INTRODUCTION TO THE FACTORIAL DESIGNS

ERSH 8310

Today's Class

- An introduction to factorial designs.
  - More than one manipulated factor in an experiment.
- Basic information available from factorial designs.
- The concept of interaction.
- The definition of an interaction.

Basic Information Available from Factorial Designs

Basic Information Available from Factorial Designs

- An experiment may be designed to focus attention on a single independent variable or factor.
- An alternative approach is to study the influence of one independent variable in conjunction with variations in one or more additional independent variables.
- We can study not only the effects of the two independent variables separately but also how they combine to influence the dependent variable.

Simple Effects of an Independent Variable

- A factorial design actually consists of a set of single-factor experiments.
  - See Table 10.1 for an example of a factorial design that investigates the format of the books (i.e., three line lengths and three letter-paper contrasts) on the reading speed.

- In the context of the factorial design, the results of the component single-factor experiments are called the simple effects of an independent variable.

Factorial Design Example (Table 10.1)

Table 10.1: A factorial experiment (left) and its interpretation as a set of single-factor experiments (right).

A Factorial Design

Contrast	Line Length: 3 in.	5 in.	7 in.
Low			
Medium			
High			

Three Single-Factor Designs

	3 in.	5 in.	7 in.
Low Contrast			

	3 in.	5 in.	7 in.
Medium Contrast			

	3 in.	5 in.	7 in.
High Contrast			

Interaction Effects

- A comparison among the simple effects of the component experiments (i.e., differences in the simple effects) is called the analysis of interaction.

- If the outcomes of the different component experiments within each set are the same, interaction is absent.

- If the outcomes are different, interaction is present.

Main Effects

- The main effects of an independent variable refer to the average of the component single-factor experiments making up the factorial design.
- We obtain the main effect by combining the individual treatment means from each component experiment.
- Note that main effects are most easily interpreted when interaction is absent.

Summary

- A factorial design produces three important pieces of information; the simple effects, the interaction effects, and the main effects.

The Concept of Interaction

Interaction is a new concept introduced by the factorial experiment.

An Example of No Interaction

- Table 10.3 presents some hypothetical results for the experiment on reading speed.
- Assume that equal numbers of children are included in each of the nine conditions and the values presented in the table represent the average reading scores found in the experiment.
- Factor A is the line length variable (a_1, a_2, a_3) and factor B is the contrast variable (b_1, b_2, b_3).
- If the plot of the marginal means and the plot of the cell means of the component single-factor experiments are parallel, there is no interaction (see Figure 10.1).

Table 10.3

Table 10.3: A hypothetical set of means for the reading experiment that shows no interaction.

	Line Length (Factor B)			
Contrast (Factor A)	3 in. (b_1)	5 in. (b_2)	7 in. (b_3)	Mean
Low (a_1)	0.89	2.22	2.89	2.00
Medium (a_2)	3.89	5.22	5.89	5.00
High (a_3)	4.22	5.55	6.22	5.33
Mean	3.00	4.33	5.00	4.11

