Multivariate Models: Assessment of Absolute Model Fit

EPSY 905: Fundamentals of Multivariate Modeling

Online Lecture #11

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In This Lecture

- How to assess the fit of a multivariate linear model to the data, in an absolute sense

EXAMPLE DATA SET

Today's Data Example

- Data are simulated based on the results reported in:

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: a path analysis. *Journal of Educational Psychology, 86*, 193-203.

- Sample of 350 undergraduates (229 women, 121 men)
  - In simulation, 10% of variables were missing (using missing completely at random mechanism)

- Note: simulated data characteristics differ from actual data (some variables extend beyond their official range)
  - Simulated using Multivariate Normal Distribution
    - Some variables had boundaries that simulated data exceeded
  - Results will not match exactly due to missing data and boundaries

Variables of Data Example

- Female (sex variable: 0 = male; 1 = female)
- Math Self-Efficacy (MSE)
  - Reported reliability of .91
  - Assesses math confidence of college students
- Perceived Usefulness of Mathematics (USE)
  - Reported reliability of .93
- Math Anxiety (MAS)
  - Reported reliability ranging from .86 to .90
- Math Self-Concept (MSC)
  - Reported reliability of .93 to .95
- Prior Experience at High School Level (HSL)
  - Self report of number of years of high school during which students took mathematics courses
- Prior Experience at College Level (CC)
  - Self report of courses taken at college level
- Math Performance (PERF)
  - Reported reliability of .788
  - 18-item multiple choice instrument (total of correct responses)

METHODS OF EXAMINING MODEL FIT

Picking Up From Last Lecture…

- To demonstrate how models may vary in terms of model fit (and to set up a discussion of model fit and model comparisons) we will estimate a model where we set the covariance between PERF and USE to zero
  - Zero covariance implies zero correlation – which is unlikely to be true given our previous analysis
- You likely would not use this model in a real data analysis
  - If anything, you may start with a zero covariance and then estimate one
- But, this will help to introduce come concepts needed to assess the quality of the multivariate model

Model Notation

- The multivariate model for PERF and USE is given by two regression models, which are estimated simultaneously:

$$PERF_i = \beta_{0,PERF} + e_{i,PERF}$$
$$USE_i = \beta_{0,USE} + e_{i,USE}$$

- As there are two variables, the error terms have a joint distribution that will be multivariate normal:

$$\begin{bmatrix} e_{i,PERF} \\ e_{i,USE} \end{bmatrix} \sim N_2\left(\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} \sigma^2_{e,PERF} & 0 \\ 0 & \sigma^2_{e,USE} \end{bmatrix}\right)$$

- Each error term has its own variance and we do not estimate the covariance

Lavaan Syntax

- The lavaan() syntax for setting the covariance to zero is:

```
model03.syntax = "
#Variances:
  perf ~~ perf
  use  ~~ use

#Covariance:
  perf ~~ 0*use

#Means:
  perf ~ 1
  use  ~ 1
"
```

- The only difference is 0* in front of USE in the ~~ section
  - You can set a parameter equal to a value this way

