Random Slopes, Cross-level Interactions, and Analysis Interpretations

Applied Multilevel Models for Cross-Sectional Data
Lecture 7

ICPSR Summer Workshop
University of Colorado Boulder

Covered this Section

- Reinforcing MLM through examples:
  - Random slopes
  - Cross-level interactions

- A new example with actual data
  - Class-guided analysis

AS SEEN LAST TIME…

Guiding Example

- Imagine you are interested in studying the effects of socioeconomic status (SES) on student achievement
  - What do you think the relationship between student achievement and SES happens to be?

- You are interested in predicting achievement from SES
  - Your guiding research question

Your Study…

- Let's imagine you are able to get data from 7 elementary schools around Boulder
  - You sample 50 students from each elementary school
  - You record a measure of their SES (scale with a mean of 50)
  - You record a measure of their achievement (scale with a mean of 100)

- Both scales magically have absolutely perfect reliability

Previously Called Model #3…

- Adding cluster-mean centered student SES and grand-mean centered school mean SES yields the following model:

- Level-1: $Y_{is} = \beta_{0s} + \beta_{1s}(X_{is} - \bar{X}_s) + e_{is}$
  - Where $e_{is} \sim N(0, \sigma^2)$

- Level-2: $\beta_{0s} = \gamma_{00} + \gamma_{01}\left(\bar{X}_s - \bar{X}_g\right) + U_{0s}$

$$\beta_{1s} = \gamma_{10}$$

  - Where $U_{0s} \sim N(0, \tau_0^2)$

- Combined:

$$Y_{is} = \gamma_{00} + \gamma_{01}(\bar{X}_s - \bar{X}_g) + \gamma_{10}(X_{is} - \bar{X}_s) + U_{0s} + e_{is}$$

Model #3 Results- Model Fit

- Model Fit:

Model #2 (old model)

Value	Estimate
Deviance	2,202.5

Model #3 (new)

Value	Estimate
Deviance	2,147.9

- Question: Is model 3 preferred to model 2?

- Answer: Deviance test (two new parameters:
  - Test statistic: 2,202.5-2,147.9 = 57.6
  - Degrees of freedom = 2 (new parameters γ_{10} and γ_{01})
  - P-value: < 0.0001

- Conclusion: Model #3 is preferred

Model #3 Results – Fixed Effects (Means)

- Model parameter estimates:
  - $\gamma_{00} = 101.38\ (1.41); p < 0.0001$
    - The overall intercept – the value of achievement for a student with SES equal to their school mean SES at a school with mean SES equal to the grand mean SES
    - Is the average value of achievement
  - $\gamma_{10} = -0.99\ (0.14); p < 0.0001$
    - The slope for student SES (minus school mean SES)
    - Represents the change in achievement for each unit a student SES differs from their school mean
      - Given school mean SES is held constant
  - $\gamma_{01} = 1.63\ (0.42); p = 0.0060$
    - The slope for school mean SES (minus grand mean SES)
    - Represents the change in achievement for each unit the school mean SES differs from the grand mean
      - Given student SES is held constant

Model #3 Results – Variance Parameters

- Our estimated variance parameters were:

Parameter	**Estimate**
σ^2 (Error)	25.35 (1.93)
τ_0^2(Random Intercept)	13.35 (7.41)

- Meaning, our intraclass correlation was:

$$\rho = \frac{13.35}{13.35 + 25.35} = .345$$

- This means:
  - Student's achievement scores within a school had a correlation of .345
  - 34.5% of the total variability in achievement scores came from between school variability
  - Our linear regression assumption of uncorrelated residuals is violated
    - Should be using the mixed model with a random intercept

More on Variances

- In comparing Model #3 to Model #2, an important distinction is of how much each variance component is reduced because of addition of the predictors
  - Called a pseudo- R^2

- Level-2 Variance (τ_0^2):
  - Model #2 = 43.25; Model #3 = 13.35
  - Reduction: 43.25 – 13.35 = 29.9
  - Proportion of Model #2 Variance Explained: 29.9/43.25 = .69
  - Explanation: School Mean SES explains 69% of the variance in school achievement (random intercept variance)

Level-1 Variance Reduction

- Level-1 Variance (σ^2):
  - Model #2 = 29.04; Model #3 = 25.35
  - Reduction: 29.04 – 25.35 = 3.69
  - Proportion of Model #2 Variance Explained: 3.69/29.04 = .13
  - Explanation: Student SES explains 13% of the variance in student achievement

UP NEXT: RANDOM SLOPES

More On Our Analysis

- So far, we've been concerned that there are differences in the relationship between SES and achievement at the differing levels of our analysis
  - Between-schools: Positive relationship
  - Within-schools: Negative relationship

- We also have investigated whether or not schools have differences in their intercepts
  - Random intercept variance was non-zero
  - Was able to be predicted by school-level SES

- But…what about the slope for SES
  - Shouldn't we see if there are school-level differences in slope?