Figure 10.1

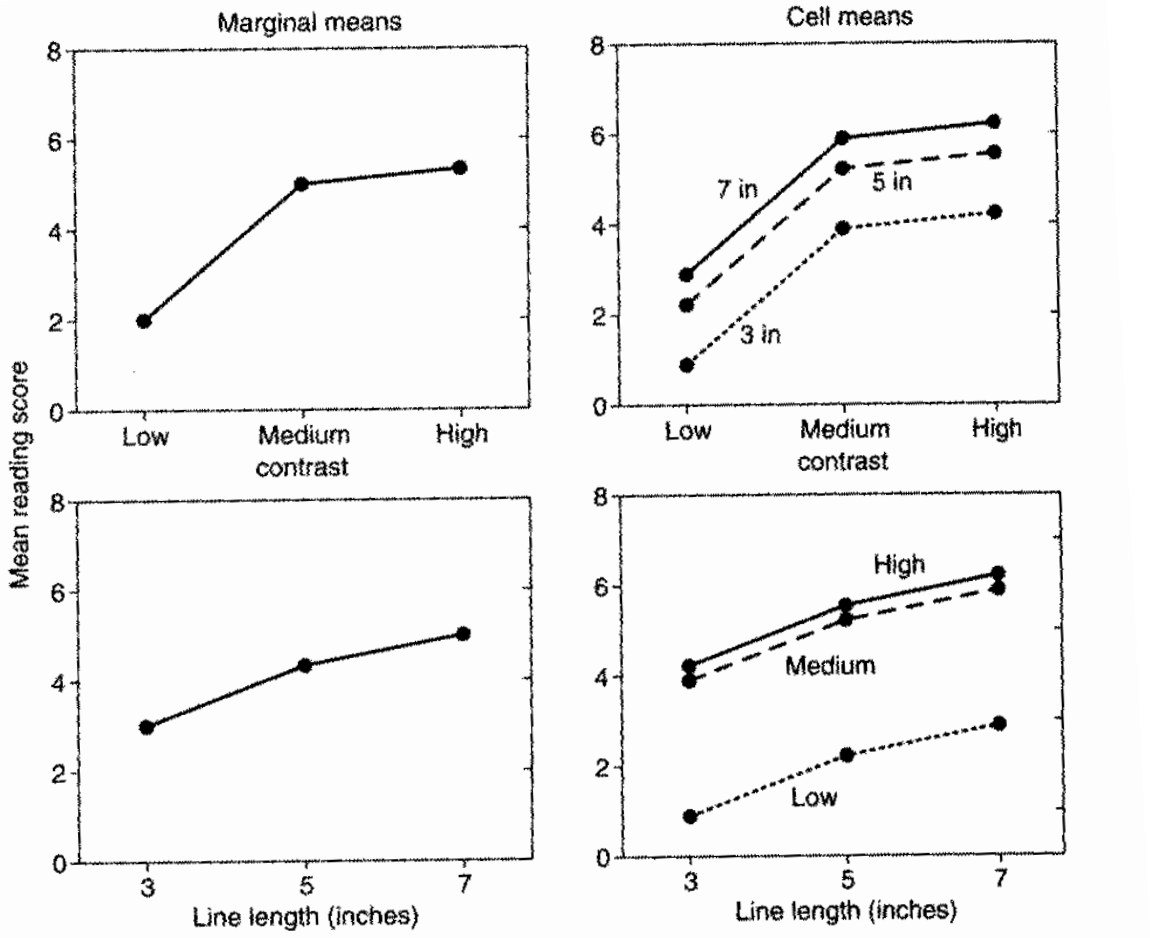

Figure 10.1: Reading scores from Table 10.3 plotted as a function of contrast (upper panel) and as a function of line length (lower panel). No interaction is present.

An Example of Interaction

- Table 10.4 presents a second set of hypothetical results using the same experimental design.
- Note that the same main effects are present.
- The patterns of differences reflected by the simple effects are not the same at all levels of the other independent variable, and interaction is present (see Figure 10.2).
- A simple way to describe the situation where interaction exists is that the cell mean lines are not parallel.

Table 10.4

Table 10.4: A hypothetical set of means for the reading experiment that shows an interaction.

	Line Length (Factor B)			
Contrast (Factor A)	3 in. (b_1)	5 in. (b_2)	7 in. (b_3)	Mean
Low (a_1)	1.00	2.00	3.00	2.00
Medium (a_2)	3.00	5.00	7.00	5.00
High (a_3)	5.00	6.00	5.00	5.33
Mean	3.00	4.33	5.00	4.11

