantly worse than the full metric model (2a)? Yep, $-2\Delta LL(df=6) = 120.95, p < .01$

Based on the local misfit of the univariate distributions (from TECH10 output, see right), it looks like dia3 is the biggest problem... so the same steps would be followed as in our other examples. That is, try to free problematic thresholds until the scalar model fits not worse than the metric model (or the configural model, if you constraint both loadings and thresholds at the same time).

RESIDUAL OUTPUT

UNIVARIATE DISTRIBUTION FIT FOR CLASS 1

Variable	Observed	Estimated	Residual (Obs.-Est.)	Stand. Residual
DIA1				
Category 1	0.404	0.352	0.052	1.591
Category 2	0.596	0.648	-0.052	-1.591
DIA2				
Category 1	0.184	0.168	0.017	0.664
Category 2	0.816	0.832	-0.017	-0.664
DIA3				
Category 1	0.313	0.229	0.084	2.939
Category 2	0.687	0.771	-0.084	-2.939
DIA4				
Category 1	0.257	0.333	-0.076	-2.362
Category 2	0.743	0.667	0.076	2.362
DIA5				
Category 1	0.249	0.335	-0.086	-2.683
Category 2	0.751	0.665	0.086	2.683
DIA6				
Category 1	0.233	0.261	-0.028	-0.946
Category 2	0.767	0.739	0.028	0.946
DIA7				
Category 1	0.066	0.062	0.004	0.233
Category 2	0.934	0.938	-0.004	-0.233

UNIVARIATE DISTRIBUTION FIT FOR CLASS 2

Variable	Observed	Estimated	Residual (Obs.-Est.)	Stand. Residual
DIA1				
Category 1	0.343	0.370	-0.027	-1.132
Category 2	0.657	0.630	0.027	1.132
DIA2				
Category 1	0.147	0.162	-0.015	-0.813
Category 2	0.853	0.838	0.015	0.813
DIA3				
Category 1	0.186	0.230	-0.044	-2.165
Category 2	0.814	0.770	0.044	2.165
DIA4				
Category 1	0.378	0.348	0.030	1.298
Category 2	0.622	0.652	-0.030	-1.298
DIA5				
Category 1	0.397	0.350	0.046	1.985
Category 2	0.603	0.650	-0.046	-1.985
DIA6				
Category 1	0.284	0.267	0.017	0.785
Category 2	0.716	0.733	-0.017	-0.785
DIA7				
Category 1	0.052	0.053	-0.001	-0.078
Category 2	0.948	0.947	0.001	0.078