# Longitudinal Invariance CFA (using MLR) Example in Mplus v. 7.4 (*N* = 151; 6 items over 3 occasions)

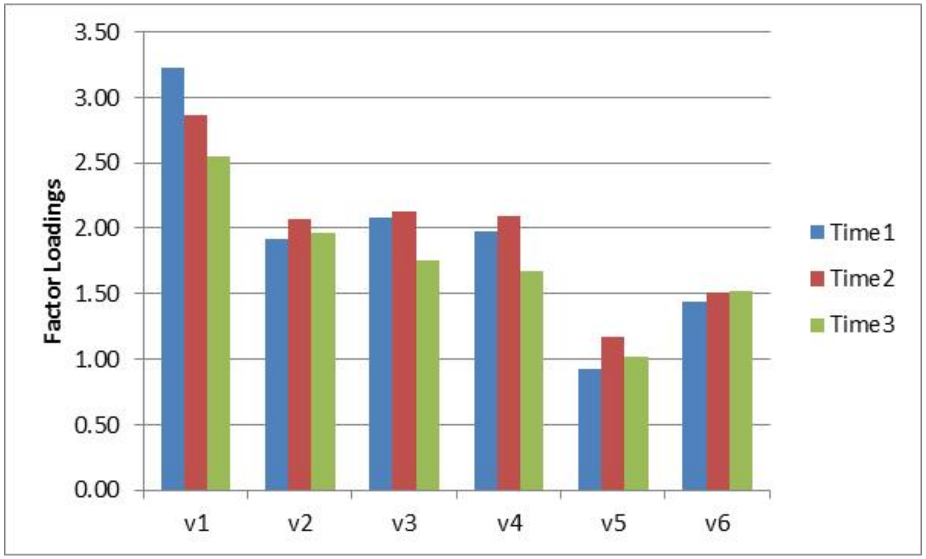
These data measuring a latent trait of social functioning were collected at a Psychiatric Rehabilitation center, in which time 1 was admittance, and times 2 and 3 were collected at six-month intervals. There were six subscales that were completed by the hospital staff for each patient, including positively-oriented measures of Social Competence, Social Interest, and Personal Neatness, and negatively-oriented measures of Psychoticism, Motor Retardation, and Irritability. The negatively-oriented subscales were reflected (*−1) prior to analysis. Initial models examined the fit of one-factor versus two-factor models given the two valences of the subscales, but the fit of the two-factor model was not a significant improvement, and thus a one-factor model with all six items was used here.

## Mplus Code to Read in Data:

```
TITLE:      Longitudinal Invariance
DATA:       FILE = CAF.dat;                     ! Don't need path if in same folder
            FORMAT = free; TYPE = INDIVIDUAL;   ! Defaults

VARIABLE:   NAMES = ID v1T1 v1T2 v1T3 v2T1 v2T2 v2T3         ! Every variable in data set
                       v3T1 v3T2 v3T3 v4T1 v4T2 v4T3
                       v5T1 v5T2 v5T3 v6T1 v6T2 v6T3;

            USEVARIABLES = v1T1 v1T2 v1T3 v2T1 v2T2 v2T3     ! Every variable in MODEL
                               v3T1 v3T2 v3T3 v4T1 v4T2 v4T3
                               v5T1 v5T2 v5T3 v6T1 v6T2 v6T3;

           MISSING = ALL (9999);     ! Make sure to specify all missing values
           IDVARIABLE = ID;          ! ID variable to be included in output files

! Reverse-coding items so that higher = better
DEFINE:      v4T1 = v4T1*(-1);
             v4T2 = v4T2*(-1);
             v4T3 = v4T3*(-1);
             v5T1 = v5T1*(-1);
             v5T2 = v5T2*(-1);
             v5T3 = v5T3*(-1);
             v6T1 = v6T1*(-1);
             v6T2 = v6T2*(-1);
             v6T3 = v6T3*(-1);

ANALYSIS: ESTIMATOR = MLR;    ! For continuous items whose residuals may not be normal

OUTPUT:   MODINDICES(3.84);  ! For modification indices of p<.05 for df=1
          STDYX RESIDUAL;    ! Fully standardized solution, local model fit

MODEL:    ! Model syntax goes here, to be changed for each model
```

## Model 1. Configural Longitudinal Invariance Model (everything separate across time)

```
MODEL:
!!!!! Model 1: Configural Longitudinal Invariance

! Factor loadings all freely estimated, not labeled
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1*;
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2*;
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3*;
! Item intercepts all freely estimated, not labeled
  [v1T1* v1T2* v1T3*]; [v2T1* v2T2* v2T3*];
  [v3T1* v3T2* v3T3*]; [v4T1* v4T2* v4T3*];
  [v5T1* v5T2* v5T3*]; [v6T1* v6T2* v6T3*];
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variances all fixed=1 for identification
  Time1@1 Time2@1 Time3@1;
! Factor means all fixed=0 for identification
  [Time1@0 Time2@0 Time3@0];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```




```
MODEL FIT INFORMATION

Number of Free Parameters                       75

Loglikelihood
          H0 Value                       -4430.302
          H0 Scaling Correction Factor      1.4617
            for MLR
          H1 Value                       -4284.045
          H1 Scaling Correction Factor      1.2029
            for MLR

Information Criteria
          Akaike (AIC)                    9010.604
          Bayesian (BIC)                  9236.900
          Sample-Size Adjusted BIC        8999.533
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            283.247*
          Degrees of Freedom                   114
          P-Value                           0.0000
          Scaling Correction Factor         1.0327
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.099
          90 Percent C.I.                    0.085  0.114
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.903
          TLI                                0.870

Chi-Square Test of Model Fit for the Baseline Model
          Value                           1896.788
          Degrees of Freedom                   153
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.089
```

Although the fit is not great, attempts to improve it logically were unsuccessful, so we proceed from here with this as the configural invariance mode. The plot of factor loadings on the left foreshadows what will happen when testing metric invariance next…

**UNSTANDARDIZED MODEL RESULTS – NOTE ALL MEASUREMENT PARAMETERS DIFFER ACROSS TIME**

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| **FACTOR LOADINGS PER OCCASION** | | | | |
| TIME1 BY | | | | |
| V1T1 | 3.222 | 0.267 | 12.063 | 0.000 |
| V2T1 | 1.915 | 0.274 | 6.997 | 0.000 |
| V3T1 | 2.080 | 0.209 | 9.956 | 0.000 |
| V4T1 | 1.975 | 0.271 | 7.298 | 0.000 |
| V5T1 | 0.931 | 0.148 | 6.281 | 0.000 |
| V6T1 | 1.441 | 0.119 | 12.101 | 0.000 |
| TIME2 BY | | | | |
| V1T2 | 2.863 | 0.305 | 9.372 | 0.000 |
| V2T2 | 2.072 | 0.197 | 10.490 | 0.000 |
| V3T2 | 2.133 | 0.185 | 11.509 | 0.000 |
| V4T2 | 2.098 | 0.322 | 6.514 | 0.000 |
| V5T2 | 1.175 | 0.239 | 4.921 | 0.000 |
| V6T2 | 1.512 | 0.129 | 11.749 | 0.000 |
| TIME3 BY | | | | |
| V1T3 | 2.550 | 0.288 | 8.865 | 0.000 |
| V2T3 | 1.961 | 0.230 | 8.539 | 0.000 |
| V3T3 | 1.751 | 0.210 | 8.323 | 0.000 |
| V4T3 | 1.678 | 0.260 | 6.448 | 0.000 |
| V5T3 | 1.021 | 0.170 | 6.012 | 0.000 |
| V6T3 | 1.523 | 0.159 | 9.574 | 0.000 |
| TIME1 WITH **(ESTIMATED FACTOR COVARIANCES)** | | | | |
| TIME2 | 0.786 | 0.042 | 18.827 | 0.000 |
| TIME3 | 0.707 | 0.084 | 8.456 | 0.000 |
| TIME2 WITH | | | | |
| TIME3 | 0.671 | 0.089 | 7.532 | 0.000 |
| ***** Residual covariances among same item across time ****** | | | | |
| V1T1 WITH | | | | |
| V1T2 | -0.214 | 0.250 | -0.855 | 0.393 |
| V1T3 | -0.004 | 0.247 | -0.016 | 0.987 |
| V1T2 WITH | | | | |
| V1T3 | 0.113 | 0.231 | 0.488 | 0.626 |
| ........... | | | | |
| **Variances (FACTOR VARIANCES FIXED=1 FOR IDENTIFICATION)** | | | | |
| TIME1 | 1.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 1.000 | 0.000 | 999.000 | 999.000 |
| TIME3 | 1.000 | 0.000 | 999.000 | 999.000 |

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| **Means (FACTOR MEANS FIXED=0 FOR IDENTIFICATION)** | | | | |
| TIME1 | 0.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 0.000 | 0.000 | 999.000 | 999.000 |
| TIME3 | 0.000 | 0.000 | 999.000 | 999.000 |
| **Intercepts (ARE EXPECTED OUTCOME WHEN FACTOR IS AT 0)** | | | | |
| V1T1 | 16.077 | 0.276 | 58.220 | 0.000 |
| V1T2 | 17.226 | 0.245 | 70.294 | 0.000 |
| V1T3 | 17.756 | 0.220 | 80.620 | 0.000 |
| V2T1 | 8.672 | 0.298 | 29.132 | 0.000 |
| V2T2 | 9.981 | 0.263 | 37.921 | 0.000 |
| V2T3 | 10.442 | 0.281 | 37.204 | 0.000 |
| V3T1 | 11.970 | 0.225 | 53.108 | 0.000 |
| V3T2 | 12.467 | 0.218 | 57.264 | 0.000 |
| V3T3 | 13.029 | 0.213 | 61.157 | 0.000 |
| V4T1 | -3.037 | 0.271 | -11.216 | 0.000 |
| V4T2 | -3.211 | 0.260 | -12.349 | 0.000 |
| V4T3 | -2.738 | 0.249 | -11.014 | 0.000 |
| V5T1 | -1.283 | 0.138 | -9.293 | 0.000 |
| V5T2 | -1.664 | 0.200 | -8.338 | 0.000 |
| V5T3 | -1.247 | 0.166 | -7.511 | 0.000 |
| V6T1 | -2.871 | 0.164 | -17.508 | 0.000 |
| V6T2 | -2.413 | 0.158 | -15.316 | 0.000 |
| V6T3 | -2.075 | 0.152 | -13.618 | 0.000 |
| **Residual Variances (VARIANCE PER ITEM THAT IS NOT THE FACTOR)** | | | | |
| V1T1 | 0.241 | 0.395 | 0.610 | 0.542 |
| V1T2 | 0.511 | 0.268 | 1.907 | 0.056 |
| V1T3 | 0.523 | 0.349 | 1.497 | 0.134 |
| V2T1 | 8.672 | 1.022 | 8.484 | 0.000 |
| V2T2 | 5.913 | 0.617 | 9.581 | 0.000 |
| V2T3 | 5.142 | 0.806 | 6.379 | 0.000 |
| V3T1 | 2.413 | 0.398 | 6.067 | 0.000 |
| V3T2 | 2.202 | 0.369 | 5.972 | 0.000 |
| V3T3 | 2.381 | 0.430 | 5.542 | 0.000 |
| V4T1 | 7.199 | 1.036 | 6.950 | 0.000 |
| V4T2 | 6.765 | 0.990 | 6.834 | 0.000 |
| V4T3 | 6.456 | 1.078 | 5.988 | 0.000 |
| V5T1 | 1.824 | 0.446 | 4.093 | 0.000 |
| V5T2 | 4.676 | 1.439 | 3.251 | 0.001 |
| V5T3 | 2.944 | 0.752 | 3.913 | 0.000 |
| V6T1 | 1.694 | 0.243 | 6.974 | 0.000 |
| V6T2 | 1.103 | 0.166 | 6.643 | 0.000 |
| V6T3 | 0.751 | 0.162 | 4.630 | 0.000 |

**Model 2a. Metric Invariance Model (ALL loadings held equal across time – identified model using Time1 Factor Variance = 1)**

