THE ANALYSIS OF INTERACTION COMPARISONS

ERSH 8310

Keppel and Wickens Chapter 13

Today's Class

- The Analysis of Interaction Comparisons
- Types of Interaction Components
- Analyzing Interaction Contrasts
- Orthogonal Interaction Contrasts
- Testing Contrast-by-Factor Interactions
- Contrasts Outside the Factorial Structure.
- Multiple Tests and Type I Error

The Analysis of Interaction Comparisons

The Analysis of Interaction Comparisons

- The presence of an interaction leads to the analysis of the simple effects and simple comparisons.
- One drawback to this approach is the proliferation of tests when an interaction is found.
- We need a statistical test to tell us when simple contrasts are different.
- Moreover, except in a 2×2 table, the $A \times B$ interaction is a composite effect with several degrees of freedom.
- We need a focused strategy to interpret the interaction.

Types of Interaction Components

Types of Interaction Components

- Whenever the omnibus A $\times$B interaction has more than one degree of freedom, individual components that express precise questions, called interaction components, can be extracted.
- The most exact questions are expressed by single-df interaction components, called the interaction contrasts.

Example

- Suppose that in a 3 ×3 factorial design 45 fifth-grade schoolchildren are randomly assigned to ab = 9 groups, with n = 5 subjects per group (see Table 13.1).
- The factor A has three levels:
  - a_1: The subjects receive no verbal feedback.
  - a_2: The subjects receive positive comments following each training trial.
  - a_3: The subjects receive negative comments following each training trial.
- The factor B has three levels:
  - b_1: Low-frequency words with low emotional content.
  - b_2: High-frequency words with low emotional content.
  - b_3: High-frequency works with high emotional content.

Interaction of a Contrast and a Factor

- Suppose we form the same simple contrast $\psi_{A \text{ at } bk}$ at every level of factor B.
  - We may want to determine whether the contrast interacts with factor B (see Table 13.2).
- Similarly, we can perform the same simple contrast $\psi_{B \text{ at } aj}$ at every level of factor A.
  - We may want to determine whether the contrast interacts with factor A.
- The $\psi_A \times B$ or $A \times \psi_B$ interaction components is one portion (i.e., partial interaction) of the A×B interaction.
  - When the interacting factor has more than two levels, these interactions are still composites.

Possible Factor A Contrasts

- Some possible contrasts for factor A are:
  - ψ_{A1}: A comparison between positive and negative feedback.
  - ψ_{A2}: A comparison between no feedback and positive feedback.
  - ψ_{A3}: A comparison between no feedback and negative feedback.
  - ψ_{A4}: A comparison between no feedback and some (i.e., either positive or negative) feedback.

Possible Factor B Contrasts

- Some possible contrasts for factor B are:
  - ψ_{B1}: A comparison of the recall for low- and high-frequency words with the same low emotional content (i.e., b_1 and b_2, respectively).
  - ψ_{B2}: A comparison of the recall for words of low and high emotional content with same high frequency (b_2 and b_3, respectively).

Interaction of Two Contrasts: Interaction Contrasts

- A more precise interaction effect is created by viewing both factors as contrasts.
- The resulting interaction contrast is formed by crossing the two contrasts.
  - This $\psi_A \times \psi_B$ interaction helps us pinpoint the source(s) of an omnibus $A \times B$ effect.
  - These interaction contrasts have one degree of freedom.
- We will use, for example, $\psi_{A1} \times \psi_{B2}$ or ψ_{A1B2} (see Table 13.3).

Analyzing Interaction Contrasts

Analyzing Interaction Contrasts

- A test of the null hypothesis that an interaction contrast is zero is fundamentally similar to the other single-df tests.

Testing Interaction Contrasts

The key to testing interaction contrasts is to set up

$$\psi_{AB} = \sum_{jk} c_{jk}\, \mu_{jk} \qquad (1)$$

with

$$\sum_{jk} c_{jk} = 0, \qquad (2)$$

where

$$c_{jk} = c_{Aj}\, c_{Bk}, \qquad (3)$$

where

$$\psi_A = \sum_{j} c_{Aj}\, \mu_{j} \qquad (4)$$

and

$$\psi_B = \sum_{k} c_{Bk}\, \mu_{k}. \qquad (5)$$

Testing Interaction Contrasts

The estimate of the contrast is obtained using the sample cell means (i.e., [$\bar{Y}]_{jk}$) and the corresponding c_{jk} (see Table 13.4). Hence,

$$\hat{\psi}_{AB} = \sum_{jk} c_{jk} \bar{Y}_{jk} \, . \qquad (6)$$

Note that

$$\sum_{jk} c_{jk}^2 = \left(\sum_{j} c_{Aj}^2 \right) \left(\sum_{k} c_{Bk}^2 \right) \qquad (7)$$

and

$$SS_{\psi AB} = \frac{n \, \hat{\psi}_{AB}^2}{\sum_{jk} c_{jk}^2} \, . \qquad (8)$$

