Analysis of the Raven's Progressive Matrices (RPM) Scale Using Skills Assessment

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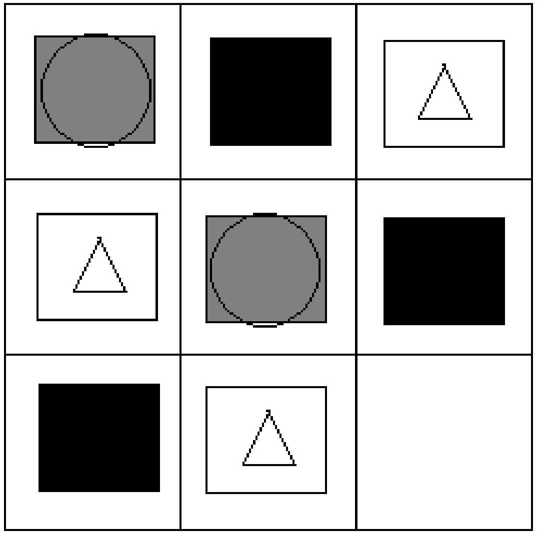
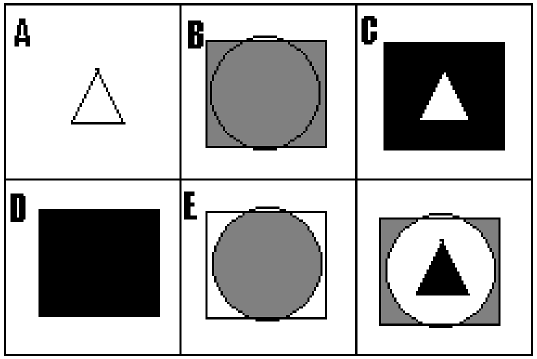
Overview

- Abstract Reasoning
  - Raven’s Progressive Matrices Test
  - Rules needed to provide successful responses.
- Cognitive Diagnosis Approaches to Measurement
  - The DINA Model with the RPMT data
- Current projects and future directions