Model #4…Adding a Random Slope

- We will now expand our Level-2 Model to include a random effect
  - Called a random slope
- We will add a level-2 predictor of the slope, too…
  - Leading to a cross-level interaction.

- Level-1: $Y_{is} = \beta_{0s} + \beta_{1s}(X_{is} - \bar{X}_s) + e_{is}$
  - Where $e_{is} \sim N(0, \sigma^2)$

- Level-2: $\beta_{0s} = \gamma_{00} + \gamma_{01}\left(\bar{X}_s - \bar{X}_g\right) + U_{0s}$

  $\beta_{1s} = \gamma_{10} + U_{1s}$

  - Where $\begin{bmatrix} U_{0s} \\ U_{1s} \end{bmatrix} \sim N_2\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_0^2 & \tau_{01} \\ \tau_{01} & \tau_1^2 \end{bmatrix}\right)$

- Combined:

$$Y_{is} = \gamma_{00} + \gamma_{01}\left(\bar{X}_s - \bar{X}_g\right) + \gamma_{10}(X_{is} - \bar{X}_s) + U_{0s} + U_{1s}(X_{is} - \bar{X}_s) + e_{is}$$

The Process

- Just as with our transition from or previous models, we must first determine if the addition of a random slope (and accompanying covariance between random intercept and slope) are needed in the model.

Model #3 (Old Model)

Value	Estimate
Deviance	2,147.9

Model #4 (Random Slope)

Value	Estimate
Deviance	2,143.8

- Test statistic: 2,147.9-2,143.8 = 3.9
- Degrees of freedom = 2 (new parameters τ_{01} and τ_1^2)
- P-value: p = 0.1423
- Conclusion: No variation in slopes across schools

So…No Random Slopes

- Our analysis resulted in not finding any variation in the random slopes
  - Model #3 (just random intercept) was preferred to Model #4 (random intercepts + random slopes)

- The Model #4 estimates for the random effect variance components were:
  - $\tau_0^2 = 20.47$
  - $\tau_1^2 = 0$
  - $\tau_{01} = -1.34*$
    - Note this covariance is not possible (think of correlation)
- So…schools do not vary in how student SES is related to student achievement

One More Analysis…Investigating Cross-Level Interactions

- One question remains: does the effect of student SES on achievement vary as a function of school-level SES
  - Perhaps effect is stronger in lower-SES schools?

- This would be what is considered a **cross-level interaction**
  - Level-2 (school SES) and level-1 (student SES) variables interacting to produce an effect on outcome (achievement)

- Usually is tested when random slope is present…
  - …but can be without (different df used)

Model #5…Adding a Cross-Level Interaction

- We will now expand our Level-2 Model to include a random effect
  - Called a random slope
- We will add a level-2 predictor of the slope, too…
  - Leading to a cross-level interaction.

- Level-1: $Y_{is} = \beta_{0s} + \beta_{1s}(X_{is} - \bar{X}_s) + r_{is}$
  - Where $r_{is} \sim N(0, \sigma^2)$

- Level-2: $\beta_{0s} = \gamma_{00} + \gamma_{01}(\bar{X}_s - \bar{X}_g) + U_{0s}$

  $\beta_{1s} = \gamma_{10} + \gamma_{11}(\bar{X}_s - \bar{X}_g)$

  - Where $U_{0s} \sim N(0, \tau_0^2)$

- Combined:

$$Y_{is} = \gamma_{00} + \gamma_{01}(\bar{X}_s - \bar{X}_g) + \gamma_{10}(X_{is} - \bar{X}_s) +$$

The Process

Model #3 (Old Model)

Value	Estimate
Deviance	2,147.9

Model #5 (Cross-Level Interaction)

Value	Estimate
Deviance	2,147.4

- Test statistic: 2,147.9-2,147.40 = 0.5
- Degrees of freedom = 1 (new parameter: γ_{11})
- P-value: p = 0.4795
- Conclusion: No cross-level interaction present