Figure 10.2

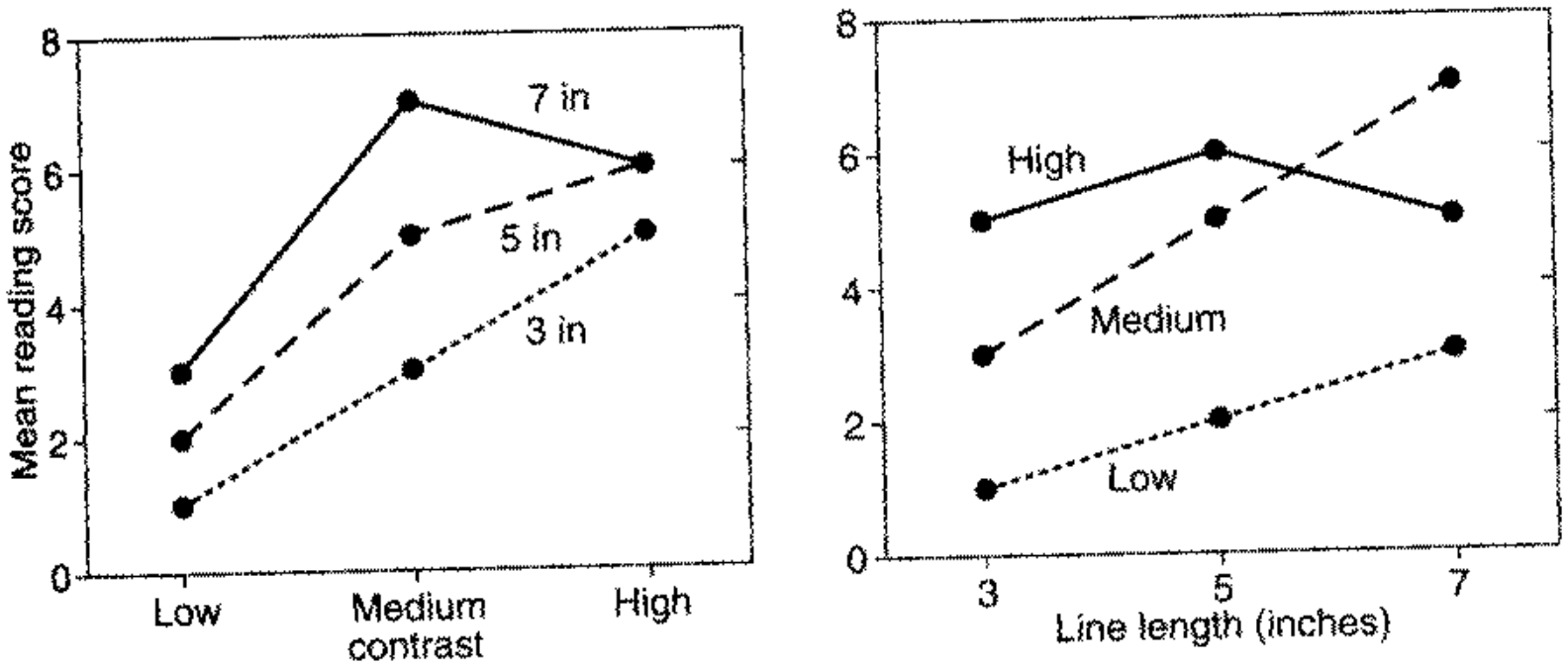

Figure 10.2: Reading scores from Table 10.4. An interaction is present.

The Definition of an Interaction

The Definition of an Interaction

1. An interaction is present when the effects of one independent variable on behavior change at the different levels of the second independent variable.
2. An interaction is present when the values of one or more contrasts in one independent variable changes at the different levels of the other independent variable.
3. An interaction is present when the simple effects of one independent variable are not the same at all levels of the second independent variable.

The Definition of an Interaction

4. An interaction is present when the main effect of an independent variable is not representative of the simple effects of that variable.
5. An interaction is present when the differences among the cell means representing the effect of factor A at one level of factor B do not equal the corresponding differences at another level of factor B.
6. An interaction is present when the effects of one of the independent variables are conditionally related to the levels of the other independent variable.

Interaction and Theoretical Analysis

- When interaction is found, there exist the complexity of post hoc explanations of a set of data.
- If behavior is complexly determined, we need factorial experiments to isolate and to tease out the complexities.
- The factorial design allows us to manipulate two or more independent variables concurrently and to obtain some idea of how the variables combine to produce the behavior.
- An assessment of the interaction provides a hint to the rules of combination.

Further Examples of Interaction

- Examples using a 2 ×2 factorial design are presented in Table 10.5 and Figure 10.3.

- The following are some the basic definitions:
  - A factorial design contains the conditions formed by combining each level of one independent variable with each level of another.
  - A simple effect expresses the differences among the means of one independent variable at a fixed level of the other independent variable.
  - The main effect expresses the difference among the means for one independent variable averaged over the levels of the other independent variable.
  - An interaction is present when the simple effects of one independent variable are not the same at all levels of the other independent variable.

Table 10.5

Table 10.5: Eight different outcomes of a two-factor experiment.