Raven’s Progressive Matrices



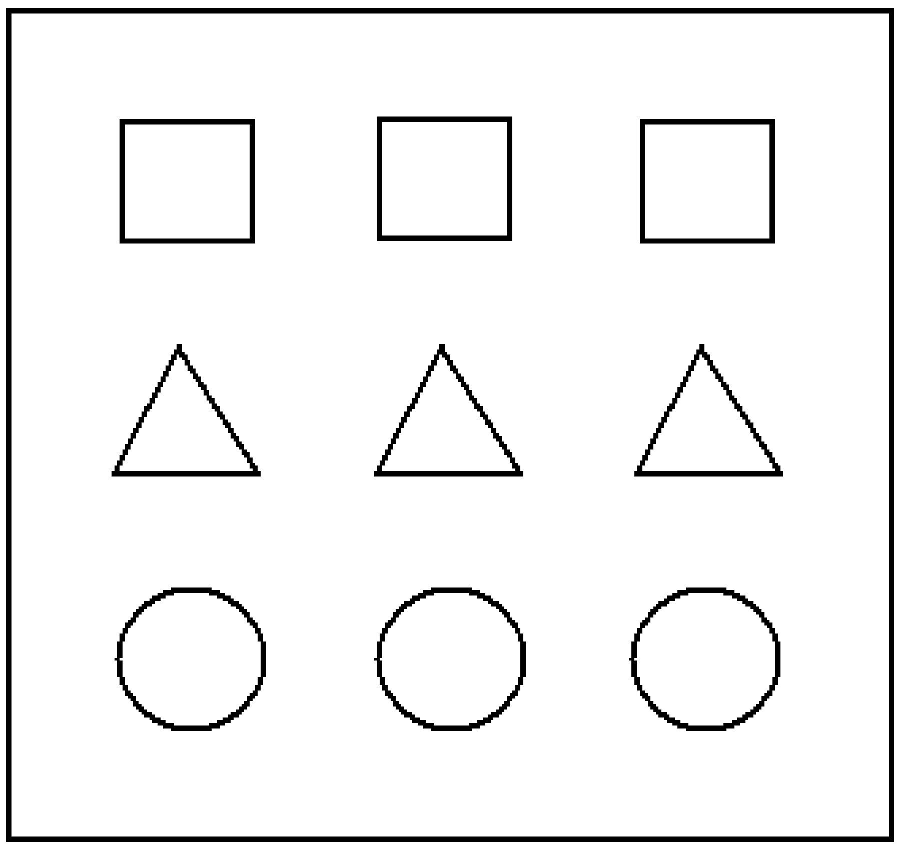


Rules for Solving RMT

- Carpenter, et al. (1990)
  1. Identity
  2. Progression
  3. Figure Addition/Subtraction
  4. Distribution of Three
  5. Distribution of Two

