Test Score Equating
Using IRT

University of Kansas
Item Response Theory
Stats Camp ‘07
Overview

• Test Score Equating Using IRT
  – How do we get the results from separate calibrations onto the same scale, so that they can be compared?
  – Tomorrow we’ll discuss DIF in detail, which will be more relevant once you have a better understanding of the scale comparability issues
Test Score Equating

- It is common for scores obtained from different forms of a test to be compared, just as if they came from the same test, but this can be problematic if tests differ in difficulty!

- Example: No one asks what year you took the SAT, only your score!
CTT Weaknesses

• Item statistics (e.g., $p$ and $r$) depend on the group of examinees used to estimate them, and therefore do not generalize to a larger group.
• Examinee ability estimates depend on the particular test items used, and therefore do not generalize to anything but a strictly parallel test.
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Test Score Equating

• Test score equating is the process by which we ensure that comparisons across forms of a test are meaningful
  – Multiple forms of a single test in a given administration, or scores from tests administered across years
Purpose of Equating

• The purpose of equating is to adjust for differences in test form difficulty so that the forms can be used interchangeably.

• After successful equating, examinees can be expected to earn the same score regardless of the test form administered.

• Thus, scores from various forms can be meaningfully compared.
Importance of Equating

• It’s important whenever scores from two or more tests are to be compared and the tests themselves are unequal in difficulty and/or the test score distributions are not equivalent to each other.
  – Entrance exams: SAT, GRE, LSAT
  – Credentialing exams: AICPA, NBME
  – National, statewide standardized achievement tests: NAEP, Kansas
Practical Importance…

• LOTS of decisions made based on student test scores, growth in student ability, etc.

• NCLB Act of 2001 calls for schools to show “adequate yearly progress”

• If we make decisions about changes in pass rates, there is a critical need to ensure scale comparability.
Important Points

• To be considered “forms of a test,” forms must be developed from the same content and statistical specifications.

• Forms built to different content or difficulty specifications can’t be equated.

• You wouldn’t “equate” SAT and ACT scores, because the tests are built with different (though similar) purposes, and each is built to its own specifications.
Important Points

• Equating is necessary, in practice, because it is essentially impossible to construct test forms that are precisely equal in overall difficulty.
• So, if we want (or need) to have multiple forms of an exam, then we need to statistically equate the scores from these forms.
Practical Sidebar

• There is almost always a need to equate forms, because tests very regularly change over time. For example, many operational testing programs release items after score reporting…and they certainly can’t administer them again after that!
Related Issues

1) **Equate**: Convert scores from alternate forms to the same scale
2) **Vertical Scaling**: Place tests of different difficulty but closely related content on the same scale
3) **Concordance**: Place scores from different tests used for a particular purpose onto the same scale
Score Equivalence Issues

• Let’s say that for all examinees, we report the raw scores from the forms, regardless of their relative difficulty.

• **PROBLEMS:**
  – Examinees administered an easier form are advantaged; those administered a harder form are disadvantaged.
  – Trends in examinee ability over time are confounded with the test form difficulty.
Score Equivalence Issues

• Let’s say we convert raw scores such that the examinee ability distribution is always the same across forms (i.e., transform all scores to have the same mean and SD).

• PROBLEMS:
  – Examinees tested with a lower achieving group are advantaged; those tested with a higher achieving group are disadvantaged.
  – Trends in examinee ability over time cannot be addressed using these scores.
Score Equivalence Issues

• If we use equating and report equated scores, we adjust for differences in form difficulty and account for differences in examinee groups, so that successfully equated scores are not affected by the previously mentioned problems.

• PROBLEM:
  – “Successful” equating depends on meeting assumptions; if not met…
Topics

• Review some fundamentals, assumptions, and model features.
• Steps in conducting IRT Equating
  – Common persons, common items
  – Characteristics of Anchor Items
• Advantages of IRT Equating
• Examples…
Recall Fundamentals

- We assume that the construct we are attempting to measure exists and influences test performance.
- This “ability” or “trait” relates to item performance, and that relationship can be specified in an ICC.
- This should be true regardless of whether an item is on Form A or B.
Ability Scores

• “Ability” is the label used to describe what the test measures. It is used to define what is being measured by the pool of items from which the test items were drawn.

• Thus, even though there are some different items across forms, they measure the same “ability” construct.
Recall Parameter Invariance

• IF THE MODEL FITS…

• Item parameters are invariant over samples of examinees from the population for whom the test is intended.