```
MODEL:
!!!!! Model 2a: Metric Longitudinal Invariance

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts all freely estimated, not labeled
  [v1T1* v1T2* v1T3*]; [v2T1* v2T2* v2T3*];
  [v3T1* v3T2* v3T3*]; [v4T1* v4T2* v4T3*];
  [v5T1* v5T2* v5T3*]; [v6T1* v6T2* v6T3*];
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor means all fixed=0 for identification
  [Time1@0 Time2@0 Time3@0];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

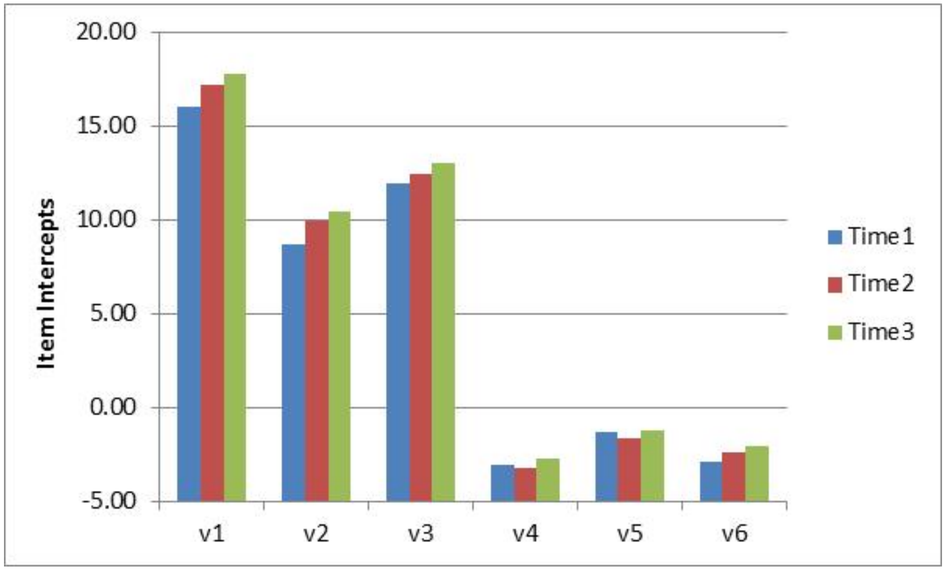
**Does the metric model (2a) fit *worse* than the configural model (1)?**
Yes, $-2\Delta LL(df=10) = 19.14$, $p = .04$

```
MODEL FIT INFORMATION
Number of Free Parameters                        65

Loglikelihood
          H0 Value                        -4442.401
          H0 Scaling Correction Factor       1.4921
            for MLR
          H1 Value                        -4284.045
          H1 Scaling Correction Factor       1.2029
            for MLR

Information Criteria
          Akaike (AIC)                     9014.803
          Bayesian (BIC)                   9210.926
          Sample-Size Adjusted BIC         9005.208
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                             301.234*
          Degrees of Freedom                    124
          P-Value                            0.0000
          Scaling Correction Factor          1.0514
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.097
          90 Percent C.I.                     0.083  0.111
          Probability RMSEA <= .05            0.000

CFI/TLI
          CFI                                 0.898
          TLI                                 0.875

SRMR (Standardized Root Mean Square Residual)
          Value                               0.094


MODEL MODIFICATION INDICES (relevant for testing invariance)
BY Statements
                       M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
BY Statements
TIME1    BY V1T1      10.377     0.182      0.182        0.058
TIME1    BY V5T1       6.062    -0.054     -0.054       -0.033
TIME3    BY V6T3       7.603     0.201      0.175        0.105
```

Modification indices suggest that freeing the loading for v1 at Time1 would help, and that matches our observations, so let's try that.

**Model 2b. Partial Metric Invariance Model with loading for v1 at Time 1 free**

```
MODEL:
! Model 2b: Partial Metric Invariance without v1T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts all freely estimated, not labeled
  [v1T1* v1T2* v1T3*]; [v2T1* v2T2* v2T3*];
  [v3T1* v3T2* v3T3*]; [v4T1* v4T2* v4T3*];
  [v5T1* v5T2* v5T3*]; [v6T1* v6T2* v6T3*];
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor means all fixed=0 for identification
  [Time1@0 Time2@0 Time3@0];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```




```
MODEL FIT INFORMATION
Number of Free Parameters                        66

Loglikelihood
          H0 Value                        -4435.669
          H0 Scaling Correction Factor       1.4980
            for MLR
          H1 Value                        -4284.045
          H1 Scaling Correction Factor       1.2029
            for MLR

Information Criteria
          Akaike (AIC)                     9003.337
          Bayesian (BIC)                   9202.478
          Sample-Size Adjusted BIC         8993.595
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                             290.301*
          Degrees of Freedom                    123
          P-Value                            0.0000
          Scaling Correction Factor          1.0446
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.095
          90 Percent C.I.                     0.081  0.109
          Probability RMSEA <= .05            0.000