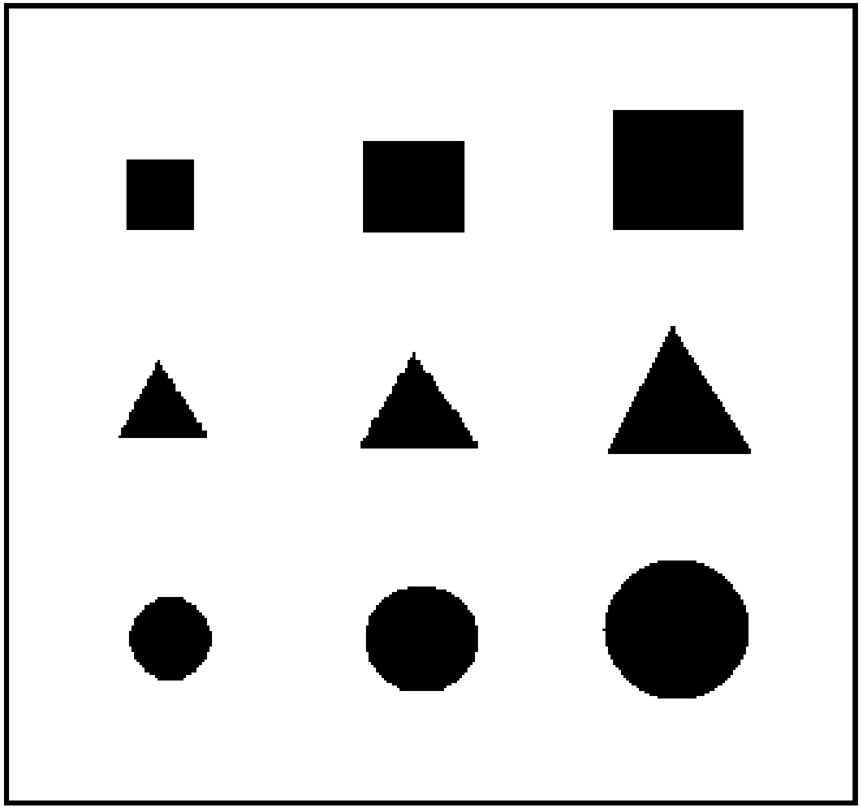
Rules for Solving RMT

1. Identity – Same figure across rows/columns



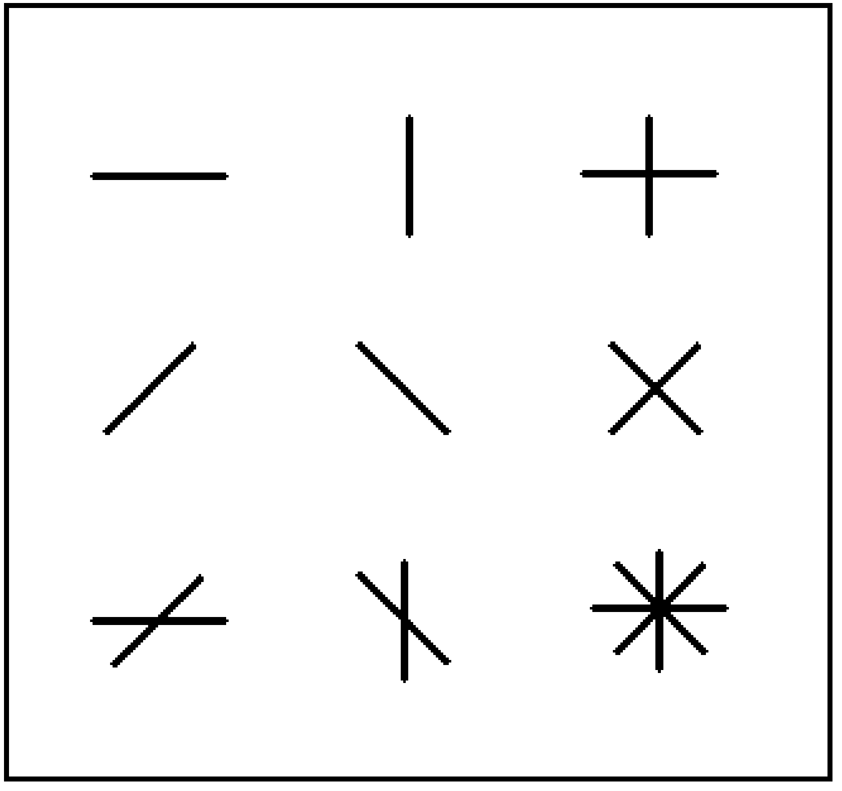
Rules for Solving RMT

2. Progression – Attributes change by a degree across rows/columns



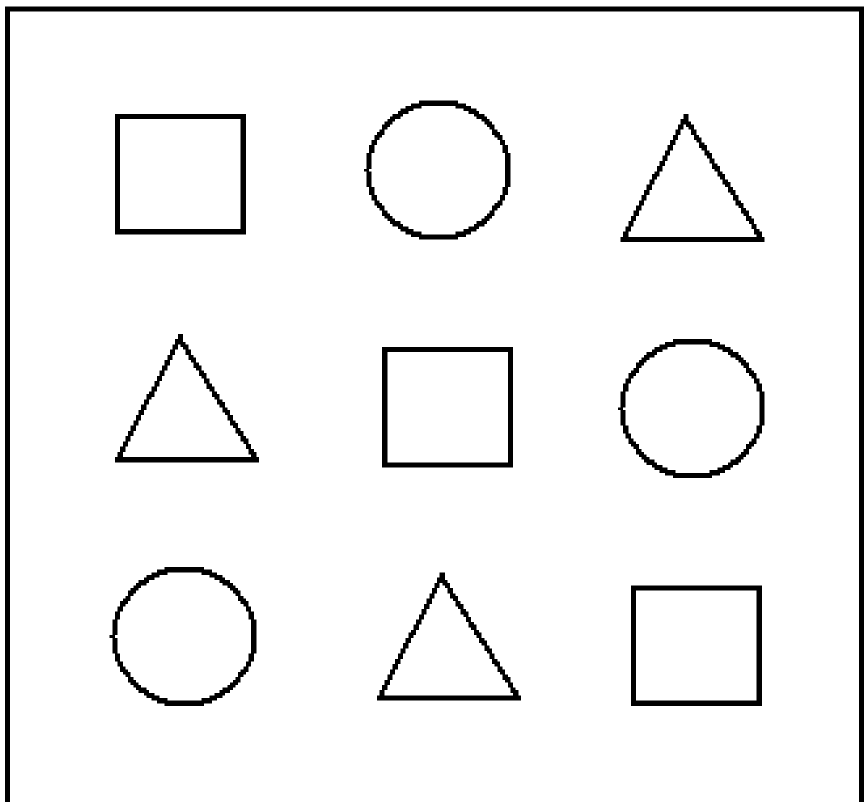
Rules for Solving RMT

3. Figure Addition/Subtraction – Attributes of first two elements are added/subtracted to make third element



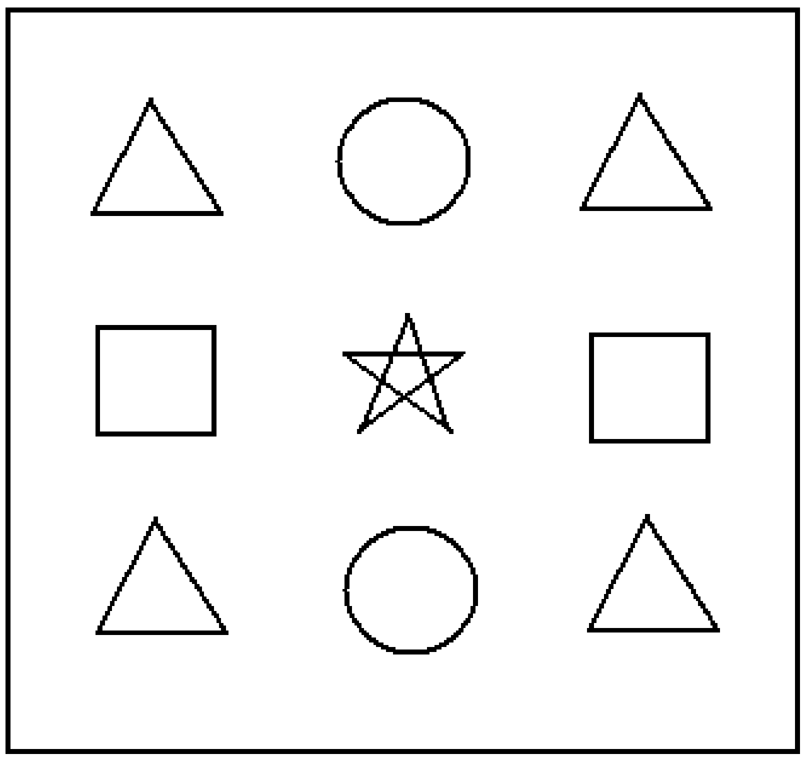
Rules for Solving RMT

4. Distribution of Three – 3 different elements are distributed evenly among the rows *and* columns