• Ability parameters are invariant over samples of test items from the population of items measuring the ability of interest.
Arbitrariness of the scale

- The scale chosen to identify a solution in IRT modeling is only defined up to a linear transformation.
- Choosing a mean of 0 and SD of 1 identifies a scale for interpretation, and determines the scale of item parameters, but any linear transformation of $\theta$, with a corresponding transformation for items, would provide the same ICC results.
Parameter Invariance

• This assumption basically states that parameters are invariant \textit{up to a linear transformation}, which accounts for the arbitrariness of the scale chosen to identify a solution.

• Once the scale is chosen, this assumption can then be tested.
\[
\text{if } \quad \theta_{new} = x\theta + y
\]
\[
\text{then } \quad b_{new} = xb + y
\]
\[
a_{new} = \frac{a}{x}
\]
\[
c_{new} = c
\]

These transformations preserve the probability:
\[
P(u = 1 \mid \theta_{new}) = P(u = 1 \mid \theta)
\]
Ability Scale

• Because response probabilities are maintained through a linear transformation, the ability scale can be (and often is) transformed after calibration to create a more convenient scale for interpretation, usage, and score reporting

• Example: GRE (μ = 500, σ = 100)
Scale Transformations

• The ability or item scale can also be transformed after calibration to make it match the scale from another calibration, as long as there is some “link” across the forms which creates the basis for the transformation.

• For this reason, equating is often referred to as “linking”
Steps in IRT Equating

1) Choose an equating design (includes data collection)

2) Place item parameter estimates from the tests being equated onto a common scale

3) Complete the equating according to the procedure chosen
1) Choose an Equating Design
Data Collection Designs

• Single Group or Common Person
• Randomly Equivalent Groups
• Common Item Non-equivalent Groups
Common Person Design

• The same group of examinees takes both test forms (Forms A & B)
• To minimize ordering effects, counterbalance administration order
• **Link** $\Rightarrow$ ability ($\theta$) should be the same for either form; transform $\theta_B$ to match $\theta_A$ and do corresponding transformation to item parameters.
Common Person Design

• Practical Problems
  – Hard to control for practice and fatigue
  – Testing time is essentially doubled
  – Two or more administrations (days) are typically required
Random Groups Design

- Popular with states and test publishers; field test forms are distributed randomly to students in classrooms and schools
- Groups taking each form are treated as “randomly equivalent” → assumption is made of equal ability
- Do IRT analysis of each test, and use item statistics interchangeably
Random Groups Design

• Randomly equivalent groups or samples of examinees are administered one of the test forms
• With large samples, differences in test difficulties are attributed to tests, not samples, because samples are considered equivalent.
Random Groups Design

• Preferable to a Single Group Design if samples are large because examinees take only a single test
• Gets around the problem of practice effects and unwillingness to participate
• Large samples minimize sampling errors in the equating
• Difficult assumption to test (equivalence)
Common Item Design

• Most common method of equating
• Often called “anchor test” design
• Common items are included on every test form, and parameters for those items are treated as equivalent across all forms.
• Common items may be internal (scored) or external (not scored)
Common Item Design

• Popular design because it represents a common situation: tests of unequal difficulty are administered to groups which may differ in ability, and it is assumed that they do differ.

• Ability differences are assessed through anchor test (common items) and then adjustments are made for test difficulties
Practical Sidebar

- Because many operational testing programs release items after score reporting, external anchors are quite common...if the items weren’t used to determine scores, they don’t have to be released, and can therefore be used again on subsequent forms.
Common Item Design

• Practical problem
  – Administrative flexibility (examinees take only one test) gained at the expense of strong statistical assumptions needed to separate group and test differences.
  – Common Items must be ‘behave’ the same way on each form.
Common Administration

• Forms administered via *matrix sampling*
  – 1 Common scoring block
  – $X$ linking blocks $\rightarrow$ $X$ forms
  – These blocks comprise the ANCHOR TEST
  – Blocks composed to match average difficulty, information from the scoring test.
• Forms are linked within a particular year based on common scoring items
• Forms are linked across years based on common linking blocks
• Subsequent years: new scoring items
Scoring Items (Different items across Years)  

Linking Items (Same items across Years)  

Year 1  

n items  

Year 2  

n items
Anchor Tests

• Anchor items should look like mini-versions of the tests they are linking (same content and similar statistically)
• Better results obtained with more items (equating is more stable)
• Anchor items should occur approximately in the same place on each form
2) Place item parameters onto a common scale
Two Approaches

• Transformation procedures
  – Following separate calibrations, determine a linear transformation that puts item parameters from one calibration on same scale as another

• Calibration procedures
  – Scale equivalence is achieved through the actual calibration process
Parameter Invariance

• This assumption basically states that parameters are invariant *up to a linear transformation*, which accounts for the arbitrariness of the scale chosen to identify a solution.

