

Graded Response Polytomous IFA-IRT Models in Mplus version 7.4

Example data: 635 older adults (age 80-100) self-reporting on 7 items assessing the Instrumental Activities of Daily Living (IADL) as follows:

1. Housework (cleaning and laundry)
2. Bedmaking
3. Cooking
4. Everyday shopping
5. Getting to places outside of walking distance
6. Handling banking and other business
7. Using the telephone

Item	0=Can't Do It	1=Big Problems	2=Some Problems	3=Can Do It
1	0.09	0.08	0.26	0.58
2	0.07	0.04	0.12	0.77
3	0.09	0.05	0.15	0.72
4	0.10	0.09	0.19	0.62
5	0.06	0.16	0.21	0.57
6	0.06	0.08	0.12	0.74
7	0.01	0.03	0.08	0.88

Graded Response Model Syntax for 2PL-ish model (left) and 1PL-ish model (right) using ML and a logit scale:

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TITLE: Assess polytomous IADL items using GRM
DATA: FILE IS ADL.dat;

VARIABLE: NAMES ARE case dial-dia7 cial-cia7;
USEVARIABLES ARE cial-cia7;
CATEGORICAL ARE cial-cia7;
MISSING ARE .;
IDVARIABLE IS case;

ANALYSIS: ESTIMATOR IS ML;
LINK IS LOGIT;

MODEL:
! Factor loadings all estimated
IADL BY cial-cia7*;
! Item thresholds all estimated
[cial$1-cia7$1*];
[cial$2-cia7$2*];
[cial$3-cia7$3*];
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

OUTPUT: STDYX; ! Standardized solution
RESIDUAL TECH10; ! Local fit info

SAVEDATA: SAVE = FSCORES; ! Save factor scores (thetas)
FILE = IADL_42Thetas.dat; ! File factor scores saved to

PLOT: TYPE IS PLOT1; ! PLOT1 gets you sample descriptives
TYPE IS PLOT2; ! PLOT2 gets you the IRT-relevant curves
TYPE IS PLOT3; ! PLOT3 gets you descriptives for theta

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TITLE: Assess polytomous IADL items using constrained GRM
DATA: FILE IS ADL.dat;

VARIABLE: NAMES ARE case dial-dia7 cial-cia7;
USEVARIABLES ARE cial-cia7;
CATEGORICAL ARE cial-cia7;
MISSING ARE .;
IDVARIABLE IS case;

ANALYSIS: ESTIMATOR IS ML;
LINK IS LOGIT;

MODEL:
! Factor loadings all constrained equal
IADL BY cial-cia7* (loading);
! Item thresholds all estimated
[cial$1-cia7$1*];
[cial$2-cia7$2*];
[cial$3-cia7$3*];
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

OUTPUT: STDYX; ! Standardized solution
RESIDUAL TECH10; ! Local fit info

SAVEDATA: SAVE = FSCORES; ! Save factor scores (thetas)
FILE = IADL_41Thetas.dat; ! File factor scores saved to

