Measurement of Psychological Disorders Using Cognitive Diagnosis Models

Jonathan Templin
University of Illinois at Urbana-Champaign

ETS External Diagnostic Research Group
Outline

■ Models for Cognitive Diagnosis
■ Psychological Assessment Application
  ♦ Study of Pathological Gambling
  ♦ Model Development
  ♦ Estimation
  ♦ Results
■ Current and Future Research
Imagine a test covering basic math:

\[
\begin{align*}
1.) & \quad 2 + 3 - 1 \\
2.) & \quad 4 \div 2 \\
3.) & \quad 3 \times (4 - 2)
\end{align*}
\]

- Using CTT or IRT, an individual’s score, or general math ability, could be estimated.
- Instead, math ability can be expressed as a set of basic skills (commonly called attributes):
  - Add
  - Subtract
  - Multiply
  - Divide
- Cognitive diagnosis models estimate a profile of the skills an individual has mastered.
Table 1: Math Test Example Q-matrix

<table>
<thead>
<tr>
<th></th>
<th>Add</th>
<th>Sub</th>
<th>Mult</th>
<th>Div</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 + 3 - 1$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$4/2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$3 \times (4 - 2)$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Possible Attribute Patterns

<table>
<thead>
<tr>
<th></th>
<th>Add</th>
<th>Sub</th>
<th>Mult</th>
<th>Div</th>
<th>Correct Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>#1</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>#1, #2</td>
</tr>
</tbody>
</table>
Example

Notation:

$I$ denotes the total number of examinees.

$J$ denotes the total number of items.

$K$ denotes the total number of attributes.

$Q$ has elements $q_{jk}$ that indicate whether mastery of the $k^{th}$ attribute is required by the $j^{th}$ item.

$\alpha_i$ is a set of indicators for examinee $i$'s attribute mastery for the $K$ attributes.
The DINA Model

- Deterministic Input; Noisy “And” Gate
  (Macready and Dayton, 1977; Haertel, 1989; Junker and Sijstma, 2001)

\[
P(X_{ij} = 1|\xi_{ij}) = (1 - s_j)\xi_{ij} g_j^{(1-\xi_{ij})}.
\]

where

\[
\xi_{ij} = \prod_{k=1}^{K} \alpha_{ik}^{q_{jk}}
\]

\[
s_j = P(X_{ij} = 0|\xi_{ij} = 1) - \text{“slip” parameter}
\]

\[
g_j = P(X_{ij} = 1|\xi_{ij} = 0) - \text{“guess” parameter}
\]
Cognitive Diagnosis Models...

- Are special cases of latent class models.
  - Latent classes are defined by a set of dichotomous skills (commonly called attributes).

- Provide **why** students are not performing well in addition to **which** individuals are not performing well.
Cognitive Diagnosis Models

- Concepts
- Example
- The DINA Model
- Definitions
- Technical Details
- CDM Applications
- Pathological Gambling
- Model Development
- Model Estimation
- Future Directions
- Acknowledgements

---

**CDM are a Subset of Latent Class Models**

A latent class model for the response vector of individual $i$:

$$P(X_i = x_i) = \sum_{\alpha} P(\alpha) \prod_{j=1}^{J} P(X_{ij} = x_{ij}|\alpha)$$

- For $K$ attributes, total of $2^K$ classes are defined.
- Equality constraints determined by:
  - Choice of model.
  - Q-matrix specifications.
CDM Structural Model

- Attribute pattern probabilities: $P(\alpha)$.
  - Saturated model has $2^K - 1$ parameters.

- One way to restrict the number of parameters is to use an underlying distribution (tetrachoric association):
  $$\tilde{\alpha} \sim N_K(0, \Xi)$$
  - Tetrachoric structural model has $K$ cut-point parameters:
    $$P(\alpha_{ik} = 1) = P(\tilde{\alpha}_{ik} > \kappa_k)$$
  - Covariance structure model is for elements of $\Xi$ representing attribute tetrachoric correlations.

- General tetrachoric model has $K + \frac{K(K-1)}{2}$ parameters
Interdisciplinary CDM Applications

- Common applications of CDM are found in educational measurement.

- Because of the information CDM can provide, such models may have benefits in other disciplines:
  - Diagnostic estimates for each individual.
  - Ability to estimate population tendencies.
    - Proportion of individuals having a given trait.
    - Structural relationship of latent traits.
Psychological Assessment using CDM

- As an example, consider the assessment of pathological gambling using CDM.

