Introduction to Multilevel Models

Applied Multilevel Models for Cross-Sectional Data

Lecture 1

ICPSR Summer Workshop
University of Colorado Boulder
Covered This Lecture

• Features of multilevel models and hierarchical data

• What can multilevel models do for you?

• Workshop overview
HIERARCHICAL/MULTILEVEL DATA STRUCTURES
Multilevel or Hierarchical Data Structures

• In the social and educational sciences there are many examples of *Hierarchical Data*

• I will use the terms *Hierarchical* and *Multilevel* synonymously
  - *Hierarchical* is often used in educational sciences
  - *Multilevel* is often used in the social sciences

• Natural question: What are *Hierarchical Data*?
  - Two or more different dimensions of sampling units
  - Unit are nested within the other units
  - Later we will call units levels
Multilevel Data

- An educational example is when the two different units are classes and students

For any given classroom, we will have students. Here we would say that students are nested within classroom. We could have variables at the classroom level and at the student level.
Multilevel Data

• In this particular example we may be interested in
  - Performance of each student as predicted by a set of variables such as gender, socio-economic status (SES), ...

• But it also makes sense that different classroom characteristics can have an impact

• It could also be the case that students from the same classroom have observations that are more highly related to each other than students from different classrooms
  - Literally: a non-zero intraclass correlation
  - In practice: these data have additional dependencies
Multilevel Data

• So we could have several different classes with different characteristics that impact the students differently
Longitudinal Data: A Different Type of Nesting

We could have subject variables e.g., gender, job, ...

And we could have variables about each time point e.g., energy level, stress level...
Multilevel Data and Models

• Repeated measures models and longitudinal models are different types of multilevel models
  - Trials/observations are nested within subjects

• Subjects may also be nested within classes
  - We would have three levels of data

  - Time (1), Subject (2), Class (3)
Data Requirements for Models Featured in Workshop

• We need multiple **OUTCOMES** from the same sampling unit
  - Multiple level-1 people within level-2 clusters

  • 2 in a cluster is the minimum, but just 2 can lead to problems:
    • Can’t distinguish “real” individual differences from error

• More data is better (with diminishing returns)
  - More people means we can get a better understanding of level-1 effects

    • More clusters/sampling units means we can get better estimates of amount of differences across units; better prediction of those differences

    • More items/stimuli means we have more power to show effects of differences between items/stimuli/conditions
Types of Outcome Data Covered in this Workshop

• For our workshop, an outcome (dependent) variable:
  ➢ Has an interval scale
    • A one-unit difference means the same thing across all scale points
  ➢ Has scores with the same meaning over observations
    • Includes meaning of construct
    • Includes how items relate to the scale
    • Implies measurement invariance

• We will note the following up front:

  FANCY STATISTICAL MODELS CANNOT SAVE BADLY MEASURED VARIABLES
  or
  BADLY DESIGNED MULTILEVEL STUDIES
Levels of Analysis in Longitudinal Data

- **Between-Cluster (BC) Variation:**
  - **Level-2** – “**INTER**-cluster Differences” – Person-Invariant

- **Within-Cluster (WC) Variation:**
  - **Level-1** – “**INTRA**-cluster Differences” – Person-Varying

- Multilevel studies allow examination of both types of relationships simultaneously (and their interactions)

- Any variable measured **WC usually has both BC and WC variation**
  - BC = more/less than other clusters
  - WC = more/less than a cluster’s average
Options for Longitudinal Models

- Although models and software are logically separate, multilevel data can be analyzed via multiple analytic frameworks:
  - “Multilevel/Mixed Models”
    - Dependency over time, persons, groups, etc. is modeled via random effects (multivariate through “levels” using stacked/long data)
    - Builds on GLM, generalizes easier to additional levels of analysis
  - “Structural Equation Models”
    - Dependency only is modeled via latent variables (single-level analysis using multivariate/wide data)
    - Generalizes easier to broader analysis of latent constructs, mediation

- Because random effects and latent variables are the same thing, many multilevels models can be specified/estimated either way
  - And now “Multilevel Structural Equation Models” can do it all...
WHAT CAN MLM DO FOR YOU?
What can MLM do for you?

• **Model dependency across observations**
  - Longitudinal, clustered, and/or cross-classified data? No problem!
  - Tailor your model of sources of correlation to your data

• **Include categorical or continuous predictors at any level**
  - Person-level, group-level predictors for each variance
  - Explore reasons for dependency, don’t just control for dependency

• **Does not require same data structure for each person**
  - Unbalanced or missing data? No problem!

• **You already know how (or you will soon)!**
  - Use SPSS Mixed, SAS Mixed, Stata, Mplus, R, HLM, MlwiN...
  - What’s an intercept? What’s a slope? What’s a variance component?
1. Model Dependency

- Sources of dependency depend on the sources of variation created by your sampling design: residuals for outcomes from the same unit are likely to be related, which violates the general linear model “independence” assumption.

- “Levels” for dependency = “levels of random effects”
  - Sampling dimensions can be nested
    - e.g., time within person, person within group, school within district
  - If you can’t figure out the direction of your nesting structure, odds are good you have a crossed sampling design instead
    - e.g., persons crossed with items, raters crossed with targets
  - To have a “level”, there must be random outcome variation due to sampling that remains after including the model’s fixed effects
    - e.g., treatment vs. control does not create another level of “group”
Why care about dependency?

• In other words, what happens if we have how we model dependencies wrong?