Model Results

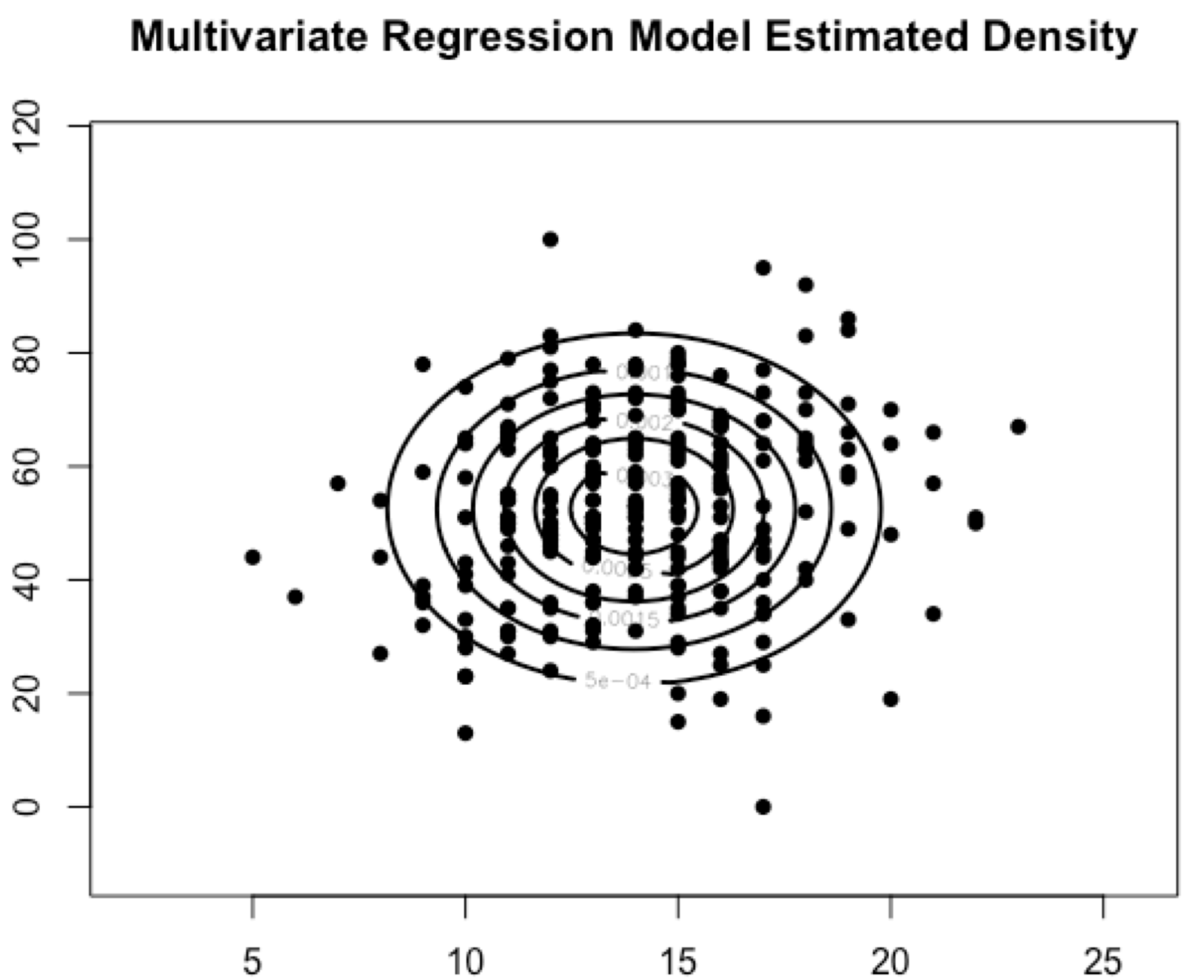
Model 3

```
Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  perf ~~
    use               0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
    perf             13.966    0.174   80.397    0.000
    use              52.500    0.874   60.047    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    perf              8.751    0.756   11.581    0.000
    use             249.201   19.212   12.971    0.000
```

Examining Model/Data Fit



Methods of Model Fit

- Model-data fit is of utmost concern when building models with multivariate outcomes

- If a model does not fit the data:
  - Parameter estimates may be biased
  - Standard errors of estimates may be biased
  - Inferences made from the model may be wrong
  - If the saturated model fit is wrong, then the LRTs will be inaccurate

- Examining model fit is the first step in multivariate models

- That said, not all "good-fitting" models are useful…
  - …model fit just allows you to talk about your model…there may be nothing of significance (statistically or practically) in your results, though