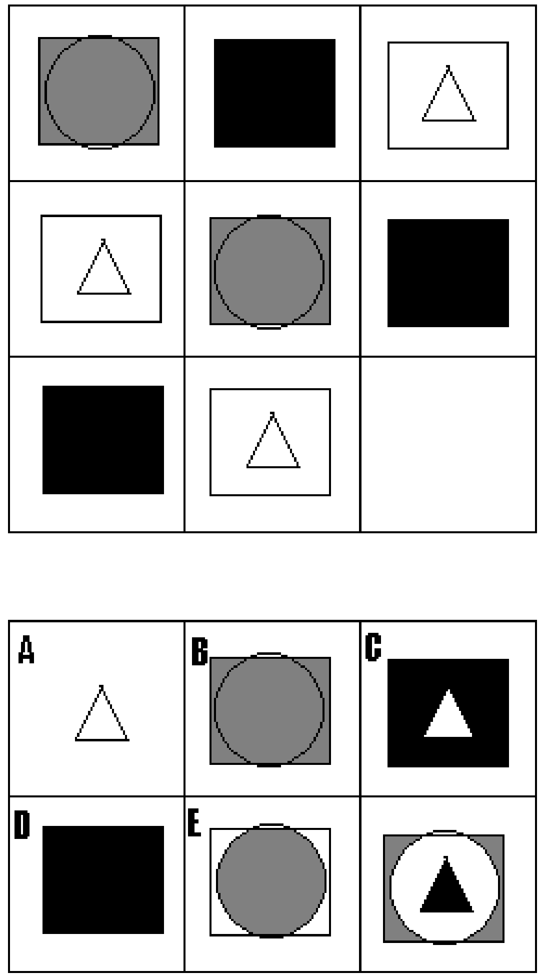


Rules for Solving RMT

5. Distribution of Two – 2 of the same element are found in each row/column with the third being a null value



Raven's Progressive Matrices



Raven’s Progressive Matrices

- Matrix completion task
  - Non-verbal intelligence measure
  - Speeded test
  - $N_{items} = 23$
  - Multiple-choice format with 6 choices
  - 1,364 6^{th} grade students

Q-Matrix

- Rules
  1. Identity (N_i = 10)
  2. Progression (N_i = 7)
  3. Add/Subtract (N_i = 9)
  4. Distribution of 3 (N_i = 6)
  5. *Distribution of 2 (N_i = 0)*