- Application overview:
  - **Background.**
  - How pathological gambling is commonly assessed.
  - The development and application of a new cognitive diagnosis model.
The Gambling Explosion

- Exponential increase in accessibility of gambling opportunities:
  - State lotteries
  - Native American Tribal Casinos
  - Riverboat gambling
  - Internet gambling

- Incidences of pathological gambling have increased (Volberg, 2002).

- In order to limit the detrimental effects of gambling on a community:
  - Easily identify potential pathological gamblers and provide treatment interventions.
  - Understand the underlying causes of the disorder.
Pathological Gambling

- The DSM-IV-TR defines pathological gambling as an impulse-control disorder (not elsewhere classified).
- To be classified as a pathological gambler, an individual must meet 5 of 10 defined criteria:

  C1  Is preoccupied with gambling.
  C2  Needs to gamble with increasing amounts of money in order to achieve the desired excitement.
  C3  Has repeated unsuccessful efforts to control, cut back, or stop gambling.
  C4  Is restless or irritable when attempting to cut down or stop gambling.
  C5  Gambles as a way of escaping from problems or of relieving a dysphoric mood.
  C6  After losing money gambling, often returns another day to get even.
  C7  Lies to family members, therapist, or others to conceal the extend of involvement with gambling.
  C8  Has committed illegal acts such as forgery, fraud, theft, or embezzlement to finance gambling.
  C9  Has jeopardized or lost a significant relationship, job, or educational or career opportunity because of gambling.
  C10 Relies on others to provide money to relieve a desperate financial situation caused by gambling.
South Oaks Gambling Screen

- Lesieur & Blume, (1987)
  - Twenty dichotomous items (yes/no) used for diagnosis
    - Five (or more) affirmative answers indicates a probable pathological gambler.

- In development, great effort was placed on validation.
  - The SOGS is useful for diagnosing pathological gambling.

- The SOGS has been used in a wide variety of settings (e.g. Volberg, 2002; The WAGER, 2003).

- Pathological gambling has low incidence rate in the general population (0.4% to 3.4%).
  - Individuals provide very few affirmative responses.

- Limited ability for use to study of traits underlying gambling pathology.
Gambling Research Instrument

- Forty-one Likert scale items.
  - Six response levels (Strongly Agree - Strongly Disagree).
- Increased variability in responses.
  - Enabled structure of pathological gambling to be studied.

GRI Example Items

- I would like to cut back on my gambling. (C1, C2, C3, C6)
- I worry that I am spending too much money gambling. (C2, C3, C6)
- I find it difficult to stop gambling. (C3, C4)
GRI Structural Model

- GRI was used to measure each criterion on a latent continuum.
- Henson, Feasel, & Jones (2000) hypothesized that underlying the 10 criteria were three latent factors: Dependence, Disruption, Loss of Control:

![Diagram showing GRI Structural Model with factors and criteria]

- Dependence
- Disruption
- Loss of Control

Criteria:
- C1
- C2
- C3
- C4
- C5
- C6
- C7
- C8
- C9
- C10
Analysis of the GRI with Cognitive Diagnosis

- Take each of the 10 criteria to be the dichotomous latent attributes.

- Applying a CDM would simultaneously provide:
  - Diagnostic information for each individual (like the SOGS).
  - Underlying structural model parameters (like the GRI).
    - Loadings of underlying factors onto latent criteria.
    - Proportion of individuals satisfying each criterion.

- Data from GRI calibration was used:
  - Study included 112 experienced gamblers.
  - Participants provided responses to the GRI and SOGS.
Several assumptions of the DINA model are inconsistent with the structure of the GRI:

♦ Separates into classes based on mastery of all needed skills.

• Missing a single skill means an individual is likely to score poorly on an item (i.e., a conjunctive model).

♦ Model is defined only for dichotomous (0/1) responses.
A Disjunctive Model

- Imagine an item that could be rated highly for multiple reasons.

- For Example:

22. Gambling has hurt my financial situation.

- An individual may rate this item highly if they satisfied at least one of:

  C8 Has committed illegal acts such as forgery, fraud, theft, or embezzlement to finance gambling.