GLOBAL MEASURES OF MODEL FIT

Types of Model Fit Information

- Model fit information for models where outcomes are <u>conditionally MVN*</u> come in several types, but all are based on the premise that any model mean and covariance structure must fit <u>as well as</u> the saturated mean vector and covariance matrix model

  ***If model outcomes are not conditionally MVN, model fit is very different**

- All possible models/structures **are nested within** the saturated mean vector and covariance matrix model
  - Most model fit statistics come from comparing any model/structure with the saturated model

- Indices shown first are called "global" model fit indices
  - Report fit of model globally (as opposed to locally for specific parameters)

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Example lavaan Model Fit Output

```
> summary(model03.fit, fit.measures=TRUE)
lavaan (0.5-23.1097) converged normally after  18 iterations

                                                  Used       Total
  Number of observations                           348         350

  Number of missing patterns                         3

  Estimator                                         ML      Robust
  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value (Chi-square)                           0.014       0.018
  Scaling correction factor                                  1.088
    for the Yuan-Bentler correction (Mplus variant)

Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.000       0.000
  Tucker-Lewis Index (TLI)                      -0.000      -0.000

  Robust Comparative Fit Index (CFI)                         0.000
  Robust Tucker-Lewis Index (TLI)                           -0.000

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2088.064   -2088.064
  Scaling correction factor                                  1.012
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -2085.032   -2085.032
  Scaling correction factor                                  1.028
    for the MLR correction

  Number of free parameters                          4           4
  Akaike (AIC)                                4184.128    4184.128
  Bayesian (BIC)                              4199.537    4199.537
  Sample-size adjusted Bayesian (BIC)         4186.848    4186.848

Root Mean Square Error of Approximation:

  RMSEA                                          0.121       0.115
  90 Percent Confidence Interval          0.044  0.220       0.040  0.210
  P-value RMSEA <= 0.05                          0.063       0.073

  Robust RMSEA                                               0.120
  90 Percent Confidence Interval                             0.039  0.224

Standardized Root Mean Square Residual:

  SRMR                                           0.066       0.066
```

The fit.measures=TRUE Model Fit Statistics

- **Unlabeled section**
  - **Likelihood ratio test versus the saturated model**
  - **Testing if your model fits as well as the saturated model**
- Model test baseline model
  - Likelihood ratio test pitting the saturated model against the independent variables model
  - Testing whether any variables have non-zero covariances (significant correlations)
- User model versus baseline model
  - CFI
  - TLI
- Loglikelihood and Information Criteria
  - Likelihood ratio tests (nested models)
  - Information criteria comparisons (non-nested models)
- Root Mean Square Error of Approximation
  - How far off a model is from the saturated model, per degree of freedom
- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

Indices of Global Model Fit

- Primary: obtained model $\boldsymbol{\chi^2}$ (from Model test baseline model) – here we use the MLR rescaled $\boldsymbol{\chi^2}$ from the "Robust" Column (next lecture)
  - χ^2 is evaluated based on model df (difference in parameters between your CFA model and the saturated model)
  - Tests null hypothesis that ***this*** model (H_0) fits equally to ***saturated model*** (H_1) so significance is undesirable (smaller χ^2, bigger p-value is better)
    - Means saturated model is estimated **automatically** for each model analyzed
  - Just using χ^2 is insufficient, however:
    - Distribution doesn't behave like a true χ^2 if sample sizes are small (or, if not using MLR, if items are non-normally distributed)
    - Obtained χ^2 depends largely on sample size
    - Some mention this is an unreasonable null hypothesis (perfect fit??)