• Check parameter invariance of difficulty through b-plots
Parameter Invariance

b-plot
Transformation Methods

• The argument is that the item parameter estimates (apart from sampling error) should be the same on both tests, regardless of the calibration sample (they’re the same items!).

• Systematic differences in item parameters are therefore a result of differences in the calibration samples.
Transformation Methods

- A linear transformation can be found that will place the item parameters from common items on one test to the same scale as the other test.
  - Shift in mean b-parameter: intercept
    - Accounts for mean difference in ability
  - Shift in scale of b-parameters: slope
    - Accounts for variability difference in ability
if \( \theta_{\text{new}} = x\theta + y \)

then \( b_{\text{new}} = xb + y \)

\[
\begin{align*}
\alpha_{\text{new}} &= \frac{a}{x} \\
\gamma_{\text{new}} &= c
\end{align*}
\]

These transformations preserve the probability:

\[
P(u = 1 \mid \theta_{\text{new}}) = P(u = 1 \mid \theta)
\]
Achieving a Common Scale

- Determine the set of equating constants (x and y) to place the b-parameter estimates from Form B onto the scale of Form A estimates.
- These constants will contain some error (not every pair of values falls perfectly on a line); best solution: make anchor test and sample sizes as large as possible
May have to remove items: Parameter Drift
Transformation procedures

- **Mean & Sigma (M-S)**
  After separate calibrations, determine the linear transformation that matches the mean and SD of anchor item b-values across administrations

- **Test Characteristic Curve (TCC)**
  (Stocking & Lord, 1983)
  After separate calibrations, determine the linear transformation that minimizes the average squared difference between anchor item TCCs across administrations
Mean & Sigma

• After separate calibrations, determine the linear transformation that matches the mean and SD of anchor item b-values across administrations:

\[ \chi = \frac{\sigma_{b-FormA}}{\sigma_{b-FormB}} \quad y = \mu_{b-FormA} - x \mu_{b-FormB} \]
Mean & Sigma

• This transformation places the scale of item parameters from “Form B” onto the scale of item parameters from “Form A”:

\[ x = \frac{\sigma_{b-\text{FormA}}}{\sigma_{b-\text{FormB}}} \quad y = \mu_{b-\text{FormA}} - x \mu_{b-\text{FormB}} \]
Mean & Sigma Example

• 25 Linking items
  – Form A: $\mu_{b-\text{FormA}} = -0.06$, $\sigma_{b-\text{FormA}} = 1.03$
  – Form B: $\mu_{b-\text{FormB}} = -0.25$, $\sigma_{b-\text{FormB}} = 1.12$

• $x = \frac{1.03}{1.12} = 0.92$

• $y = -0.06 - (0.92 \times -0.25) = 0.17$

• $b_{\text{new}} = x \ b_{\text{old}} + y$

• $b_{\text{new}} = 0.92 \ b_{\text{old}} + 0.17$
M & S Transformation of Form B
linking b-parameters

\[ b_{\text{new}} = x \, b_{\text{old}} + y \]
\[ b_{\text{new}} = 0.92 \, b_{\text{old}} + 0.17 \]

Intercept (y) is above zero (i.e., common items were easier).
This indicates that Group B had higher ability than Group A.
if
\[
\begin{align*}
\theta_{new} &= x\theta + y \\
b_{new} &= xb + y
\end{align*}
\]
then
\[
\begin{align*}
a_{new} &= a \\
c_{new} &= c
\end{align*}
\]

After item parameters are adjusted, the same transformation of b is done for all \(\theta\)...now the Group B examinees will look more able (as they should).

These transformations preserve the probability:
\[
P(u = 1 \mid \theta_{new}) = P(u = 1 \mid \theta)
\]
Mean & Sigma Equating Example

• To demonstrate the Mean and Sigma equating method, we will now work through an example.

• For the example, data from two tests were collected:
  – Form A
  – Form B

• Let’s assume the tests were constructed to be roughly parallel in terms of content and that they both measure a single latent trait.
Test Form Specifics

• For both forms:
  – Simulated data (for example purposes).
  – 60 multiple choice items total.
    • All scored correct/incorrect.
  – 10 anchor items common to each form.
    • These will be used for equating.
  – 10,000 examinees took each test.
    • No one took both tests.

• The data for each exam can be found in the files FormA.dat and FormB.dat.
Analysis Steps

1. Run a 3PL model in BILOG on Form A.
   - Save the item and examinee parameters.
2. Run a 3PL model in BILOG on Form B.
   - Again, save the item and examinee parameters.
3. Load the estimated item parameter values into Excel.
   - Find the parameters for the last 10 items for each exam.
Analysis Steps

4. Using the estimated b parameters, compute:

\[ x = \frac{\sigma_{b-FormA}}{\sigma_{b-FormB}} \]

\[ y = \mu_{b-FormA} - x \mu_{b-FormB} \]

5. Transform the a, b, and c parameters from Form B.

6. Transform the \( \theta \) parameters from Form B.
Notes

• We DON’T simply do an OLS regression of b(B) on b(A) → the solution is not symmetric

• If we just regressed b(B) on b(A):
  – Different $\theta$ estimates would be obtained depending on whether we equated A to B or B to A.