CFI/TLI
          CFI                                 0.904
          TLI                                 0.881

SRMR (Standardized Root Mean Square Residual)
          Value                               0.091
```

**Does the partial metric model (2b) fit *better* than the full metric model (2a)?** Yes, $-2\Delta LL(df=1) = 7.16$, $p < .01$

**Does the partial metric model (2b) fit *worse* than the configural model (1)?** No, $-2\Delta LL(df=9) = 8.98$, $p = .44$

No invariance-related modification indices were found, so we'll call it good! Onto the next model! The plot of intercepts on the left foreshadow what we will find with testing scalar invariance…

**2b UNSTANDARDIZED PARTIAL METRIC MODEL RESULTS – ALL FACTOR LOADINGS ARE HELD EQUAL EXCEPT v1T1**

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| TIME1 BY | | | | |
| **V1T1** | **3.233** | **0.261** | **12.362** | **0.000** |
| V2T1 | 1.950 | 0.201 | 9.706 | 0.000 |
| V3T1 | 1.967 | 0.198 | 9.910 | 0.000 |
| V4T1 | 1.899 | 0.224 | 8.481 | 0.000 |
| V5T1 | 0.968 | 0.137 | 7.055 | 0.000 |
| V6T1 | 1.476 | 0.131 | 11.247 | 0.000 |
| TIME2 BY | | | | |
| **V1T2** | **2.644** | **0.234** | **11.315** | **0.000** |
| V2T2 | 1.950 | 0.201 | 9.706 | 0.000 |
| V3T2 | 1.967 | 0.198 | 9.910 | 0.000 |
| V4T2 | 1.899 | 0.224 | 8.481 | 0.000 |
| V5T2 | 0.968 | 0.137 | 7.055 | 0.000 |
| V6T2 | 1.476 | 0.131 | 11.247 | 0.000 |
| TIME3 BY | | | | |
| **V1T3** | **2.644** | **0.234** | **11.315** | **0.000** |
| V2T3 | 1.950 | 0.201 | 9.706 | 0.000 |
| V3T3 | 1.967 | 0.198 | 9.910 | 0.000 |
| V4T3 | 1.899 | 0.224 | 8.481 | 0.000 |
| V5T3 | 0.968 | 0.137 | 7.055 | 0.000 |
| V6T3 | 1.476 | 0.131 | 11.247 | 0.000 |
| TIME1 WITH | | | | |
| TIME2 | 0.847 | 0.078 | 10.837 | 0.000 |
| TIME3 | 0.682 | 0.124 | 5.508 | 0.000 |
| TIME2 WITH | | | | |
| TIME3 | 0.699 | 0.128 | 5.473 | 0.000 |
| ***** Residual covariances among same item across time ****** | | | | |
| V1T1 WITH | | | | |
| V1T2 | -0.225 | 0.249 | -0.904 | 0.366 |
| V1T3 | -0.012 | 0.236 | -0.049 | 0.961 |
| V1T2 WITH | | | | |
| V1T3 | 0.132 | 0.230 | 0.573 | 0.566 |
| ........... | | | | |
| Variances (FACTOR VARIANCE AT TIME1=1 FOR IDENTIFICATION) | | | | |
| TIME1 | 1.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 1.162 | 0.185 | 6.270 | 0.000 |
| TIME3 | 0.941 | 0.157 | 5.999 | 0.000 |

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| Means (FACTOR MEANS FIXED=0 FOR IDENTIFICATION) | | | | |
| TIME1 | 0.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 0.000 | 0.000 | 999.000 | 999.000 |
| TIME3 | 0.000 | 0.000 | 999.000 | 999.000 |
| Intercepts – SCALED SO SHOULD BE EQUAL ACROSS TIME | | | | |
| V1T1 | 16.078 | 0.276 | 58.267 | 0.000 |
| V1T2 | 17.225 | 0.245 | 70.282 | 0.000 |
| V1T3 | 17.756 | 0.222 | 80.036 | 0.000 |
| V2T1 | 8.672 | 0.298 | 29.071 | 0.000 |
| V2T2 | 9.980 | 0.264 | 37.872 | 0.000 |
| V2T3 | 10.434 | 0.280 | 37.245 | 0.000 |
| V3T1 | 11.978 | 0.225 | 53.192 | 0.000 |
| V3T2 | 12.468 | 0.217 | 57.325 | 0.000 |
| V3T3 | 13.041 | 0.212 | 61.441 | 0.000 |
| V4T1 | -3.034 | 0.267 | -11.343 | 0.000 |
| V4T2 | -3.210 | 0.260 | -12.365 | 0.000 |
| V4T3 | -2.720 | 0.254 | -10.720 | 0.000 |
| V5T1 | -1.288 | 0.137 | -9.377 | 0.000 |
| V5T2 | -1.663 | 0.199 | -8.340 | 0.000 |
| V5T3 | -1.246 | 0.169 | -7.373 | 0.000 |
| V6T1 | -2.871 | 0.164 | -17.506 | 0.000 |
| V6T2 | -2.414 | 0.158 | -15.319 | 0.000 |
| V6T3 | -2.087 | 0.154 | -13.571 | 0.000 |
| Residual Variances – ITEM VARIANCE THAT IS NOT THE FACTOR | | | | |
| V1T1 | 0.170 | 0.374 | 0.454 | 0.650 |
| V1T2 | 0.548 | 0.265 | 2.070 | 0.038 |
| V1T3 | 0.509 | 0.314 | 1.618 | 0.106 |
| V2T1 | 8.702 | 1.026 | 8.483 | 0.000 |
| V2T2 | 5.895 | 0.605 | 9.746 | 0.000 |
| V2T3 | 5.177 | 0.795 | 6.514 | 0.000 |
| V3T1 | 2.502 | 0.386 | 6.484 | 0.000 |
| V3T2 | 2.178 | 0.352 | 6.183 | 0.000 |
| V3T3 | 2.309 | 0.416 | 5.548 | 0.000 |
| V4T1 | 7.172 | 1.021 | 7.021 | 0.000 |
| V4T2 | 6.759 | 0.967 | 6.990 | 0.000 |
| V4T3 | 6.613 | 1.128 | 5.860 | 0.000 |
| V5T1 | 1.829 | 0.443 | 4.131 | 0.000 |
| V5T2 | 4.678 | 1.430 | 3.272 | 0.001 |
| V5T3 | 2.944 | 0.760 | 3.872 | 0.000 |
| V6T1 | 1.707 | 0.242 | 7.059 | 0.000 |
| V6T2 | 1.090 | 0.165 | 6.599 | 0.000 |
| V6T3 | 0.784 | 0.170 | 4.618 | 0.000 |

**Model 3a. Scalar Invariance Model (all intercepts held equal across over time except v1T1); identified by Time1 mean=0**

```
MODEL:
! Model 3a: Full Scalar Invariance without v1T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  [v1T1*]; [v1T2* v1T3*] (I1);
  [v2T1*    v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*    v4T2* v4T3*] (I4);
  [v5T1*    v5T2* v5T3*] (I5);
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

**Does the full scalar model (3a) fit worse than the partial metric model (2b)?** Yes, $-2\Delta LL(df=9) = 55.13$, $p < .01$

Modification indices suggest that freeing these intercepts would help, so let's try v5T1 first (biggest $\chi^2$ change suggested).

```
MODEL FIT INFORMATION
Number of Free Parameters                       57

Loglikelihood
          H0 Value                       -4461.842
          H0 Scaling Correction Factor      1.5846
            for MLR
          H1 Value                       -4284.045
          H1 Scaling Correction Factor      1.2029
            for MLR

Information Criteria
          Akaike (AIC)                    9037.685
          Bayesian (BIC)                  9209.670
          Sample-Size Adjusted BIC        9029.271
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            342.530*
          Degrees of Freedom                   132
          P-Value                           0.0000
          Scaling Correction Factor         1.0381
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.103
          90 Percent C.I.                    0.089  0.116
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.879
          TLI                                0.860

SRMR (Standardized Root Mean Square Residual)
          Value                              0.093


MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds

                          M.I.   E.P.C.  Std E.P.C.  StdYX E.P.C.
[ V2T1     ]            14.761   -0.696      -0.696        -0.189
[ V2T2     ]             5.578    0.307       0.307         0.094
[ V4T1     ]            10.400    0.366       0.366         0.113
[ V4T2     ]             5.167   -0.271      -0.271        -0.084
[ V5T1     ]            20.890   -0.027      -0.027        -0.017
[ V5T2     ]            14.191   -0.596      -0.596        -0.241
```

**Model 3b. Partial Scalar Invariance Model (all intercepts held equal across over time except v1T1 and v5T1)**

```
MODEL: ! Model 3b: Partial Scalar Invariance, no v1T1 v5T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME, no v1T1 v5T1
  [v1T1*]; [v1T2* v1T3*] (I1);
  [v2T1*    v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*    v4T2* v4T3*] (I4);
  [v5T1*]; [v5T2* v5T3*] (I5);
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

**Does the partial scalar model (3b) fit *better* than the full scalar model (3a)?**
Yes, $-2\Delta LL(df=1) = 15.16$, $p < .01$

**Does the partial scalar model (3b) fit *worse* than the partial metric model (2b)?** Yes, $-2\Delta LL(df=8) = 27.84$, $p < .01$

Modification indices still suggest that freeing these intercepts would help, so let's try v4T1 next (biggest $\chi^2$ change suggested).