PLOT: TYPE IS PLOT1; ! PLOT1 gets you sample descriptives
TYPE IS PLOT2; ! PLOT2 gets you the IRT-relevant curves
TYPE IS PLOT3; ! PLOT3 gets you descriptives for theta

```

Graded Response Model 2PL-ish Model Fit (left) and 1PLish Model Fit (right) using ML logit:

MODEL FIT INFORMATION		MODEL FIT INFORMATION	
Number of Free Parameters	28	Number of Free Parameters	22
Loglikelihood H0 Value	-2523.585	Loglikelihood H0 Value	-2591.310
Information Criteria		Information Criteria	
Akaike (AIC)	5103.171	Akaike (AIC)	5226.620
Bayesian (BIC)	5227.828	Bayesian (BIC)	5324.565
Sample-Size Adjusted BIC	5138.931	Sample-Size Adjusted BIC	5254.717
(n* = (n + 2) / 24)		(n* = (n + 2) / 24)	
Chi-Square Test of Model Fit for the Binary and Ordered Categorical (Ordinal) Outcomes**		Chi-Square Test of Model Fit for the Binary and Ordered Categorical (Ordinal) Outcomes**	
Pearson Chi-Square		Pearson Chi-Square	
Value	1876.488	Value	2650.119
Degrees of Freedom	16317	Degrees of Freedom	16321
P-Value	1.0000	P-Value	1.0000
Likelihood Ratio Chi-Square		Likelihood Ratio Chi-Square	
Value	676.937	Value	803.028
Degrees of Freedom	16317	Degrees of Freedom	16321
P-Value	1.0000	P-Value	1.0000
** Of the 48600 cells in the latent class indicator table, 38 were deleted in the calculation of chi-square due to extreme values.		** Of the 48600 cells in the latent class indicator table, 40 were deleted in the calculation of chi-square due to extreme values.	
This error message indicates that these 2 sets of chi-squares are not on the same scale. We need to test the -2LL difference instead.			

Does the 2PL-ish version of the GRM fit better than the 1PL-ish version?

$-2523.585^* - 2 = 5047.170$ $-2\Delta LL = 135.45$, df = 6, p < .0001
 $-2591.310^* - 2 = 5182.620$ AIC and BIC are smaller for 2PL, too

3 differently scaled solutions from ML logit (2 given, 1 calculated in excel) – all provide the exact same predictions!

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS = CHANGE IN LOGIT(Y) PER UNIT CHANGE IN THETA				
IADL BY				
CIA1	6.846	0.841	8.140	0.000
CIA2	5.200	0.555	9.363	0.000
CIA3	4.613	0.456	10.119	0.000
CIA4	5.701	0.612	9.312	0.000
CIA5	3.556	0.298	11.950	0.000
CIA6	2.897	0.261	11.094	0.000
CIA7	1.778	0.209	8.512	0.000
THRESHOLDS = EXPECTED LOGIT(Y=0) WHEN THETA IS 0 (MEAN OF SAMPLE)				
CIA1\$1	-9.808	1.138	-8.620	0.000
CIA1\$2	-6.460	0.799	-8.088	0.000
CIA1\$3	-1.238	0.384	-3.226	0.001