  C10 Relies on others to provide money to relieve a desperate situation caused by gambling.
The DINO Model

(Deterministic Input; Noisy “Or” Gate)

\[
P(X_{ij} = 1|\omega_{ij}) = (1 - s_j)^{\omega_{ij}} g_j^{(1-\omega_{ij})}
\]

where

\[
\omega_{ij} = 1 - \prod_{k=1}^{K} (1 - \alpha_{ik})^{q_{jk}}
\]

\[
s_j = P(X_{ij} = 0|\omega_{ij} = 1) - “slip” \text{ parameter}
\]

\[
g_j = P(X_{ij} = 1|\omega_{ij} = 0) - “guess” \text{ parameter}
\]
Next, consider a DINO model that allows for polytomous item responses.

Such a model parameterized the probability of a score \( m \), with \( m = 0, \ldots, M \).

Possible model extensions:
- Nominal Response Model.
- Graded Response Model.
- Binomial Model.
The Binomial DINO Model

- When compared to the dichotomous version of the DINO, the Graded Response DINO adds an additional $2 \times (M - 1)$ parameters.

- As an alternative, assume that an item’s score is the sum of $M$ dichotomous items (pseudo-items).

- Responses to the $M$ pseudo-items are assumed to be independent.

- Each dichotomous item is assumed to have the probability of a correct response defined by the DINO.
The Binomial DINO Model (cont.)

- **DINO Model:**

\[ P = (1 - s_j)\omega_{ij} g_j^{1-\omega_{ij}} \]

- **Binomial DINO:**

\[ P(X_{ij} = m | \omega_{ij}) = \binom{M}{m} [P]^m [1 - P]^{M-m} \]

where

\[ \omega_{ij} = 1 - \prod_{k=1}^{K} (1 - \alpha_{ik}) q_j^k \]

\[ s_j = P(x_{ij} = 0 | \xi_{ij} = 1) \]

\[ g_j = P(x_{ij} = 1 | \xi_{ij} = 0) \]
Model Response Distributions

Example Conditional Distributions: $s = 0.15$, $g = 0.25$

<table>
<thead>
<tr>
<th>$x_{ij}$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Gambler $\omega_{ij} = 0$</td>
<td>0.50</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Gambler $\omega_{ij} = 1$</td>
<td>0.50</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Disagree</td>
<td>Disagree</td>
<td>Slightly Disagree</td>
<td>Slightly Agree</td>
<td>Agree</td>
<td>Strongly Agree</td>
</tr>
</tbody>
</table>
Model Response Distributions

Possible Marginal Distributions (depending on population parameters):

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strongly Disagree</td>
<td>Disagree</td>
<td>Slightly Disagree</td>
<td>Slightly Agree</td>
<td>Agree</td>
<td>Strongly Agree</td>
</tr>
</tbody>
</table>

- **Few Gamblers**
  - Strongly Disagree
  - Disagree
  - Slightly Disagree
  - Slightly Agree
  - Agree

- **Mix of Gamblers**
  - Strongly Disagree
  - Disagree
  - Slightly Disagree
  - Slightly Agree
  - Agree

- **Many Gamblers**
  - Strongly Disagree
  - Disagree
  - Slightly Disagree
  - Slightly Agree
  - Agree
An algorithm was developed to estimate the hypothesized latent structural model for the underlying factors of the 10 criteria.
Estimation Algorithm

- Markov Chain Monte Carlo estimation algorithm.
- Uniform prior for all DINO item parameters \((s, g)\).
- Latent traits \((\alpha)\) modeled with prior defined by structural model.
- Uniform prior for all mean and covariance structural parameters \((\kappa, \lambda, \phi)\).
- Chain length of 50,000.
- Convergence check:
  - Geweke test.
  - Visual inspection of timeseries plots.
Individual Diagnosis Estimates

Individual A:

C1 ➤ 0.00
C2 ➤ 1.00
C3 ➤ 0.25
C4 ➤ 0.50
C5 ➤ 0.75
C6 ➤ 0.00
C7 ➤ 0.90
C8 ➤ 0.58
C9 ➤ 0.79
C10 ➤ 0.00

Estimated probability of being a pathological gambler (satisfying five or greater): **0.040**

SOGS Score: **2**
Individual Diagnosis Estimates

Individual B:

Estimated probability of being a pathological gambler (satisfying five or greater): 0.960

SOGS Score: 6
SOGS Concordance

<table>
<thead>
<tr>
<th>CDM Classification</th>
<th>SOGS Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-PG</td>
<td>Non-PG</td>
</tr>
<tr>
<td>78</td>
<td>7</td>
</tr>
<tr>
<td>PG</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>83</td>
</tr>
</tbody>
</table>