```
MODEL FIT INFORMATION
Number of Free Parameters                       58

Loglikelihood
          H0 Value                       -4450.001
          H0 Scaling Correction Factor      1.5626
            for MLR
          H1 Value                       -4284.045
          H1 Scaling Correction Factor      1.2029
            for MLR

Information Criteria
          Akaike (AIC)                    9016.001
          Bayesian (BIC)                  9191.004
          Sample-Size Adjusted BIC        9007.440
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            318.018*
          Degrees of Freedom                   131
          P-Value                           0.0000
          Scaling Correction Factor         1.0437
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.097
          90 Percent C.I.                    0.084  0.111
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.893
          TLI                                0.875

SRMR (Standardized Root Mean Square Residual)
          Value                              0.086

MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds

                      M.I.     E.P.C.  Std E.P.C.  StdYX
E.P.C.
[ V2T1     ]        11.529    -0.599     -0.599      -0.164
[ V2T2     ]         4.390     0.278      0.278       0.085
[ V4T1     ]        13.795     0.425      0.425       0.132
[ V4T2     ]         6.398    -0.306     -0.306      -0.096
```

## Model 3c. Partial Scalar Invariance Model (all intercepts held equal across over time except v1T1, v5T1, v4T1)

```
MODEL: ! Model 3c: Partial Scalar Invariance, no v1T1 v5T1 v4T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1
  [v1T1*]; [v1T2* v1T3*] (I1);
  [v2T1*    v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4);
  [v5T1*]; [v5T2* v5T3*] (I5);
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

**Does the partial scalar model (3c) fit *better* than the partial scalar model (3b)?** Yes, −2ΔLL(df=1) = 9.24, $p$ <.01

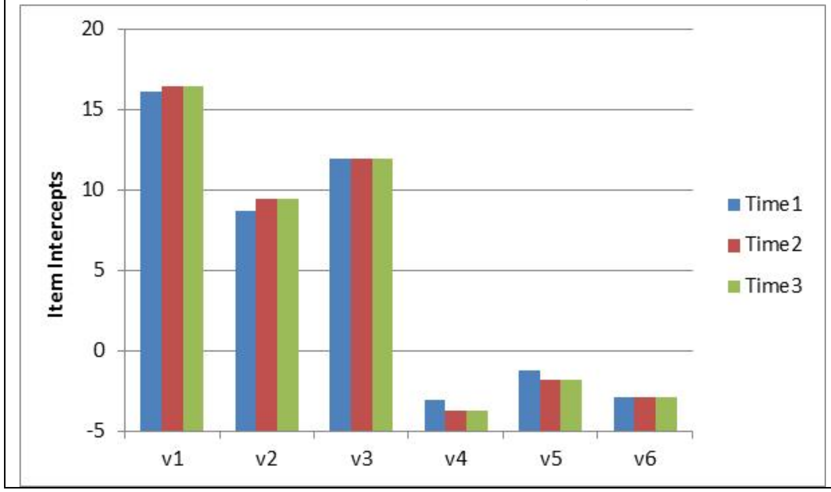
**Does the partial scalar model (3c) fit *worse* than the partial metric model (2b)?** Eh, −2ΔLL(df=7) = 13.99, $p$ =.05

Although fit is close to not worse than the partial metric model, there is a significant modification index for v2T1, suggesting localized strain. So let's see what happens if we free that one, too.

```
MODEL FIT INFORMATION
Number of Free Parameters                       59

Loglikelihood
          H0 Value                       -4442.214
          H0 Scaling Correction Factor      1.5647
            for MLR
          H1 Value                       -4284.045
          H1 Scaling Correction Factor      1.2029
            for MLR

Information Criteria
          Akaike (AIC)                    9002.427
          Bayesian (BIC)                  9180.447
          Sample-Size Adjusted BIC        8993.718
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            304.537*
          Degrees of Freedom                   130
          P-Value                           0.0000
          Scaling Correction Factor         1.0387
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.094
          90 Percent C.I.                    0.081  0.108
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.900
          TLI                                0.882

SRMR (Standardized Root Mean Square Residual)
          Value                              0.092

MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds

                    M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
[ V2T1     ]       8.560     -0.497      -0.497        -0.137
```

## Model 3d. Partial Scalar Invariance Model (all intercepts held equal across over time except v1T1, v5T1, v4T1, v2T1)

```
MODEL: ! Model 3d: Partial Scalar Invariance,
        ! no v1T1 v5T1 v4T1 v2T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1 v2T1
  [v1T1*]; [v1T2* v1T3*] (I1);
  [v2T1*]; [v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4);
  [v5T1*]; [v5T2* v5T3*] (I5);
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```




```
MODEL FIT INFORMATION
Number of Free Parameters                        60

Loglikelihood
          H0 Value                        -4437.665
          H0 Scaling Correction Factor       1.5560
            for MLR
          H1 Value                        -4284.045
          H1 Scaling Correction Factor       1.2029
            for MLR

Information Criteria
          Akaike (AIC)                     8995.330
          Bayesian (BIC)                   9176.366
          Sample-Size Adjusted BIC         8986.473
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                             295.789*
          Degrees of Freedom                    129
          P-Value                            0.0000
          Scaling Correction Factor          1.0387
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.093
          90 Percent C.I.                     0.079  0.106
          Probability RMSEA <= .05            0.000
CFI/TLI
          CFI                                 0.904
          TLI                                 0.887

Chi-Square Test of Model Fit for the Baseline Model
          Value                            1896.788
          Degrees of Freedom                    153
          P-Value                            0.0000
SRMR (Standardized Root Mean Square Residual)
          Value                               0.091
```

**Does the partial scalar model (3d) fit *better* than the partial scalar model (3c)?** Yes, −2ΔLL(df=1) = 8.73, $p$ <.01

**Does the partial scalar model (3d) fit *worse* than the partial metric model (2b)?** No, −2ΔLL(df=6) = 4.35, $p$ =.63

No invariance-related modification indices remain, so we are done! The intercepts at the end of Model 3d are shown on the left.

**3d UNSTANDARDIZED PARTIAL SCALAR MODEL RESULTS**

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| TIME1 BY | | | | |
| **V1T1** | **3.231** | **0.262** | **12.330** | **0.000** |
| V2T1 | 1.953 | 0.201 | 9.739 | 0.000 |
| V3T1 | 1.974 | 0.198 | 9.989 | 0.000 |
| V4T1 | 1.904 | 0.220 | 8.656 | 0.000 |
| V5T1 | 0.983 | 0.138 | 7.097 | 0.000 |
| V6T1 | 1.477 | 0.130 | 11.353 | 0.000 |
| TIME2 BY | | | | |
| V1T2 | 2.629 | 0.232 | 11.317 | 0.000 |
| V2T2 | 1.953 | 0.201 | 9.739 | 0.000 |
| V3T2 | 1.974 | 0.198 | 9.989 | 0.000 |
| V4T2 | 1.904 | 0.220 | 8.656 | 0.000 |
| V5T2 | 0.983 | 0.138 | 7.097 | 0.000 |
| V6T2 | 1.477 | 0.130 | 11.353 | 0.000 |
| TIME3 BY | | | | |
| V1T3 | 2.629 | 0.232 | 11.317 | 0.000 |
| V2T3 | 1.953 | 0.201 | 9.739 | 0.000 |
| V3T3 | 1.974 | 0.198 | 9.989 | 0.000 |
| V4T3 | 1.904 | 0.220 | 8.656 | 0.000 |
| V5T3 | 0.983 | 0.138 | 7.097 | 0.000 |
| V6T3 | 1.477 | 0.130 | 11.353 | 0.000 |
| TIME1 WITH | | | | |
| TIME2 | 0.850 | 0.079 | 10.809 | 0.000 |
| TIME3 | 0.686 | 0.124 | 5.543 | 0.000 |
| TIME2 WITH | | | | |
| TIME3 | 0.706 | 0.128 | 5.519 | 0.000 |
| ***** Residual covariances among same item across time ****** | | | | |
| V1T1 WITH | | | | |
| V1T2 | -0.206 | 0.246 | -0.838 | 0.402 |
| V1T3 | -0.010 | 0.233 | -0.043 | 0.966 |
| V1T2 WITH | | | | |
| V1T3 | 0.130 | 0.231 | 0.561 | 0.575 |
| ........... | | | | |
| Variances (FACTOR VARIANCE AT TIME1=1 FOR IDENTIFICATION) | | | | |
| TIME1 | 1.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 1.167 | 0.187 | 6.252 | 0.000 |
| TIME3 | 0.947 | 0.156 | 6.054 | 0.000 |

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| Means (FACTOR MEAN AT TIME1 FIXED=0 FOR IDENTIFICATION) | | | | |
| TIME1 | 0.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 0.293 | 0.081 | 3.625 | 0.000 |
| TIME3 | 0.521 | 0.093 | 5.612 | 0.000 |
| Intercepts | | | | |
| **V1T1** | **16.090** | **0.274** | **58.684** | **0.000** |
| V1T2 | 16.425 | 0.281 | 58.364 | 0.000 |
| V1T3 | 16.425 | 0.281 | 58.364 | 0.000 |
| **V2T1** | **8.674** | **0.294** | **29.540** | **0.000** |
| V2T2 | 9.413 | 0.261 | 36.036 | 0.000 |
| V2T3 | 9.413 | 0.261 | 36.036 | 0.000 |
| V3T1 | 11.950 | 0.225 | 53.099 | 0.000 |
| V3T2 | 11.950 | 0.225 | 53.099 | 0.000 |
| V3T3 | 11.950 | 0.225 | 53.099 | 0.000 |
| **V4T1** | **-3.024** | **0.267** | **-11.334** | **0.000** |
| V4T2 | -3.744 | 0.299 | -12.535 | 0.000 |
| V4T3 | -3.744 | 0.299 | -12.535 | 0.000 |
| **V5T1** | **-1.215** | **0.131** | **-9.277** | **0.000** |
| V5T2 | -1.802 | 0.207 | -8.706 | 0.000 |
| V5T3 | -1.802 | 0.207 | -8.706 | 0.000 |
| V6T1 | -2.854 | 0.161 | -17.688 | 0.000 |
| V6T2 | -2.854 | 0.161 | -17.688 | 0.000 |
| V6T3 | -2.854 | 0.161 | -17.688 | 0.000 |
| Residual Variances (ITEM VARIANCE THAT IS NOT THE FACTOR) | | | | |
| V1T1 | 0.170 | 0.374 | 0.454 | 0.650 |
| V1T2 | 0.548 | 0.265 | 2.070 | 0.038 |
| V1T3 | 0.509 | 0.314 | 1.618 | 0.106 |
| V2T1 | 8.702 | 1.026 | 8.483 | 0.000 |
| V2T2 | 5.895 | 0.605 | 9.746 | 0.000 |
| V2T3 | 5.177 | 0.795 | 6.514 | 0.000 |
| V3T1 | 2.502 | 0.386 | 6.484 | 0.000 |
| V3T2 | 2.178 | 0.352 | 6.183 | 0.000 |
| V3T3 | 2.309 | 0.416 | 5.548 | 0.000 |
| V4T1 | 7.172 | 1.021 | 7.021 | 0.000 |
| V4T2 | 6.759 | 0.967 | 6.990 | 0.000 |
| V4T3 | 6.613 | 1.128 | 5.860 | 0.000 |
| V5T1 | 1.829 | 0.443 | 4.131 | 0.000 |
| V5T2 | 4.678 | 1.430 | 3.272 | 0.001 |
| V5T3 | 2.944 | 0.760 | 3.872 | 0.000 |
| V6T1 | 1.707 | 0.242 | 7.059 | 0.000 |
| V6T2 | 1.090 | 0.165 | 6.599 | 0.000 |
| V6T3 | 0.784 | 0.170 | 4.618 | 0.000 |

## Model 4a. Residual Variance Invariance Model (error variances held equal for all except non-invariant items)