(1)

	b_1	b_2	$\bar{Y}_A$
a_1	5	5	5
a_2	5	5	5
$\bar{Y}_B$	5	5	

(2)

	b_1	b_2	$\bar{Y}_A$
a_1	4	4	4
a_2	6	6	6
$\bar{Y}_B$	5	5	

(3)

	b_1	b_2	$\bar{Y}_A$
a_1	7	3	5
a_2	7	3	5
$\bar{Y}_B$	7	3	

(4)

	b_1	b_2	$\bar{Y}_A$
a_1	6	2	4
a_2	8	4	6
$\bar{Y}_B$	7	3	

(5)

	b_1	b_2	$\bar{Y}_A$
a_1	6	4	5
a_2	4	6	5
$\bar{Y}_B$	5	5	

(6)

	b_1	b_2	$\bar{Y}_A$
a_1	5	3	4
a_2	5	7	6
$\bar{Y}_B$	5	5	

(7)

	b_1	b_2	$\bar{Y}_A$
a_1	8	2	5
a_2	6	4	5
$\bar{Y}_B$	7	3	

(8)

	b_1	b_2	$\bar{Y}_A$
a_1	7	1	4
a_2	7	5	6
$\bar{Y}_B$	7	3	

Figure 10.3

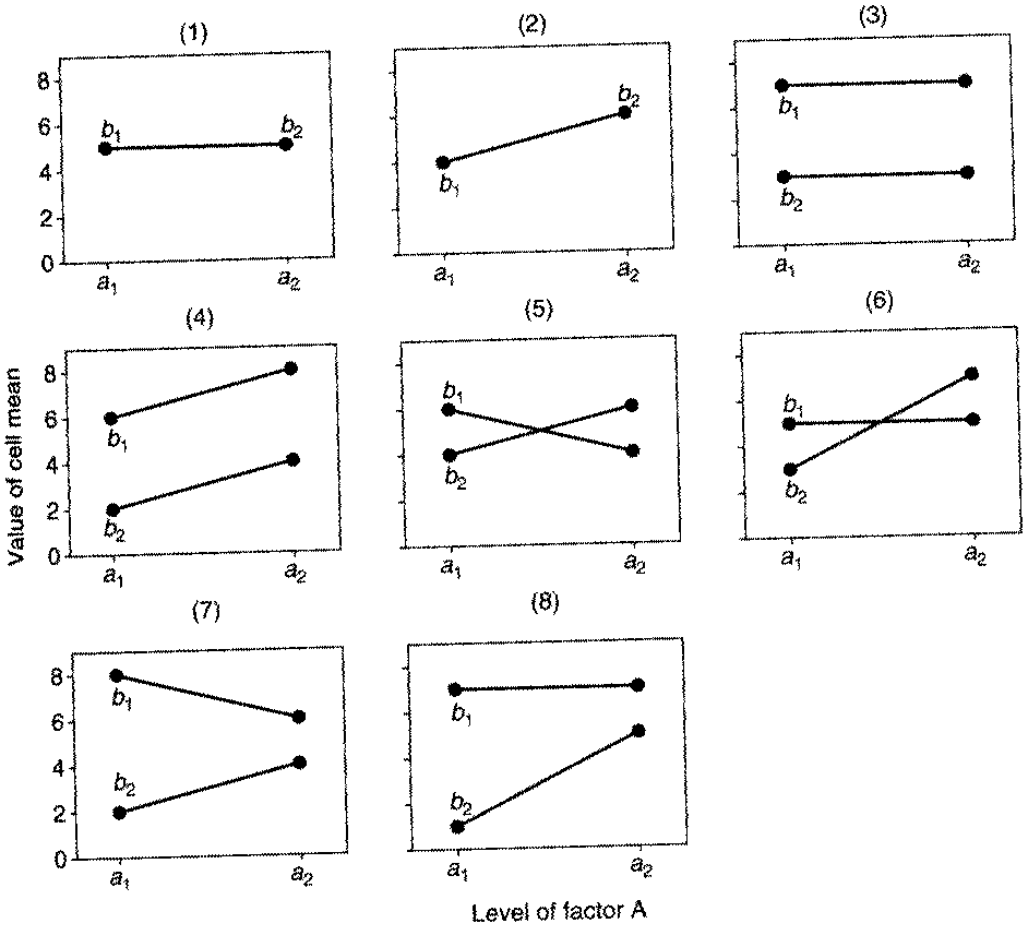

Figure 10.3: Plot of the cell means from Table 10.5.

Measurement of the Dependent Variable

- For the most part, measurement issues does not have a substantial impact on the one-way procedures.

- For two-factor designs, the choice of how we measure a concept may determine the size and nature of an interaction, and even its presence or absence (cf. nonremovable/crossover vs. removable interaction).

Final Thought

- Factorial ANOVA is a more complex analysis tool for looking at multi-factor studies.

- Multi-factor studies are important in that they more appropriately mimic the conditions found outside of the experiment.

Next Class

- Chapter 11: The overall two-way ANOVA.