		Rule				
Item	Diff.	1	2	3	4	5
4	-1.160	0	0	0	1	0
5	-1.040	1	0	0	0	0
6	-0.300	0	0	0	1	0
7	0.122	1	0	0	1	0
8	0.129	0	0	1	0	0
11	1.441	0	0	1	0	0
16	-0.831	1	0	0	1	0
17	-1.100	0	1	0	0	0
18	-0.620	1	1	0	0	0
19	-0.190	1	0	1	0	0
20	0.306	0	0	1	0	0
21	0.485	1	0	1	0	0
22	0.937	0	0	0	1	0
23	1.536	1	0	1	0	0
24	1.279	0	1	0	0	0
28	-1.140	1	0	0	0	0
29	-1.080	0	1	0	0	0
30	0.114	1	1	0	0	0
31	0.253	0	0	0	1	0
32	0.420	1	1	0	0	0
33	0.460	0	1	0	0	0
34	0.695	0	0	1	0	0
35	1.279	1	1	0	0	0

Cognitive Diagnosis Modeling

- CDMs estimate profile of dichotomous skills (item attributes) an individual has mastered
- CDMs are special cases of latent class models
  - Defined by a set of dichotomous attributes
- Provides *why* students are not performing well, in addition to *which* students are not performing well

Cognitive Diagnosis Modeling

RPM Q-matrix

	Iden.	Prog.	Add/Sub	Dist. 3
Item 4	0	0	0	1
Item 5	1	0	0	0
Item 7	1	0	0	1

Possible Attribute Patterns

	Iden.	Prog.	Add/Sub	Dist.3		Expected Correct Responses
α_1	1	0	0	0	→	#5
α_2	0	1	0	0	→	None
α_3	1	0	1	1	→	#4, #5, #7

Cognitive Diagnosis Models

- Provide information regarding:
  1. Item-level information
     - High cognitive structure items separate groups more efficiently
  2. Examinee-level information (mastery profiles)
     - Most likely mastery profile
     - Probability an examinee has mastered each skill
  3. Population-level information
     - Probability distribution of skill mastery patterns
       - Can be used to determine skill hierarchies

The DINA Model

- Deterministic Input; Noisy "And" Gate
  (Macready & Dayton, 1977; Haertel, 1989; Junker & Sijstma, 2001)

  - Separates examinees into two classes per item:
    - Examinees who have mastered *all* necessary attributes
    - Examinees who have not mastered *all* necessary attributes

  - Ensures all attributes missing are treated equally, resulting in equal chance of "guessing" correctly

  - For each item, two parameters are estimated
    - For *J* items, 2 x *J* item parameters are modeled
      - A guessing parameter and a slip parameter
    - For our study, 2 x 23 = 46 item parameters are modeled

The DINA Model

- Deterministic Input; Noisy "And" Gate

$$P\left(X_{ij} = 1 \mid \xi_{ij}\right) = \left(1 - s_j\right)^{\xi_{ij}} g_j^{\left(1 - \xi_{ij}\right)}$$