```
MODEL: ! Model 4a: Residual Variances
       ! except for non-invariant items

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1 v2T1
  [v1T1*]; [v1T2* v1T3*] (I1);
  [v2T1*]; [v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4);
  [v5T1*]; [v5T2* v5T3*] (I5);
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances NOW CONSTRAINED EQUAL ACROSS TIME
  (WHEN POSSIBLE)
  v1T1*; v1T2* v1T3* (E1);
  v2T1*; v2T2* v2T3* (E2);
  v3T1*  v3T2* v3T3* (E3);
  v4T1*; v4T2* v4T3* (E4);
  v5T1*; v5T2* v5T3* (E5);
  v6T1*  v6T2* v6T3* (E6);
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

**Does the full residual model (4a) fit *worse* than the partial scalar model (3d)?** Yes, $-2\Delta LL(df=8) = 24.72$, $p < .01$

Modification indices suggest freeing v5 across Time2 and Time3, so let's try that next.

```
MODEL FIT INFORMATION
Number of Free Parameters                        52

Loglikelihood
          H0 Value                        -4454.592
          H0 Scaling Correction Factor       1.5487
            for MLR
          H1 Value                        -4284.045
          H1 Scaling Correction Factor       1.2029
            for MLR

Information Criteria
          Akaike (AIC)                     9013.185
          Bayesian (BIC)                   9170.083
          Sample-Size Adjusted BIC         9005.509
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                             318.280*
          Degrees of Freedom                    137
          P-Value                            0.0000
          Scaling Correction Factor          1.0717
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.094
          90 Percent C.I.                     0.080  0.107
          Probability RMSEA <= .05            0.000

CFI/TLI
          CFI                                 0.896
          TLI                                 0.884

SRMR (Standardized Root Mean Square Residual)
          Value                               0.095

MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds

                    M.I.    E.P.C.  Std E.P.C.  StdYX E.P.C.
Variances/Residual Variances

V5T2               12.739     0.755      0.755         0.153
V5T3               12.740    -1.125     -1.125        -0.238
V6T1               13.740     0.421      0.421         0.124
V6T3                7.815    -0.393     -0.393        -0.124
```

**Model 4b. Partial Residual Variance Invariance Model (error variances held equal for all except non-invariant items and v5T2/T3)**

```
MODEL: ! Model 4b: Residual Variances
       ! except for non-invariant items, v5T2-v5T3

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1 v2T1
  [v1T1*]; [v1T2* v1T3*] (I1);
  [v2T1*]; [v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4);
  [v5T1*]; [v5T2* v5T3*] (I5);
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances NOW CONSTRAINED EQUAL ACROSS TIME
  (WHEN POSSIBLE) except v5T2-v5T3
  v1T1*; v1T2* v1T3* (E1);
  v2T1*; v2T2* v2T3* (E2);
  v3T1*  v3T2* v3T3* (E3);
  v4T1*; v4T2* v4T3* (E4);
  v5T1*; v5T2*; v5T3*;
  v6T1*  v6T2* v6T3* (E6);
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

**Does the partial residual model (4b) fit *better* than the full residual model (4a)?** Yes, −2ΔLL(df=1) = 10.06, $p$ <.01

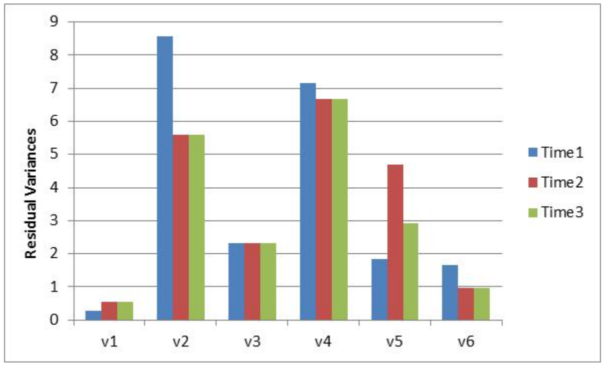
**Does the partial residual model (4b) fit *worse* than the partial scalar model (3d)?** Eh, −2ΔLL(df=7) = 14.14, $p$ =.05

Modification indices suggest freeing v6 from Time1, so let's try that next.

```
MODEL FIT INFORMATION
Number of Free Parameters                       53

Loglikelihood
          H0 Value                       -4447.259
          H0 Scaling Correction Factor      1.5823
            for MLR
          H1 Value                       -4284.045
          H1 Scaling Correction Factor      1.2029
            for MLR

Information Criteria
          Akaike (AIC)                    9000.518
          Bayesian (BIC)                  9160.434
          Sample-Size Adjusted BIC        8992.694
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            309.384*
          Degrees of Freedom                   136
          P-Value                           0.0000
          Scaling Correction Factor         1.0551
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.092
          90 Percent C.I.                    0.078  0.105
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.901
          TLI                                0.888

SRMR (Standardized Root Mean Square Residual)
          Value                              0.093

MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds

                    M.I.    E.P.C.  Std E.P.C.  StdYX E.P.C.
Variances/Residual Variances

V6T1               13.772    0.419      0.419        0.125
V6T3                7.149   -0.373     -0.373       -0.118
```

## Model 4c. Partial Residual Variance Invariance Model (error variances held equal for all except non-invariant items, v5T2/T3, v6T1)

```
MODEL: ! Model 4c: Residual Variances
       ! except for non-invariant items, v5T2-v5T3, v6T1
! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1 v2T1
  [v1T1*]; [v1T2* v1T3*] (I1);
  [v2T1*]; [v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4);
  [v5T1*]; [v5T2* v5T3*] (I5);
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances NOW CONSTRAINED EQUAL ACROSS TIME
  (WHEN POSSIBLE) except v5T2-v5T3, v6T1
  v1T1*; v1T2* v1T3* (E1);
  v2T1*; v2T2* v2T3* (E2);
  v3T1*  v3T2* v3T3* (E3);
  v4T1*; v4T2* v4T3* (E4);
  v5T1*; v5T2*; v5T3*;
  v6T1*; v6T2* v6T3* (E6);
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```




```
MODEL FIT INFORMATION
Number of Free Parameters                        54

Loglikelihood
          H0 Value                        -4439.971
          H0 Scaling Correction Factor       1.5771
            for MLR
          H1 Value                        -4284.045
          H1 Scaling Correction Factor       1.2029
            for MLR

Information Criteria
          Akaike (AIC)                     8987.942
          Bayesian (BIC)                   9150.876
          Sample-Size Adjusted BIC         8979.971
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                             296.084*
          Degrees of Freedom                    135
          P-Value                            0.0000
          Scaling Correction Factor          1.0533
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.089
          90 Percent C.I.                     0.075  0.103
          Probability RMSEA <= .05            0.000

CFI/TLI
          CFI                                 0.908
          TLI                                 0.895

SRMR (Standardized Root Mean Square Residual)
          Value                               0.092
```

**Does the partial residual model (4c) fit *better* than the partial residual model (4b)?** Yes, $-2\Delta LL(df=1) = 11.20$, $p < .01$

**Does the partial residual model (4c) fit *worse* than the partial scalar model (3d)?** No, $-2\Delta LL(df=6) = 3.38$, $p = .76$

No invariance-related modification indices remain, so we are done! The residual variances at the end of Model 4c are shown on the left. Next is structural invariance.