- Because of these issues, alternative measures of fit are usually used in conjunction with the χ^2 test of model fit
  - Absolute Fit Indices (besides χ^2)
  - Parsimony-Corrected; Comparative (Incremental) Fit Indices

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Chi-Square Test of Model Fit

- The Chi-Square Test of Model Fit provides a likelihood ratio test comparing the current model to the **saturated (unstructured) model**:
  - The value is -2 times the difference in log-likelihoods (rescaled if MLR)
  - The degrees of freedom is the difference in the number of estimated model parameters
  - The p-value is from the Chi-square distribution
- **If this test has a significant p-value:**
  - The current model (H_0) is rejected – the model fit is significantly worse than the full model
  - In latent variable models, this test is usually ignored
    - Said to be overly sensitive
- **If this test does not have a significant p-value:**
  - The current model (H_0) is not rejected – **fits equivalently to full model**

Where the Saturated Model Test Comes From

- The saturated model LRT comes from a likelihood ratio test of the current model with the saturated model
- If using MLR (Robust method), then this LRT is rescaled based on the estimated scaling factors of both models
- This same information can be obtained from:
  - Loglikelihood model output section
  - anova() function comparing fit for current and saturated models

Calculating the LRT for Global Fit Test for Model 03

- From the lavaan output:

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                6.064        5.573
Degrees of freedom                                 1            1
P-value (Chi-square)                           0.014        0.018
Scaling correction factor                                   1.088
  for the Yuan-Bentler correction (Mplus variant)
```

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2088.064   -2088.064
  Scaling correction factor                                  1.012
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -2085.032   -2085.032
  Scaling correction factor                                  1.028
    for the MLR correction
```

- Calculation:
  - 14 parameters in our model; 20 in saturated model
  - Scaling correction factor:

$$c_{LR} = \left| \frac{(q_{restricted})(c_{restricted}) - (q_{full})(c_{full})}{(q_{restricted} - q_{full})} \right| = 1.088$$

  - $\chi^2 = \frac{6.064}{1.008} = 5.573$
  - DF = 1

- Conclusion: this model fit significantly worse than the saturated model

Saturated Model LRT and Loglikelihood Output

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)                -2088.064   -2088.064
  Scaling correction factor                                    1.012
    for the MLR correction
  Loglikelihood unrestricted model (H1)        -2085.032   -2085.032
  Scaling correction factor                                    1.028
    for the MLR correction
```

- If the loglikelihoods of the current model ("User model" or H_0) are equal to the loglikelihoods of the saturated model ("Unrestriced model" or H_1), then you are running a model that is equivalent to the saturated model
  - **<u>No other model fit will be available or useful</u>**

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~

- **Model test baseline model**
  - **Likelihood ratio test pitting the saturated model against the independent variables model**
  - **Testing whether any variables have non-zero covariances (significant correlations)**

- User model versus baseline model
  - CFI
  - TLI

- Loglikelihood and Information Criteria
  - Likelihood ratio tests (nested models)
  - Information criteria comparisons (non-nested models)

- Root Mean Square Error of Approximation
  - How far off a model is from the saturated model, per degree of freedom

- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

Model Test Baseline Model

- The "model test baseline model" section provides a LRT:
  - Comparing the saturated (unstructured) model with an independent variables model (zero covariance)

```
Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018
```

- Here, the "null" model is the baseline
  - If the test is significant, this means that at least one (and likely more than one) variable has a significant covariance (and correlation)
  - If the test is not significant, this means that the independence model is appropriate
    - This is not likely to happen
    - But if it does, there are virtually no other models that will be significant
- Not often reported as it is likely variables are correlated

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~

- ~~Model test baseline model~~
  - ~~Likelihood ratio test pitting the saturated model against the independent variables model~~
  - ~~Testing whether any variables have non-zero covariances (significant correlations)~~

- **User model versus baseline model**
  - **CFI**
  - **TLI**

- Loglikelihood and Information Criteria
  - Likelihood ratio tests (nested models)
  - Information criteria comparisons (non-nested models)

- Root Mean Square Error of Approximation
  - How far off a model is from the saturated model, per degree of freedom

- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

User Model Versus Baseline Model Section

- The "User model versus baseline model" section provides two additional measures of model fit comparing the current (user) model to the baseline (independent variables) model