• Mean & Sigma will produce equivalent results regardless of which form is the reference.
Notes

• If Common Person instead of Common Item equating design was selected, then Mean & Sigma can be performed using $\theta$ instead of $b$.

• Recall that $\theta$ and $b$ are on the same scale. Whatever transformation you do to $\theta$, you do to $b$, and vice versa.
Mean & Sigma

• Two drawbacks:
  – Only b-values are used!
    • See TCC Method
  – Some b-values may be better estimated than others
    • See Robust Mean & Sigma
TCC Method

• After separate calibrations, determine the linear transformation that minimizes the average squared difference between anchor item TCCs across administrations:

\[
F = \frac{1}{N} \sum_{i=1}^{N} (\xi_{Ai} - \xi_{Bi})^2 \quad \text{“loss function”}
\]
Robust Mean & Sigma

- Proceed as with Mean & Sigma, but the contribution of each item to determining the equating constants is weighted by the standard errors of the b-parameters.
- The smaller the $SE(b_A)$ or $SE(b_B)$, the more weight that pair will have.
Calibration Procedures

• Concurrent Calibration, single group
  Joint calibration of multiple administrations; one ability distribution

• Concurrent Calibration, multiple groups
  Joint calibration of multiple administrations; multiple ability distributions

• Fixed Common Item Parameter (FCIP)
  Separate calibration for base administration; anchor item parameters fixed for subsequent administrations
Calibration Procedures

• Concurrent Calibration, single group
  – Response data from multiple forms are calibrated jointly
  – One ability distribution across administrations
  – Equivalent groups design
For a given year, we naturally assume that everyone in that administration comes from a single group.
Calibration Procedures

• Concurrent Calibration, multiple groups
  – Response data from multiple forms are calibrated jointly
  – Multiple ability distributions across administrations
  – Non-equivalent groups design
Across years, we can allow different distributions of examinee ability.

Scoring Items (Different items across Years)

Linking Items (Same items across Years)
Calibration Procedures

• Fixed Common Item Parameter (FCIP)
  – Separate calibration for base administration
  – Anchor item parameters for subsequent administrations are fixed at the values obtained in the base administration
  – This forces new item parameters to be estimated in relation to the fixed values, which are already on the same scale as the previous administration.
3) Perform the Equating
Calibration Procedures

- Equating is already done!
- The ability ($\theta$) estimates you get from the different forms as a result of such calibrations will be on the same scale as each other.
Linear Transformations

• After finding slope \((x)\) and intercept \((y)\):

\[
b_{\text{new}} = xb_{\text{old}} + y
\]

\[
a_{\text{new}} = \frac{a_{\text{old}}}{x}
\]

\[
\theta_{\text{new}} = x\theta_{\text{old}} + y
\]
Equated Scores

• Once the transformation or calibration is done, ability scores from two or more tests are comparable (i.e., they’re equated)
Conversions

• If a raw-score to raw-score conversion table is desired, then the TCC for each test (after equating) is needed to find comparable scores.

• **Ex.** A raw-score of 29 on Form B may be equivalent to a raw score of 27 on Form A (Form B is easier than Form A)

• Often estimated true scores (expected number correct scores) are used in place of actual (observed) raw scores.
Advantages of IRT Equating

• See Cook & Eignor (1991)

• Theoretical:
  – May be best when tests vary in difficulty and groups are of unequal ability
  – Equating results are independent of group or groups used to obtain them
Advantages of IRT Equating

• Practical:
  – Equating may be better at the extremes of the score distributions (don’t need extreme scores to do equating)
  – Equating many tests can be easily done once item parameter estimates are on a common scale.
Advantages of IRT Equating

• Practical:
  – Easy to redo equating if items need to be deleted from one or both forms
  – Pre-equating is possible if items are pilot-tested and placed onto a common scale
May have to remove items: Parameter Drift
Conclusion

• **Equating**: the purpose of test score “equating” is to put separate calibrations (which are scale indeterminate) on a common metric.

• This allows us to compare examinee score results from Form A to those Form B, or from Year 1 to Year 2.
Next…

- Item and Test Bias
- Differential Item Functioning
- Computerized Adaptive Testing (CAT)
- IRT Model Extensions for Special Testing Situations