**4c UNSTANDARDIZED FINAL MEASUREMENT INVARIANCE SOLUTION**

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| TIME1 BY | | | | |
| V1T1 | 3.214 | 0.259 | 12.409 | 0.000 |
| V2T1 | 1.945 | 0.200 | 9.735 | 0.000 |
| V3T1 | 1.983 | 0.196 | 10.094 | 0.000 |
| V4T1 | 1.913 | 0.219 | 8.741 | 0.000 |
| V5T1 | 0.987 | 0.138 | 7.154 | 0.000 |
| V6T1 | 1.470 | 0.123 | 11.975 | 0.000 |
| TIME2 BY | | | | |
| V1T2 | 2.644 | 0.230 | 11.473 | 0.000 |
| V2T2 | 1.945 | 0.200 | 9.735 | 0.000 |
| V3T2 | 1.983 | 0.196 | 10.094 | 0.000 |
| V4T2 | 1.913 | 0.219 | 8.741 | 0.000 |
| V5T2 | 0.987 | 0.138 | 7.154 | 0.000 |
| V6T2 | 1.470 | 0.123 | 11.975 | 0.000 |
| TIME3 BY | | | | |
| V1T3 | 2.644 | 0.230 | 11.473 | 0.000 |
| V2T3 | 1.945 | 0.200 | 9.735 | 0.000 |
| V3T3 | 1.983 | 0.196 | 10.094 | 0.000 |
| V4T3 | 1.913 | 0.219 | 8.741 | 0.000 |
| V5T3 | 0.987 | 0.138 | 7.154 | 0.000 |
| V6T3 | 1.470 | 0.123 | 11.975 | 0.000 |
| TIME1 WITH | | | | |
| TIME2 | 0.843 | 0.078 | 10.745 | 0.000 |
| TIME3 | 0.683 | 0.124 | 5.505 | 0.000 |
| TIME2 WITH | | | | |
| TIME3 | 0.692 | 0.126 | 5.489 | 0.000 |
| ***** Residual covariances among same item across time ****** | | | | |
| V1T1 WITH | | | | |
| V1T2 | -0.165 | 0.230 | -0.716 | 0.474 |
| V1T3 | 0.014 | 0.212 | 0.066 | 0.948 |
| V1T2 WITH | | | | |
| V1T3 | 0.153 | 0.230 | 0.667 | 0.505 |
| ........... | | | | |
| Variances (FACTOR VARIANCE AT TIME1=1 FOR IDENTIFICATION) | | | | |
| TIME1 | 1.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 1.159 | 0.186 | 6.231 | 0.000 |
| TIME3 | 0.934 | 0.151 | 6.171 | 0.000 |

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| Means (FACTOR MEAN AT TIME1 FIXED=0 FOR IDENTIFICATION) | | | | |
| TIME1 | 0.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 0.295 | 0.081 | 3.654 | 0.000 |
| TIME3 | 0.520 | 0.092 | 5.668 | 0.000 |
| Intercepts - V3 AND V6 ARE HOLDING THIS TOGETHER WITH TIME1 | | | | |
| **V1T1** | **16.089** | **0.275** | **58.597** | **0.000** |
| V1T2 | 16.418 | 0.283 | 58.056 | 0.000 |
| V1T3 | 16.418 | 0.283 | 58.056 | 0.000 |
| **V2T1** | **8.675** | **0.294** | **29.523** | **0.000** |
| V2T2 | 9.416 | 0.262 | 35.991 | 0.000 |
| V2T3 | 9.416 | 0.262 | 35.991 | 0.000 |
| V3T1 | 11.950 | 0.225 | 53.170 | 0.000 |
| V3T2 | 11.950 | 0.225 | 53.170 | 0.000 |
| V3T3 | 11.950 | 0.225 | 53.170 | 0.000 |
| **V4T1** | **-3.024** | **0.266** | **-11.352** | **0.000** |
| V4T2 | -3.750 | 0.298 | -12.565 | 0.000 |
| V4T3 | -3.750 | 0.298 | -12.565 | 0.000 |
| **V5T1** | **-1.213** | **0.131** | **-9.275** | **0.000** |
| V5T2 | -1.803 | 0.207 | -8.720 | 0.000 |
| V5T3 | -1.803 | 0.207 | -8.720 | 0.000 |
| V6T1 | -2.851 | 0.160 | -17.815 | 0.000 |
| V6T2 | -2.851 | 0.160 | -17.815 | 0.000 |
| V6T3 | -2.851 | 0.160 | -17.815 | 0.000 |
| Residual Variances - ITEM VARIANCE THAT IS NOT THE FACTOR | | | | |
| **V1T1** | **0.285** | **0.342** | **0.831** | **0.406** |
| V1T2 | 0.539 | 0.233 | 2.316 | 0.021 |
| V1T3 | 0.539 | 0.233 | 2.316 | 0.021 |
| **V2T1** | **8.562** | **1.004** | **8.526** | **0.000** |
| V2T2 | 5.592 | 0.502 | 11.132 | 0.000 |
| V2T3 | 5.592 | 0.502 | 11.132 | 0.000 |
| V3T1 | 2.312 | 0.271 | 8.534 | 0.000 |
| V3T2 | 2.312 | 0.271 | 8.534 | 0.000 |
| V3T3 | 2.312 | 0.271 | 8.534 | 0.000 |
| **V4T1** | **7.139** | **1.043** | **6.842** | **0.000** |
| V4T2 | 6.686 | 0.870 | 7.684 | 0.000 |
| V4T3 | 6.686 | 0.870 | 7.684 | 0.000 |
| **V5T1** | **1.829** | **0.448** | **4.078** | **0.000** |
| **V5T2** | **4.705** | **1.455** | **3.233** | **0.001** |
| **V5T3** | **2.908** | **0.749** | **3.881** | **0.000** |
| **V6T1** | **1.664** | **0.233** | **7.138** | **0.000** |
| V6T2 | 0.957 | 0.136 | 7.039 | 0.000 |
| V6T3 | 0.957 | 0.136 | 7.039 | 0.000 |

# STRUCTURAL INVARIANCE TESTS

## Model 5a. Factor Variance Invariance Model

```
MODEL: ! Model 5a: Factor Variance Invariance
(rest of code before and after is same as 4c)

! Model 5a: Factor Variance Invariance (all fixed to 1 now)
    Time1@1 Time2@1 Time3@1;

MODEL FIT INFORMATION
Number of Free Parameters                        52
Loglikelihood
          H0 Value                        -4441.238
          H0 Scaling Correction Factor       1.5848
            for MLR
          H1 Value                        -4284.045
          H1 Scaling Correction Factor       1.2029
            for MLR
Information Criteria
          Akaike (AIC)                     8986.475
          Bayesian (BIC)                   9143.374
          Sample-Size Adjusted BIC         8978.799
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                             297.152*
          Degrees of Freedom                    137
          P-Value                            0.0000
          Scaling Correction Factor          1.0580
            for MLR
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.088
          90 Percent C.I.                     0.074  0.102
          Probability RMSEA <= .05            0.000
CFI/TLI
          CFI                                 0.908
          TLI                                 0.897
SRMR (Standardized Root Mean Square Residual)
          Value                               0.100
```

**Does the factor variance model (5a) fit *worse* than the partial residual model (4c)?** No, $-2\Delta LL(df=2) = 1.84$, $p = .40$

```
Factor Covariances...
TIME1    WITH
    TIME2              0.778      0.042     18.375      0.000
    TIME3              0.713      0.087      8.214      0.000
 TIME2    WITH
    TIME3              0.662      0.095      6.929      0.000
```

## Model 6a. Factor Covariance Invariance Model

```
MODEL: ! Model 6a: Factor Covariance Invariance
(rest of code before and after is same as 5a)

! Model 6a: Factor Covariance Invariance (all fixed equal)
Time1 Time2 Time3 WITH Time1* Time2* Time3* (Fcov);

MODEL FIT INFORMATION
Number of Free Parameters                        50
Loglikelihood
          H0 Value                        -4443.654
          H0 Scaling Correction Factor       1.5649
            for MLR
          H1 Value                        -4284.045
          H1 Scaling Correction Factor       1.2029
            for MLR
Information Criteria
          Akaike (AIC)                     8987.308
          Bayesian (BIC)                   9138.172
          Sample-Size Adjusted BIC         8979.927
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                             297.568*
          Degrees of Freedom                    139
          P-Value                            0.0000
          Scaling Correction Factor          1.0728
            for MLR
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.087
          90 Percent C.I.                     0.073  0.101
          Probability RMSEA <= .05            0.000
CFI/TLI
          CFI                                 0.909
          TLI                                 0.900
SRMR (Standardized Root Mean Square Residual)
          Value                               0.100
```

**Does the factor covariance model (6a) fit *worse* than the factor variance model (5a)?** No, $-2\Delta LL(df=2) = 2.32$, $p = .31$