$$\xi_{ij} = \prod_{k=1}^{K} \alpha_{ik}^{q_{jk}}$$

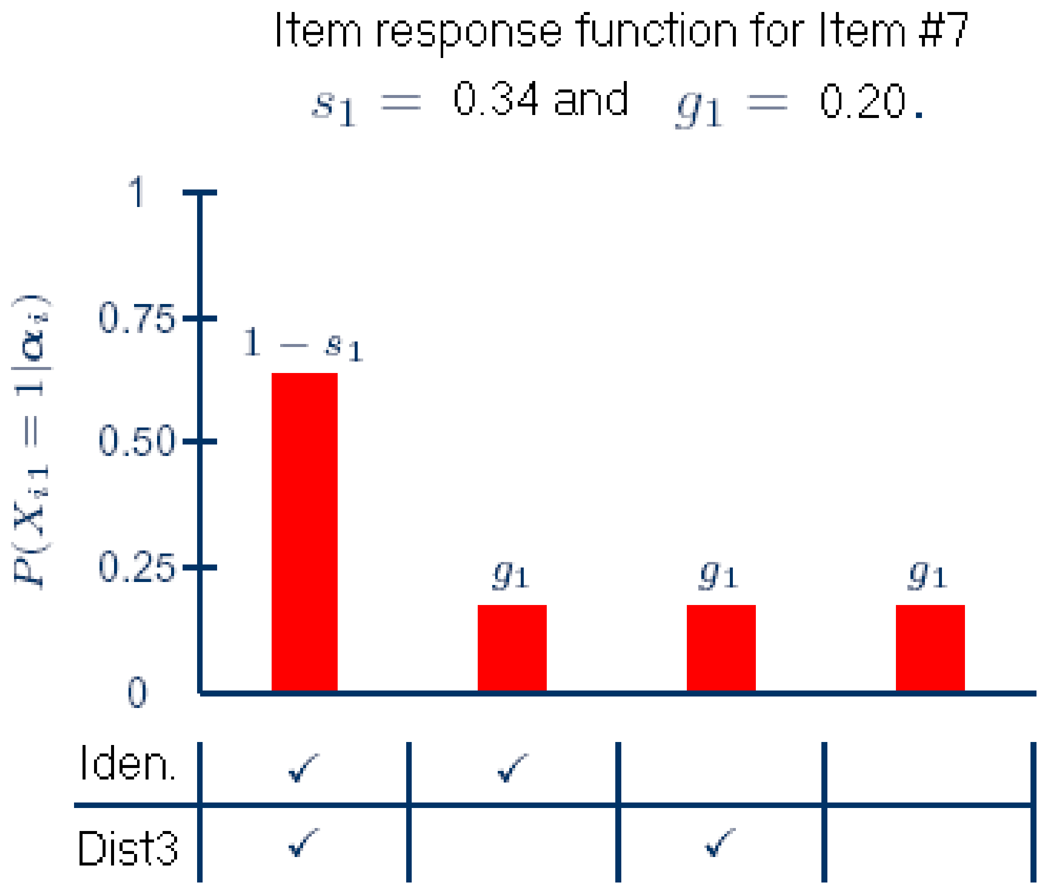
$$s_j = P\left(X_{ij} = 0 \mid \xi_{ij} = 1\right) - \text{"slip" parameter}$$

$$g_j = P\left(X_{ij} = 1 \mid \xi_{ij} = 0\right) - \text{"guess" parameter}$$

Item Attribute Assessment Estimates

- Consider item #7 {1 0 0 1}
  - Attributes necessary for success: Identity and Distribution of 3
- Imagine an examinee who has mastered both ($\xi_{i7} = 1$).
  - If $s_7 = .34$, thus $(1 - s_7) = .66$, this examinee has 66% of getting this item correct
- Imagine an examinee who has not mastered both ($\xi_{i7} = 0$).
  - If $g_7 = .20$, this examinee has a 20% chance of guessing correctly

Item Attribute Assessment Estimates



Item Results

Item	1-s	s	*se(s)*	g	*se(g)*	Diff.	p+
4	**0.984**	**0.016**	***0.006***	**0.744**	***0.028***	**-1.160**	**0.877**
5	**0.962**	**0.038**	***0.008***	**0.663**	***0.024***	**-1.040**	**0.851**
6	**0.812**	**0.188**	***0.017***	**0.376**	***0.016***	**-0.300**	**0.618**
7	**0.656**	**0.344**	***0.019***	**0.196**	***0.009***	**0.122**	**0.452**
8	**0.818**	**0.182**	***0.025***	**0.310**	***0.019***	**0.129**	**0.449**
11	**0.200**	**0.800**	***0.021***	**0.027**	***0.003***	**1.441**	**0.074**
16	**0.926**	**0.074**	***0.011***	**0.636**	***0.024***	**-0.831**	**0.798**
17	**0.991**	**0.010**	***0.006***	**0.794**	***0.038***	**-1.100**	**0.864**

- There is a significant correlation between (1-s) and percent correct
  - $r = .930$, $p < .01$
- There is a significant correlation between (1-s) and difficulty
  - $r = -.945$, $p < .01$

Item Results

Item	1-s	s	*se(s)*	g	*se(g)*	Diff.	p+
4	**0.984**	**0.016**	***0.006***	**0.744**	***0.028***	**-1.160**	**0.877**
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- Easier items have high (1-s) as well as high (g) parameters. Harder items have lower parameters. Average items tend to have high (1-s) and low (g).