```
User model versus baseline model:

  Comparative Fit Index (CFI)                    0.000       0.000
  Tucker-Lewis Index (TLI)                      -0.000      -0.000

  Robust Comparative Fit Index (CFI)                         0.000
  Robust Tucker-Lewis Index (TLI)                           -0.000
```

- CFI stands for Comparative Fit Index
  - Higher is better (above .95 indicates good fit)
- TLI stands for Tucker Lewis Index
  - Higher is better (above .95 indicates good fit)

Comparative (Incremental) Fit Indices

- Fit evaluated relative to a 'null' model (of 0 covariances)
  - Relative to that, your model should be great!

T = target (current/estimated) model
N = null (baseline/independent variables) model

- **CFI: Comparative Fit Index**
  - Based on idea of the chi-square non-centrality parameter: (χ^2 – df)
  - $CFI = 1 - \frac{\max\left(\chi_T^2 - df_T, 0\right)}{\max\left(\chi_T^2 - df_T, \chi_N^2 - df_N, 0\right)}$
  - From 0 to 1: bigger is better, > .90 = "acceptable", > .95 = "good"

- **TLI: Tucker-Lewis Index** (= **Non-Normed Fit Index**)
  - $TLI = \frac{\frac{\chi_N^2}{df_N} - \frac{\chi_T^2}{df_T}}{\frac{\chi_N^2}{df_N} - 1}$
  - From <0 to >1, bigger is better, >.95 = "good"

Information Criteria Output

- The information criteria output provides relative fit statistics:

Number of free parameters	4	4
Akaike (AIC)	4184.128	4184.128
Bayesian (BIC)	4199.537	4199.537
Sample-size adjusted Bayesian (BIC)	4186.848	4186.848

  - AIC: Akaike Information Criterion
  - BIC: Bayesian Information Criterion (also called Schwarz's criterion)
  - Sample-size Adjusted BIC

- These statistics weight the information given by the parameter values by the parsimony of the model (the number of model parameters)
  - For all statistics, the smaller number is better
- The core of these statistics is -2*log-likelihood

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~
- ~~Model test baseline model~~
  - ~~Likelihood ratio test pitting the saturated model against the independent variables model~~
  - ~~Testing whether any variables have non-zero covariances (significant correlations)~~
- ~~User model versus baseline model~~
  - ~~CFI~~
  - ~~TLI~~
- **Loglikelihood and Information Criteria**
  - **Likelihood ratio tests (nested models)**
  - **Information criteria comparisons (non-nested models)**
- Root Mean Square Error of Approximation
  - How far off a model is from the saturated model, per degree of freedom
- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

Comparing Information Criteria

- Information criteria are relative tests of fit

Number of free parameters	4	4
Akaike (AIC)	4184.128	4184.128
Bayesian (BIC)	4199.537	4199.537
Sample-size adjusted Bayesian (BIC)	4186.848	4186.848

- The are calculated based on the log-likelihood of the model, factoring in a penalty for number of parameters (plus other things)

- They should never be used to compare nested models
  - The likelihood ratio test is the most powerful test statistic to use for nested models

- When comparing non-nested models, first choose a statistic
  - AIC, BIC, or Sample-size Adjusted BIC are what are given by default

- The preferred model is the one with the lowest value of that statistic

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~
- ~~Model test baseline model~~
  - ~~Likelihood ratio test pitting the saturated model against the independent variables model~~
  - ~~Testing whether any variables have non-zero covariances (significant correlations)~~
- ~~User model versus baseline model~~
  - ~~CFI~~
  - ~~TLI~~
- ~~Loglikelihood and Information Criteria~~
  - ~~Likelihood ratio tests (nested models)~~
  - ~~Information criteria comparisons (non-nested models)~~
- **Root Mean Square Error of Approximation**
  - **How far off a model is from the saturated model, per degree of freedom**
- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

Indices of Global Model Fit

Parsimony-Corrected: **RMSEA**

- **Root Mean Square Error of Approximation**
- Uses comparison with CFA model and saturated model
  - χ^2 listed here from first part of lavaan output
- Relies on a non-centrality parameter (NCP)
  - Indexes how far off your model is → χ^2 distribution shoved over
  - NCP → $d = (\chi^2 - df) / (N-1)$ Then, $RMSEA = SQRT(d/df)$
    - df is difference between # parameters in CFA model and saturated model
  - RMSEA ranges from 0 to 1; smaller is better
    - < .05 or .06 = "good", .05 to .08 = "acceptable", .08 to .10 = "mediocre", and >.10 = "unacceptable"
  - In addition to point estimate, get 90% confidence interval
  - RMSEA penalizes for model complexity – it's discrepancy in fit per df left in model (but not sensitive to N, although CI can be)
  - Test of "close fit": null hypothesis that RMSEA ≤ .05

RMSEA (Root Mean Square Error of Approximation)

- The RMSEA is an index of model fit where 0 indicates perfect fit (smaller is better):

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.121       0.115
  90 Percent Confidence Interval          0.044  0.220       0.040  0.210
  P-value RMSEA <= 0.05                          0.063       0.073

  Robust RMSEA                                               0.120
  90 Percent Confidence Interval                             0.039  0.224
```