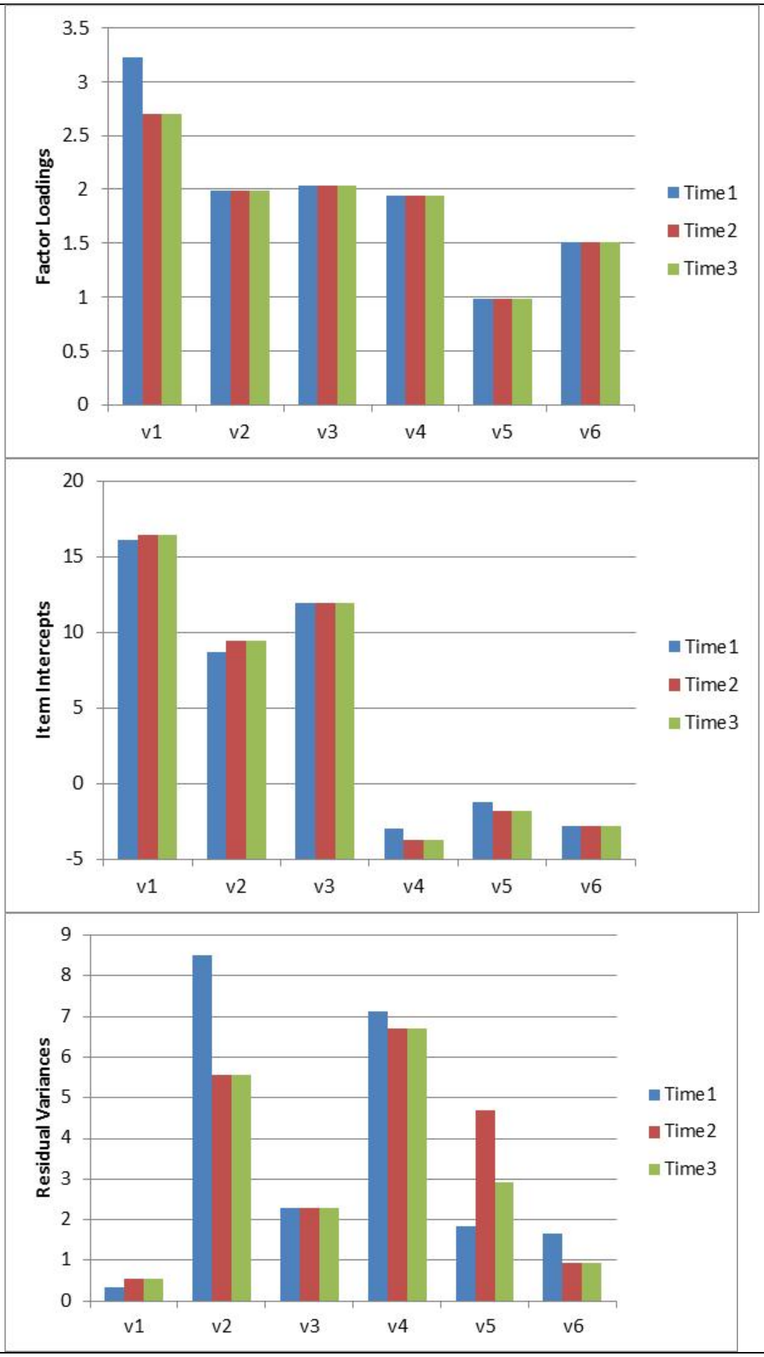
```
FACTOR COVARIANCES FROM MODEL 6a (REPRESENT CORRELATIONS):
TIME1 WITH TIME2   0.724      0.053      13.748      0.000
TIME1 WITH TIME3   0.724      0.053      13.748      0.000
TIME2 WITH TIME3   0.724      0.053      13.748      0.000
FACTOR MEANS FROM MODEL 6a (REPRESENT MEAN DIFFERENCES):
    TIME1          0.000      0.000     999.000    999.000
    TIME2          0.284      0.079       3.605      0.000
    TIME3          0.520      0.091       5.700      0.000
```

## Model 7a. Factor Mean Invariance Model

```
MODEL: ! Model 7a: Factor Mean Invariance
       ! Testing Diff between Time2 and Time3
! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1
  [v1T1*]; [v1T2* v1T3*] (I1);
  [v2T1*    v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4);
  [v5T1*]; [v5T2* v5T3*] (I5);
  [v6T1* v6T2* v6T3*] (I6);
! Residual variances NOW CONSTRAINED EQUAL ACROSS TIME
  (WHEN POSSIBLE) except v5T2-v5T3, v6T1
  v1T1*; v1T2* v1T3* (E1);
  v2T1*; v2T2* v2T3* (E2);
  v3T1*  v3T2* v3T3* (E3);
  v4T1*; v4T2* v4T3* (E4);
  v5T1*; v5T2*; v5T3*;
  v6T1*; v6T2* v6T3* (E6);
! Factor variance fixed=1 for structural invariance
  Time1@1 Time2@1 Time3@1;
! Testing factor mean difference between Time2 and Time3
  [Time1@0]; [Time2* Time3*] (Fmean);
! Factor covariances held equal for structural invariance
  Time1 Time2 Time3 WITH Time1* Time2* Time3* (Fcov);
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

**Does the factor mean model (7a) fit *worse* than the factor covariance model (6a)?** Yes, $-2\Delta LL(df=1) = 11.15$, $p < .01$, so we keep Model 6a instead.