The contrast has one degree of freedom. It is tested against the mean square from the complete design.

Orthogonal Interaction Contrasts

Orthogonal Interaction Contrasts

- An interaction contrast is a component of the overall $A \times B$ interaction.
- Sets of orthogonal interaction contrasts can completely capture the entire variability of the $A \times B$ interaction and the number of contrasts in these sets equals the degrees of freedom of the overall interaction.

Orthogonal Interaction Contrasts

- For such a set,

$$SS_{A \times B} = \sum SS_{\psi AB}.$$

The simplest way to construct a set is to base it on complete sets of orthogonal contrasts for each of the two factors (i.e., a-1 orthogonal contrasts for factor A and b-1 orthogonal contrasts for factor B).

Testing Contrast-by-Factor Interactions

Testing Contrast-by-Factor Interactions

- A partial interaction consists of a single-df comparison crossed with an interact factor.
- The interaction components have more than one degree of freedom and so are less specific than the interaction contrasts.

Computational Procedure

You may use, for example, a linear or quadratic contrast (see pp. 278-279).

Note that

$$SS_{A} + SS_{A \times B} = \sum_{k} SS_{A \text{ at } b_k} \qquad (12)$$

and that

$$SS_{\psi A} + SS_{\psi A \times B} = \sum_{k} SS_{\psi A \text{ at } b_k} \qquad (13)$$

Hence,

$$SS_{\psi A \times B} = \sum_{k} SS_{\psi A \text{ at } b_k} - SS_{\psi A} \qquad (14)$$

Computational Procedure

Hence,

$$SS_{\psi A\times B} = \sum_{k} SS_{\psi A\ \mathrm{at}\ b_k} - SS_{\psi A}. \qquad (14)$$

The degrees of freedom can be obtained by

$$df_{\psi A\times B} = df_{\psi A} \times df_{B}. \qquad (15)$$

The mean square and F ratio are obtained via

$$MS_{\psi A\times B} = \frac{SS_{\psi A\times B}}{df_{\psi A\times B}} \qquad (16)$$

and

$$F = \frac{MS_{\psi A\times B}}{MS_{S/AB}}. \qquad (17)$$

Contrasts Outside the Factorial Structure

Contrasts Outside the Factorial Structure

- The main-effect contrasts, simple contrasts, and interaction components are defined using the factorial structure of the design.
- Other contrasts can be constructed and tested using:

$$\hat{\psi} = \sum_{jk} c_{jk} \bar{Y}_{jk} \qquad (18)$$

and

$$SS_{\psi} = \frac{n\hat{\psi}^2}{\sum_{jk} c^2_{jk}}. \qquad (19)$$

Non-Factorial Contrasts

- In certain design specific comparisons that cross the factorial boundaries may be meaningful.

Predicted Patterns of the Group Means

- Contrasts that fall outside the conventional analysis also occur when they are used to measure agreement with the pattern suggested by a theory.

Multiple Tests and Type I Error

Multiple Tests and Type I Error

- The issues of multiple comparisons discussed in Chapter 6 and Section 12.7 may be relevant to interaction components and interaction contrasts.
- For example, error control in the family of interaction contrasts may be achieved with the Sidák-Bonferroni inequality (see Section 6.3).
- You may use the Scheffé criterion also,

$$F_{\text{Scheffé}} = df_{A \times B} F_{\alpha FW}(df_{A \times B}, df_{S/AB}).$$

Final Thought

- Today's class delved into what to do following the overall two-way analysis.
- Interaction effects were the focus of the discussion.

- Note that most everything discussed today came in the presence of the possible interaction between independent variables.
- Pretty much anything that you can do in a one-way ANOVA can be accomplished for interactions.

Next Class

- Chapter 14
  - The General Linear Model