• **Validity of the tests of the predictors** depends on having the “most right” dependency model (right “random effects” in the model)
  - Estimates will usually be ok as the come from “fixed effects”
  - Standard errors (and thus p-values) can be compromised

• The sources of variation that exist in your outcome will dictate **what kinds of predictors** will be useful
  - Between-Cluster variation needs Between-Cluster predictors
  - Within-Cluster variation needs Within-Cluster predictors
2. Include categorical or continuous predictors at any level of analysis

- ANOVA: test differences among discrete independent variable factor levels
  - Between-Groups: Gender, Intervention, Age Groups
  - Within-Subjects (Repeated Measures): Condition, Time
  - Test main effects of continuous covariates (ANCOVA)

- Regression: test whether slopes relating predictors to outcomes are different from zero
  - Clusters measured once, differ categorically or continuously on a set of person-invariant (Cluster-level) covariates

- What if a predictor is assessed repeatedly but can’t be characterized by ‘conditions’?
  - ANOVA or Regression won’t work per se, so there is a need for MLM
2. Include categorical or continuous predictors at any level of analysis

- Some things don’t change over measurements...
  - Sex, Ethnicity
  - Time-Invariant Predictor = Person Level

- Some things do change over measurements...
  - Health Status, Stress Levels, Living Arrangements
  - Time-Varying Predictor = Time Level

- Some predictors might be measured at higher levels
  - Family SES, length of marriage, school size

- Interactions between levels may be included, too
  - Does the effect of health status differ by gender and SES?

Level: Time Person Family
3. Does not require same data structure per person (by accident or by design)

**RM ANOVA:** uses *multivariate* (wide) data structure:

<table>
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<tr>
<th>ID</th>
<th>Sex</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
</tr>
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<tbody>
<tr>
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<td>0</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>101</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>.</td>
<td>11</td>
</tr>
</tbody>
</table>

People missing any data are excluded (data from ID 101 are not included at all)

**MLM:** uses *stacked* (long) data structure:

<table>
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<tr>
<th>ID</th>
<th>Sex</th>
<th>Time</th>
<th>Y</th>
</tr>
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<tbody>
<tr>
<td>100</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
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<td>0</td>
<td>2</td>
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</tr>
<tr>
<td>101</td>
<td>1</td>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

Only rows missing data are excluded

100 uses 4 cases
101 uses 3 cases

Time can also be *unbalanced* across people such that each person can have his or her own measurement schedule: Time “0.9” “1.4” “3.5” “4.2”…

Lecture 1: Introduction to Multilevel Models
4. You most likely already know how to use MLMs...

- If you can do general linear models (ANOVA and Regression) then you can do MLM

- How do you interpret an estimate for...
  - the intercept?
  - the effect of a continuous variable?
  - the effect of a categorical variable?
  - a variance component (think residual variance)?
WORKSHOP OVERVIEW
<table>
<thead>
<tr>
<th>Day</th>
<th>Time</th>
<th>Topic</th>
</tr>
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<tbody>
<tr>
<td>Monday</td>
<td>9:00-9:30</td>
<td>Workshop Overview and Introductions</td>
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<tr>
<td></td>
<td>9:30-10:15</td>
<td>Lecture 1: Introduction to Multilevel Models and Hierarchical Data</td>
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<tr>
<td></td>
<td>10:30-11:45</td>
<td>Lecture 2: The General Linear Model</td>
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<td>1:15-2:30</td>
<td>Lecture 3: Simple, Marginal, and Interaction Effects in GLMs</td>
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<td>2:45-4:00</td>
<td>Lecture 4: Statistical Distribution Assumptions of GLM/Maximum Likelihood</td>
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<td>4:00-5:00</td>
<td>Lab 1: Introduction to Data Manipulation in SAS</td>
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<td>Tuesday</td>
<td>9:00-10:15</td>
<td>Lecture 5: Multilevel models – a Guiding Example</td>
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<td>10:30-11:45</td>
<td>Lecture 6: Centering Predictors and Variance Decomposition</td>
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<td>1:15-2:30</td>
<td>Lecture 6, Continued – with Computing</td>
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<td>2:45-4:00</td>
<td>Lecture 7: Random Slopes, Cross-Level Interactions, Interpretations</td>
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<td>Lab 2: Fitting Single-Predictor Multilevel Models</td>
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<td>Wednesday</td>
<td>9:00-10:15</td>
<td>Lecture 8: Comprehensive Overview of Multilevel Models</td>
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<td>10:30-11:45</td>
<td>Example: Two-Level Clustered Data – Students within Schools</td>
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<td>1:15-2:30</td>
<td>Example: Two-Level Cross-Classified Data Models</td>
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<td>2:45-4:00</td>
<td>Example: Changes in Nesting over Time</td>
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<td>4:00-5:00</td>
<td>Lab 3: Fitting Multi-Predictor Multilevel Models</td>
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<td>Thursday</td>
<td>9:00-10:15</td>
<td>Lecture 9: Three Level Models</td>
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<td>10:30-11:45</td>
<td>Example: Three Level Models</td>
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<td>1:15-2:30</td>
<td>Lecture 10: Multivariate Normal Distribution and Multivariate Analyses</td>
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<td>2:45-4:00</td>
<td>Lecture 11: Multilevel Models in Matrix Form</td>
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<td>4:00-5:00</td>
<td>Lab 4: Fitting Multivariate Models</td>
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<td>Friday</td>
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<td>Lecture 12: Multivariate Multilevel Models</td>
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<td>10:30-11:45</td>
<td>Lecture 13: Generalized Multilevel Models (Non-Normal Outcomes)</td>
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<td>Lecture 14: Generalized Multivariate Multilevel Models (i.e. IRT)</td>
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<tr>
<td></td>
<td>2:45-5:00</td>
<td>Open Lab Time</td>
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Computing Information

- We will use SAS PROC MIXED for all analyses in this workshop examples

  - All examples have detailed syntax available online and in your course packet
  - Some examples are provided in SPSS and R