```
MODEL FIT INFORMATION
Number of Free Parameters                      49
Loglikelihood
          H0 Value                      -4448.472
          H0 Scaling Correction Factor     1.5792
            for MLR
Means
    TIME1              0.000      0.000    999.000    999.000
    TIME2              0.378      0.075      5.014      0.000
    TIME3              0.378      0.075      5.014      0.000
```



**6a UNSTANDARDIZED FINAL STRUCTURAL INVARIANCE SOLUTION**

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| TIME1 BY | | | | |
| V1T1 | 3.229 | 0.243 | 13.272 | 0.000 |
| V2T1 | 1.993 | 0.170 | 11.754 | 0.000 |
| V3T1 | 2.029 | 0.169 | 12.022 | 0.000 |
| V4T1 | 1.939 | 0.214 | 9.077 | 0.000 |
| V5T1 | 0.986 | 0.147 | 6.701 | 0.000 |
| V6T1 | 1.508 | 0.109 | 13.821 | 0.000 |
| TIME2 BY | | | | |
| V1T2 | 2.704 | 0.232 | 11.677 | 0.000 |
| V2T2 | 1.993 | 0.170 | 11.754 | 0.000 |
| V3T2 | 2.029 | 0.169 | 12.022 | 0.000 |
| V4T2 | 1.939 | 0.214 | 9.077 | 0.000 |
| V5T2 | 0.986 | 0.147 | 6.701 | 0.000 |
| V6T2 | 1.508 | 0.109 | 13.821 | 0.000 |
| TIME3 BY | | | | |
| V1T3 | 2.704 | 0.232 | 11.677 | 0.000 |
| V2T3 | 1.993 | 0.170 | 11.754 | 0.000 |
| V3T3 | 2.029 | 0.169 | 12.022 | 0.000 |
| V4T3 | 1.939 | 0.214 | 9.077 | 0.000 |
| V5T3 | 0.986 | 0.147 | 6.701 | 0.000 |
| V6T3 | 1.508 | 0.109 | 13.821 | 0.000 |
| TIME1 WITH | | | | |
| TIME2 | 0.724 | 0.053 | 13.748 | 0.000 |
| TIME3 | 0.724 | 0.053 | 13.748 | 0.000 |
| TIME2 WITH | | | | |
| TIME3 | 0.724 | 0.053 | 13.748 | 0.000 |
| ***** Residual covariances among same item across time ****** | | | | |
| V1T1 WITH | | | | |
| V1T2 | -0.106 | 0.225 | -0.471 | 0.638 |
| V1T3 | 0.038 | 0.215 | 0.175 | 0.861 |
| V1T2 WITH | | | | |
| V1T3 | 0.130 | 0.243 | 0.534 | |
| 0.593........... | | | | |
| Variances (FACTOR VARIANCES CONSTRAINED EQUAL) | | | | |
| TIME1 | 1.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 1.000 | 0.000 | 999.000 | 999.000 |
| TIME3 | 1.000 | 0.000 | 999.000 | 999.000 |

| | Estimate | S.E. | Est./S.E. | Two-Tailed P-Value |
|---|---|---|---|---|
| Means (FACTOR MEAN AT TIME1 FIXED=0 FOR IDENTIFICATION) | | | | |
| TIME1 | 0.000 | 0.000 | 999.000 | 999.000 |
| TIME2 | 0.284 | 0.079 | 3.605 | 0.000 |
| TIME3 | 0.520 | 0.091 | 5.700 | 0.000 |
| Intercepts – V3 AND V6 ARE HOLDING THIS TOGETHER WITH TIME1 | | | | |
| **V1T1** | **16.099** | **0.271** | **59.420** | **0.000** |
| V1T2 | 16.428 | 0.281 | 58.488 | 0.000 |
| V1T3 | 16.428 | 0.281 | 58.488 | 0.000 |
| **V2T1** | **8.681** | **0.292** | **29.694** | **0.000** |
| V2T2 | 9.423 | 0.259 | 36.368 | 0.000 |
| V2T3 | 9.423 | 0.259 | 36.368 | 0.000 |
| V3T1 | 11.956 | 0.223 | 53.706 | 0.000 |
| V3T2 | 11.956 | 0.223 | 53.706 | 0.000 |
| V3T3 | 11.956 | 0.223 | 53.706 | 0.000 |
| **V4T1** | **-3.018** | **0.263** | **-11.463** | **0.000** |
| V4T2 | -3.737 | 0.292 | -12.784 | 0.000 |
| V4T3 | -3.737 | 0.292 | -12.784 | 0.000 |
| **V5T1** | **-1.210** | **0.131** | **-9.269** | **0.000** |
| V5T2 | -1.791 | 0.203 | -8.807 | 0.000 |
| V5T3 | -1.791 | 0.203 | -8.807 | 0.000 |
| V6T1 | -2.847 | 0.159 | -17.889 | 0.000 |
| V6T2 | -2.847 | 0.159 | -17.889 | 0.000 |
| V6T3 | -2.847 | 0.159 | -17.889 | 0.000 |
| Residual Variances – ITEM VARIANCE THAT IS NOT THE FACTOR | | | | |
| **V1T1** | **0.351** | **0.331** | **1.060** | **0.289** |
| V1T2 | 0.562 | 0.231 | 2.432 | 0.015 |
| V1T3 | 0.562 | 0.231 | 2.432 | 0.015 |
| **V2T1** | **8.506** | **0.999** | **8.511** | **0.000** |
| V2T2 | 5.563 | 0.494 | 11.261 | 0.000 |
| V2T3 | 5.563 | 0.494 | 11.261 | 0.000 |
| V3T1 | 2.288 | 0.269 | 8.507 | 0.000 |
| V3T2 | 2.288 | 0.269 | 8.507 | 0.000 |
| V3T3 | 2.288 | 0.269 | 8.507 | 0.000 |
| **V4T1** | **7.134** | **1.041** | **6.853** | **0.000** |
| V4T2 | 6.694 | 0.873 | 7.666 | 0.000 |
| V4T3 | 6.694 | 0.873 | 7.666 | 0.000 |
| **V5T1** | **1.825** | **0.446** | **4.092** | **0.000** |
| **V5T2** | **4.705** | **1.454** | **3.235** | **0.001** |
| **V5T3** | **2.921** | **0.752** | **3.887** | **0.000** |
| **V6T1** | **1.656** | **0.235** | **7.054** | **0.000** |
| V6T2 | 0.942 | 0.131 | 7.188 | 0.000 |
| V6T3 | 0.942 | 0.131 | 7.188 | 0.000 |

**Example write-up for these analyses:**

The extent to which a confirmatory factor model measuring social functioning (with six observed indicators) exhibited measurement invariance and structural invariance over time (i.e., across three occasions taken at 6-month intervals) was examined using Mplus v. 7.4 (Muthén & Muthén, 1998–2015). Robust maximum likelihood (MLR) estimation was used for all analyses; accordingly, nested model comparisons were conducted using the rescaled difference in the model −2LL values as a function of the difference in model degrees of freedom. A configural invariance model was initially specified in which three correlated factors (i.e., the factor at three occasions) were estimated simultaneously; all factor means were fixed to 0 and all factor variances were fixed to 1 for identification. Residual covariances between the same indicators across occasions were estimated as well. As shown in Table 1, although the configural invariance model had marginal fit, reasonable attempts to improve the fit were unsuccessful. Thus, the analysis proceeded by applying parameter constraints in successive models to examine potential decreases in fit resulting from measurement or structural non-invariance over the three occasions.

Equality of the unstandardized indicator factor loadings across occasions was then examined in a metric invariance model. The factor variance was fixed to 1 at time 1 but was freely estimated at times 2 and 3. All factor loadings were constrained equal across time; all intercepts and residual variances were still permitted to vary across time. Factor covariances and residual covariances were estimated as described previously. The metric invariance model fit significantly worse than the configural invariance model $-2\Delta LL(10) = 19.14$, $p = .04$. The modification indices suggested that the loading of indicator 1 at time 1 was a source of misfit and should be freed. After doing so, the partial metric invariance model fit significantly better than the full metric invariance model, $-2\Delta LL(1) = 7.16$, $p < .001$, and the partial metric invariance model did not fit worse than the configural invariance model$-2\Delta LL(9) = 8.98$, $p = .44$. The fact that partial metric invariance (i.e., "weak invariance") held indicates that the indicators were related to the latent factor equivalently across time, or more simply, that the same latent factor was being measured at each of occasion (with the exception of indicator 1, which was more related to the factor at time 1 than at times 2 or 3).

Equality of the unstandardized indicator intercepts across time was then examined in a scalar invariance model. The factor mean and variance were fixed to 0 and 1, respectively, at time 1 for identification, but the factor mean and variance were then estimated at times 2 and 3. All factor loadings and indicator intercepts were constrained equal across time (except for indicator 1 at time 1); all residual variances were still permitted to differ across time. Factor covariances and residual covariances were estimated as described previously. The scalar invariance model fit significantly worse than the partial metric invariance model, $-2\Delta LL(9) = 55.13$, $p < .01$. The modification indices suggested that the intercept of indicator 5 at time 1 was the largest source of the misfit and should be freed. After doing so, although the partial scalar invariance model had significantly better fit than the full scalar invariance model, $-2\Delta LL(1) = 15.16$, $p < .01$, it still fit worse than the partial metric invariance model, $-2\Delta LL(8) = 27.84$, $p <$ 001. The modification indices suggested that the intercept of indicator 4 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, although the new partial scalar invariance model (with the intercepts for indicators 1, 4, and 5 freed at time 1) fit significantly better than the previous partial scalar invariance model (without the intercept for indicator 4 freed at time 1), $-2\Delta LL(1) = 9.24$, $p < .01$, it still fit marginally worse than the partial metric invariance model, $-2\Delta LL(7) = 13.99$, $p =$ 05. The modification indices suggested that the intercept of indicator 2 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model (with the intercepts for indicators 1, 2, 4 and 5 freed at time 1) fit significantly better than the previous partial scalar invariance model (without the intercept for indicator 2 freed at time 1), $-2\Delta LL(1) = 8.73$, $p < .01$, and it did not fit significantly worse than the partial metric invariance model, $-2\Delta LL(6) = 4.35$, $p = .63$. The fact that partial scalar invariance (i.e., "strong invariance") held indicates that times 2 and 3 have the same expected response for each indicator at the same absolute level of the trait, or more simply, that the observed differences in the indicator means between times 2 and 3 is due to factor mean differences only. However, indicators 1 and 2 had a lower expected indicator response at the same absolute level of social functioning at time 1 than at time 2 or 3, while indicators 4 and 5 had a higher expected response.

Equality of the unstandardized residual variances across time was then examined in a residual variance invariance model. As in the partial scalar invariance model, the factor mean and variance were fixed to 0 and 1, respectively, for identification at time 1, but the factor mean and variance were still estimated at times 2 and 3. All factor loadings (except for indicator 1 at time 1), item intercepts (except for indicators 1, 2, 4, and 5 at time 1), and all residual variances (except for indicators 1, 2, 4, and 5 at time 1) were constrained to be equal across groups. Factor covariances and residual covariances were estimated as described previously. The residual variance invariance model fit significantly worse than the last partial scalar invariance model, $-2\Delta LL(8) = 24.72$, $p < .01$. The modification indices suggested that the residual variance of indicator 5 at time 2 versus time 3 was the largest remaining source of the misfit and should be freed. After doing so, the partial residual variance invariance model fit significantly better than the residual invariance model, $-2\Delta LL(1) = 10.06$, $p < .01$, but still fit marginally worse than the last partial scalar invariance model, $-2\Delta LL(7) = 14.14$, $p = .05$. The modification indices suggested that the residual variance of indicator 6 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial residual variance invariance model (with residual variances for indicators 1, 2, 4, 5, and 6 free at time 1; indicator 5 free at times 2 and 3 also) fit significantly better than the partial residual invariance model (without the residual variance for indicator 6 at time 1 freed), $-2\Delta LL(1) = 11.20$, $p < .01$, and did not fit worse than the last partial scalar invariance model, $-2\Delta LL(6) = 3.38$, $p = .76$. The fact that partial residual variance invariance (i.e., "strict invariance") held indicates that the amount of indicator variance not accounted for by the factor was the same across times 2 and 3 (except for indicator 5, for which there was more residual variance at time 2). However, 5 out of 6 indicators did not have residual variance invariance at time 1 (although this was required because of a lack of metric or scalar invariance for indicators 1, 2, 4, and 5).

After achieving partial measurement invariance as was just described, structural invariance was then tested with three additional models. First, the factor variance at times 2 and 3 (which had been estimated freely) was constrained to 1 (i.e., to be equal to the factor variance at time 1), resulting in a nonsignificant decrease in fit relative to the last partial residual invariance model, $-2\Delta LL(2) = 1.84$, $p = .40$. Thus, equivalent amounts of individual differences in social functioning were found across time. Second, the factor covariances across time were constrained to be equal (which become factor correlations given a variance of 1 for each factor across time), resulting in a nonsignificant decrease in fit relative to the factor variance invariance model, $-2\Delta LL(2) = 2.32$, $p = .31$. Third, the factor means at times 2 and 3 (which had been estimated freely) was constrained to be equal to each other, resulting in a significant decrease in fit relative to the factor covariance invariance model $-2\Delta LL(1) = 11.15$, $p < .01$, indicating that the factor mean at time 3 was significantly higher than at time 2. The factor mean at time 2 was already significantly different from 0 (the factor mean at time 1), and thus, the three factor means were significantly different, increasing over time.

In conclusion, these analyses showed that partial measurement invariance was obtained over time—that is, the relationships of the indicators to the latent factor of social functioning were equivalent at times 2 and 3, although primarily not equivalent at time 1, as previous described. These analyses also showed that partial structural invariance was obtained over time, such that the same amount of individual differences variance in social functioning was observed with equal covariance over time across occasions (i.e., compound symmetry of the latent factor), although the amount of social functioning on average increased significantly over time. Model parameters from the final model are given in Table 2.

(see excel worksheet for Table 1; Table 2 would have unstandardized and standardized estimates and their SEs)