- RMSEA is based on the approximated covariance matrix

- The goal is a model with an RMSEA less than .05
  - Although there is some flexibility

- The result above indicates our model fits poorly (Robust RMSEA of .120)

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~
- ~~Model test baseline model~~
  - ~~Likelihood ratio test pitting the saturated model against the independent variables model~~
  - ~~Testing whether any variables have non-zero covariances (significant correlations)~~
- ~~User model versus baseline model~~
  - ~~CFI~~
  - ~~TLI~~
- ~~Loglikelihood and Information Criteria~~
  - ~~Likelihood ratio tests (nested models)~~
  - ~~Information criteria comparisons (non-nested models)~~
- ~~Root Mean Square Error of Approximation~~
  - ~~How far off a model is from the saturated model, per degree of freedom~~
- **Standardized Root Mean Square Residual**
  - **How far off a model's correlations are from the saturated model correlations**

Standardized Root Mean Squared Residual

- The SRMR (standardized root mean square residual) provides the average standardized difference between:
  - The estimated covariance matrix of the saturated model
  - The estimated covariance matrix of the current model

```
Standardized Root Mean Square Residual:

  SRMR                                        0.066      0.066
```

- Lower is better (some suggest less than 0.08)

LOCAL MODEL FIT MEASURES

“Local” Measures of Model (Mis)Fit

- Local measures of model (mis)fit are statistics that point to the location (typically of a covariance matrix) where a model may not fit well
  - As opposed to “global” measures that indicate a model fit overall

- Local measures of model (mis)fit are typically of two types:
  - Residual covariance matrices (unstandardized, standardized, or normalized)
    - The difference between the model’s estimated covariance matrix and the saturated model’s estimated covariance matrix
    - These were used for the SRMR
  - Model “modification indices”
    - 1-degree of freedom hypothesis tests for the improvement of the model LRT if one more parameter was allowed to be estimated

Residual Covariance Matrices

- Residual covariance matrices are used to figure out how to best improve model misfit

- The "raw" or "unstandardized" residual covariance matrix for the model literally takes the difference between model implied and saturated model covariance matrices

- I often prefer "normalized" versions of these matrices
  - We can inspect the normalized residual covariance matrix (like z-scores) to see where our biggest misfit occurs

```
> residuals(model03.fit, type = "normalized")
$type
[1] "normalized"

$cov
       perf    use
perf -0.012
use   2.403  0.002

$mean
  perf    use
-0.037 -0.069
```