Item Results

Item	1-s	s	*se(s)*	g	*se(g)*	Diff.	p+
4	**0.984**	**0.016**	***0.006***	**0.744**	***0.028***	**-1.160**	**0.877**
5	**0.962**	**0.038**	***0.008***	**0.663**	***0.024***	**-1.040**	**0.851**
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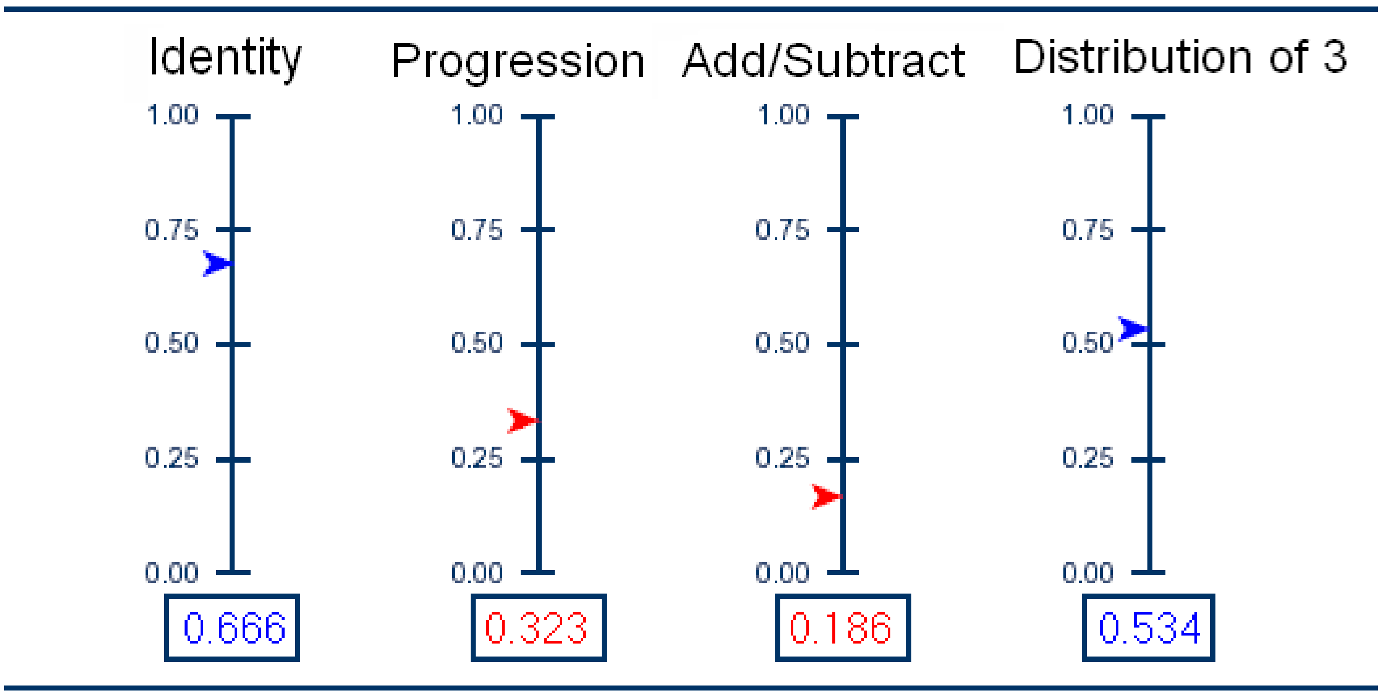
- Difference between (1-s) and g equals the discrimination of the item. So, item 4 is a low discriminating (.984-.744=.240) item. While, item 7 would be a more highly discriminating (.656-.196=.460) item.