- 89.3% matching classifications.
- Cohen’s Kappa: 0.69.
Item Parameters

- Difference in expected score per item is indicative of diagnostic ability of an item (given Q-matrix entries):
  - Average score for $\xi = 1$: 2.47
  - Average score for $\xi = 0$: 0.80
- Item 5: I find it necessary to gamble with larger amounts of money (than when I first gambled) for gambling to be exciting. [C2]
- Item 13: I find it difficult to stop gambling. [C3 or C4]
Structural Model Estimates

C1: 0.656 (0.161) (25.9%)
C4: 0.633 (0.154) (26.6%)
C2: 0.503 (0.184) (31.0%)
C5: 0.219 (0.148) (41.4%)
C8: 1.478 (0.323) (8.0%)
C9: 1.040 (0.172) (15.3%)
C10: 0.764 (0.151) (22.5%)
C3: 0.650 (0.158) (26.0%)
C6: 0.617 (0.154) (27.1%)
C7: 0.317 (0.242) (37.9%)
Structural Model Estimates

Dependence

Disruption

Loss of Control

C1 0.656 (0.161) (25.9%)
C4 0.633 (0.154) (26.6%)
C2 0.503 (0.184) (31.0%)
C5 0.219 (0.148) (41.4%)
C8 1.478 (0.323) (8.0%)
C9 1.040 (0.172) (15.3%)
C10 0.764 (0.151) (22.5%)
C3 0.650 (0.158) (26.0%)
C6 0.617 (0.154) (27.1%)
C7 0.317 (0.242) (37.9%)

C4 0.937 (0.040)
C2 0.933 (0.043)
C5 0.612 (0.101)
C8 0.488 (0.115)
C9 0.377 (0.138)
C10 0.287 (0.109)
C3 0.445 (0.247)
C6 0.894 (0.072)
C7 0.933 (0.044)
C8 0.919 (0.060)
C9 0.756 (0.119)
C10 0.798 (0.085)
Structural Model Estimates

Dependence

Disruption

Loss of Control

$D_1$

$D_2$

$D_3$

$C_1$

$C_2$

$C_3$

$C_4$

$C_5$

$C_6$

$C_7$

$C_8$

$C_9$

$C_{10}$

$\text{Loss of Control}$
Evaluation of Model Fit

- Typical distribution-based LCA measures of goodness-of-fit were unreasonable.

- To assess the fit of the model, a Monte Carlo fit index was constructed (based on Langeheine et. al, 1996).

- Root Mean Squared Error (RMSE) of the item mean was used as a criterion.

- Item mean RMSE = 0.056 ($p = 0.859$).
Concluding Remarks

- Cognitive diagnosis models seem to have extensions beyond educational measurement.
  - Criterion diagnostic estimates give rich information about the unique pattern of satisfied criteria an individual possesses.
  - Structure underlying attributes can be estimated.
- High degree of consistency in diagnosis when the CDM analysis is compared with the SOGS.
Current and Future Research

Goal:
My research focus is on the expanding the practicality of models for cognitive diagnosis while broadening the interdisciplinary applications of such models.

- There are multiple areas where I seek to pursue this goal:
  - Psychometrics
  - Pathological Gambling
  - General Psychological Assessment
Psychometrics

- Alternative estimation algorithms.
- Multi-group comparisons.
- Differential Item Functioning.
- Exploratory methods for Q-matrix construction.
- Models for mixed-type latent variables (continuous, polytomous-categorical).
- Expansion of polytomous item response models.
- Practical goodness-of-fit indices.
- Differing structural models (e.g., log-associative, disjunctive dichotomous).
Research in Pathological Gambling

- Rewrite GRI with CDM analysis as goal.
- Calibrate inventory with larger sample of subjects.
- Evaluation of DSM-definition of pathological gambling:
  - Does “five or greater” make sense?
  - Should all criterion be equally weighted?
  - Are criteria redundant/have some been omitted?
- Measurement and study of pathological gambling cross-culturally:
  - Sensitive populations in USA.
  - Study of gambling in Europe.
Psychological Assessment Applications

- Investigation into whether information given by CDM holds value in disciplines other than education.

- Evaluation of appropriateness of CDM in other types of psychological assessment situations.
  - Clinical diagnosis of other disorders.
  - Screen for job applicants.
  - Measurement of personality traits.
Acknowledgements

- Bob Henson
- Jeff Douglas
- Bill Stout
- Gambling Research Group
  - Larry Jones
  - Karen Feasel
  - Mario Moric
  - Kara Henson
  - Sara Templin

- Thank you.