Modification Indices: More Help for Fit

- As we used Maximum Likelihood to estimate our model, another useful feature is that of the modification indices
  - Modification indices, also called Score or LaGrange Multiplier tests, attempt to suggest the change in the log-likelihood for adding a given model parameter (larger values indicate a better fit for adding the parameter)

```
> modindices(model03.fit)
   lhs op rhs    mi mi.scaled   epc sepc.lv sepc.all sepc.nox
3 perf ~~ use 6.239     5.734 7.125   7.125    0.153    0.153
```

- mi column: the expected value of the LRT of the current model and a model where this parameter was added
- mi.scaled column: the scaled (robust) LRT
  - Should be bigger than 3.84 for 1 df
  - Practice is to find values that are much higher (say 10 or more)
- epc column: expected value of the parameter in the model where this parameter was added

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COMPARING INDICES

Model Fit for the Model with Covariance Estimated

```
> summary(model02.fit, fit.measures=TRUE)
lavaan (0.5-23.1097) converged normally after  29 iterations

                                                  Used       Total
  Number of observations                           348         350

  Number of missing patterns                         3

  Estimator                                         ML      Robust
  Minimum Function Test Statistic                0.000       0.000
  Degrees of freedom                                 0           0
  Minimum Function Value               0.0000000000000
  Scaling correction factor                                     NA
    for the Yuan-Bentler correction (Mplus variant)

Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018

User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000       1.000
  Tucker-Lewis Index (TLI)                       1.000       1.000

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA
```

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)          -2085.032   -2085.032
  Loglikelihood unrestricted model (H1)  -2085.032   -2085.032

  Number of free parameters                      5           5
  Akaike (AIC)                            4180.064    4180.064
  Bayesian (BIC)                          4199.325    4199.325
  Sample-size adjusted Bayesian (BIC)     4183.464    4183.464

Root Mean Square Error of Approximation:

  RMSEA                                      0.000       0.000
  90 Percent Confidence Interval      0.000  0.000       0.000  0.000
  P-value RMSEA <= 0.05                         NA          NA

  Robust RMSEA                                           0.000
  90 Percent Confidence Interval                         0.000  0.000

Standardized Root Mean Square Residual:

  SRMR                                       0.000       0.000
```

Model Fit for the Univariate Model

```
> summary(model01.fit, fit.measures=TRUE)
lavaan (0.5-23.1097) converged normally after   8 iterations

                                                  Used       Total
  Number of observations                           290         350

  Number of missing patterns                         1

  Estimator                                         ML      Robust
  Minimum Function Test Statistic                0.000       0.000
  Degrees of freedom                                 0           0
  Scaling correction factor                                     NA
    for the Yuan-Bentler correction (Mplus variant)

Model test baseline model:

  Minimum Function Test Statistic                0.000       0.000
  Degrees of freedom                                 0           0
  P-value                                           NA          NA

User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000       1.000
  Tucker-Lewis Index (TLI)                       1.000       1.000

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA
```

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)               -726.014    -726.014
  Loglikelihood unrestricted model (H1)       -726.014    -726.014

  Number of free parameters                          2           2
  Akaike (AIC)                                1456.028    1456.028
  Bayesian (BIC)                              1463.367    1463.367
  Sample-size adjusted Bayesian (BIC)         1457.025    1457.025

Root Mean Square Error of Approximation:

  RMSEA                                          0.000       0.000
  90 Percent Confidence Interval          0.000  0.000       0.000  0.000
  P-value RMSEA <= 0.05                             NA          NA

  Robust RMSEA                                               0.000
  90 Percent Confidence Interval                             0.000  0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.000       0.000
```

WRAPPING UP

Wrapping Up

- The biggest difference between univariate and multivariate models is with respect to model fit
  - Univariate models are saturated models that fit perfectly
- Global model fit helps determine absolute model fit overall
- Local model fit helps determine where model fit can be improved by adding additional parameters
- But what about model comparisons? That’s next…