Examinee Attribute Assessment Estimates

Posterior probabilities of attribute mastery:



Color Key:
➤ Probable Master
➤ Probable Non-master

Examinee Results

Examinee	Identity	Progress	Add/Sub	Dist of 3	Pattern
13	0.9995	0.9955	0.9393	0.9940	[1111]
14	0.7941	0.5564	0.5119	0.6986	[1111]
15	0.0215	0.0000	0.0000	0.0025	[0000]
16	0.0253	0.0000	0.0000	0.0093	[0000]
17	0.0651	0.0000	0.0000	0.0497	[0000]
18	0.0021	0.0000	0.0000	0.001	[0000]
19	0.0201	0.0000	0.0000	0.0013	[0000]
20	0.0652	0.0000	0.0001	0.0498	[0000]
21	0.5239	0.1396	0.0112	0.3468	[0000]
22	0.9927	0.9546	0.5632	0.9825	[1111]
23	0.8684	0.0005	0.0002	0.8240	[1001]
24	0.0201	0.0000	0.0000	0.0013	[0000]
25	0.5670	0.4029	0.0035	0.5559	[0000]
26	0.9537	0.8959	0.7165	0.9042	[1111]
27	0.9936	0.0644	0.0026	0.9920	[1001]
28	0.2668	0.0129	0.0002	0.0050	[0000]

- Posterior probabilities of mastery for each attribute for each examinee

Examinee Results

Examinee	Identity	Progress	Add/Sub	Dist of 3	Prob.
13	1	1	1	1	0.9334
14	1	1	1	1	0.4637
15	0	0	0	0	0.9783
16	0	0	0	0	0.9717
17	0	0	0	0	0.932
18	0	0	0	0	0.9975
19	0	0	0	0	0.9795
20	0	0	0	0	0.9319
21	0	0	0	0	0.4752
22	1	1	1	1	0.5588
23	1	0	0	1	0.8229
24	0	0	0	0	0.9795
25	0	0	0	0	0.4316
26	1	1	1	1	0.6912
27	1	0	0	1	0.9263
28	0	0	0	0	0.7331
Means	0.63	0.36	0.27	0.56	

- The Maximum a posteriori estimate of the most likely attribute pattern for an examinee.
- Most often patterns for this data [0000], [1111], and [1001] (p=.369, .251, and .193, respectively).

Population-level Results

α	Prob.
[0000]	0.369
[0001]	0.001
[0010]	0
[0011]	0
[0100]	0
[0101]	0
[0110]	0
[0111]	0
[1000]	0.062
[1001]	0.193
[1010]	0.001
[1011]	0.016
[1100]	0.006
[1101]	0.096
[1110]	0.006
[1111]	0.251

- The probability of possessing any attribute but not Identity is virtually 0.

Population-level Results

α	Prob.
[0000]	0.369
[0001]	0.001
[0010]	0
[0011]	0
[0100]	0
[0101]	0
[0110]	0
[0111]	0
[1000]	0.062
[1001]	0.193
[1010]	0.001
[1011]	0.016
[1100]	0.006
[1101]	0.096
[1110]	0.006
[1111]	0.251

- The probability of possessing any attribute but not Identity is virtually 0.
- The probability of possessing no attributes or possessing all attributes is more likely than possessing only some attributes.

Population-level Results

α	Prob.
[0000]	0.369
[0001]	0.001
[0010]	0
[0011]	0
[0100]	0
[0101]	0
[0110]	0
[0111]	0
[1000]	0.062
[1001]	0.193
[1010]	0.001
[1011]	0.016
[1100]	0.006
[1101]	0.096
[1110]	0.006
[1111]	0.251

- The probability of possessing any attribute but not Identity is virtually 0.
- The probability of possessing no attributes or possessing all attributes is more likely than possessing only some attributes.
- Possessing the attribute Identity and Dist. of 3 is more likely than Identity and Progression or Identity and Add/Subtraction.

Population-level Results

	Identity	Progress	Add/Sub	Dist of 3
Identity	1.000			
Progress	0.573**	1.000		
Add/Sub	0.470**	0.742**	1.000	
Dist of 3	0.854**	0.617**	0.517**	1.000

- Correlations between attributes
  - All significant, though much stronger between Distribution of 3 and Identity and between Progress and Addition/Subtraction

Summary

- CDM provides more than just an overall score
  - The likelihood that someone with a particular skill set will be able to solve an item
  - The most likely skill set that a person has
  - The likelihood that someone has mastered each skill
  - An overall picture of the skill sets of the population of interest