CIA2\$1	-8.145	0.794	-10.257	0.000
CIA2\$2	-6.313	0.618	-10.219	0.000
CIA2\$3	-3.737	0.441	-8.480	0.000
CIA3\$1	-6.841	0.613	-11.162	0.000
CIA3\$2	-5.194	0.480	-10.810	0.000
CIA3\$3	-2.572	0.330	-7.792	0.000
CIA4\$1	-7.454	0.747	-9.975	0.000
CIA4\$2	-4.635	0.514	-9.026	0.000
CIA4\$3	-1.426	0.327	-4.366	0.000
CIA5\$1	-6.578	0.494	-13.314	0.000
CIA5\$2	-3.041	0.273	-11.155	0.000
CIA5\$3	-0.681	0.203	-3.354	0.001
CIA6\$1	-5.538	0.411	-13.486	0.000
CIA6\$2	-3.583	0.285	-12.554	0.000
CIA6\$3	-2.044	0.219	-9.344	0.000
CIA7\$1	-5.810	0.472	-12.315	0.000
CIA7\$2	-4.398	0.322	-13.673	0.000
CIA7\$3	-2.951	0.237	-12.457	0.000
STDYX MODEL RESULTS (IFA MODEL SOLUTION)				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS IN STANDARDIZED METRIC = loading*SD(Theta)/SD(Y)				
IADL BY				
CIA1	0.967	0.008	124.093	0.000
CIA2	0.944	0.011	86.315	0.000
CIA3	0.931	0.012	75.583	0.000
CIA4	0.953	0.009	101.294	0.000
CIA5	0.891	0.015	57.872	0.000
CIA6	0.848	0.022	39.402	0.000
CIA7	0.700	0.042	16.689	0.000
(rest omitted)				

USING RESULTS FROM IFA MODEL (LEFT PANEL):

IFA model: $\text{Logit}(y=1) = -\text{threshold} + \text{loading}(\Theta)$
 Threshold = expected logit of ($y=0$) for someone with $\Theta=0$
 When $*-1$, threshold becomes intercept: expected logit for ($y=1$) instead
 Loading = regression of item logit on Θ

For 4-category responses, the sub-models look like this:
 $\text{Logit}(y=0 \text{ vs } 123) = -\text{threshold}_1 + \text{loading}(\Theta)$
 $\text{Logit}(y=01 \text{ vs } 23) = -\text{threshold}_2 + \text{loading}(\Theta)$
 $\text{Logit}(y=012 \text{ vs } 3) = -\text{threshold}_3 + \text{loading}(\Theta)$

IFA Models:

\$1 Logit(CIA1=0 vs 123) = 9.808 + 6.846(\Theta) → if $\Theta=0$, prob=.99994
 \$2 Logit(CIA1=01 vs 23) = 6.460 + 6.846(\Theta) → if $\Theta=0$, prob=.99844
 \$3 Logit(CIA1=012 vs 3) = 1.238 + 6.846(\Theta) → if $\Theta=0$, prob=.77522
 \$1 → if $\Theta=-1$, logit= 2.962, prob=.95083
 \$2 → if $\Theta=-1$, logit= -0.386, prob=.40468
 \$3 → if $\Theta=-1$, logit= -5.608 prob=.00365

RESULTS FROM IRT MODEL MUST BE CALCULATED BY YOU!

IRT model: $\text{Logit}(y) = a(\theta - \text{difficulty})$
 a = discrimination (rescaled slope) = loading
 b = difficulty (location on latent metric) = threshold/loading

My calculations (see spreadsheet):

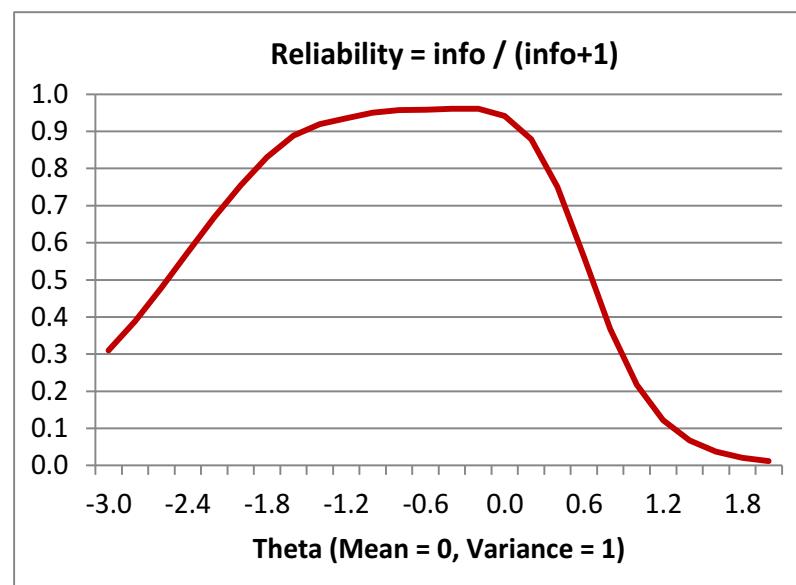
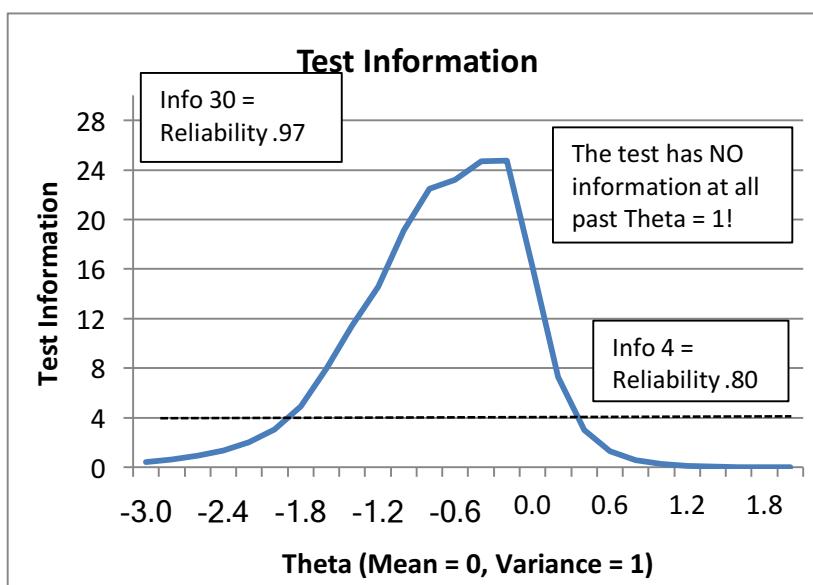
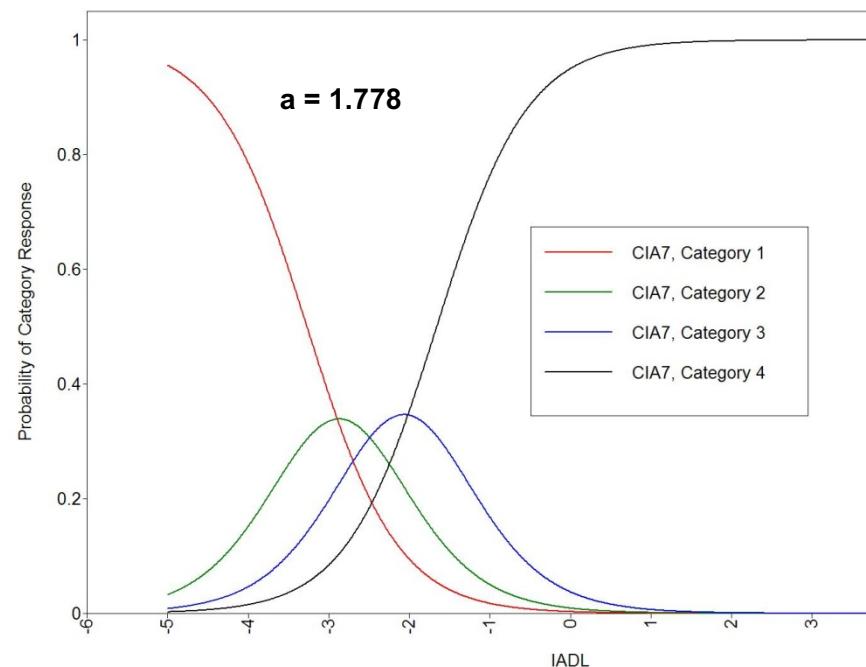
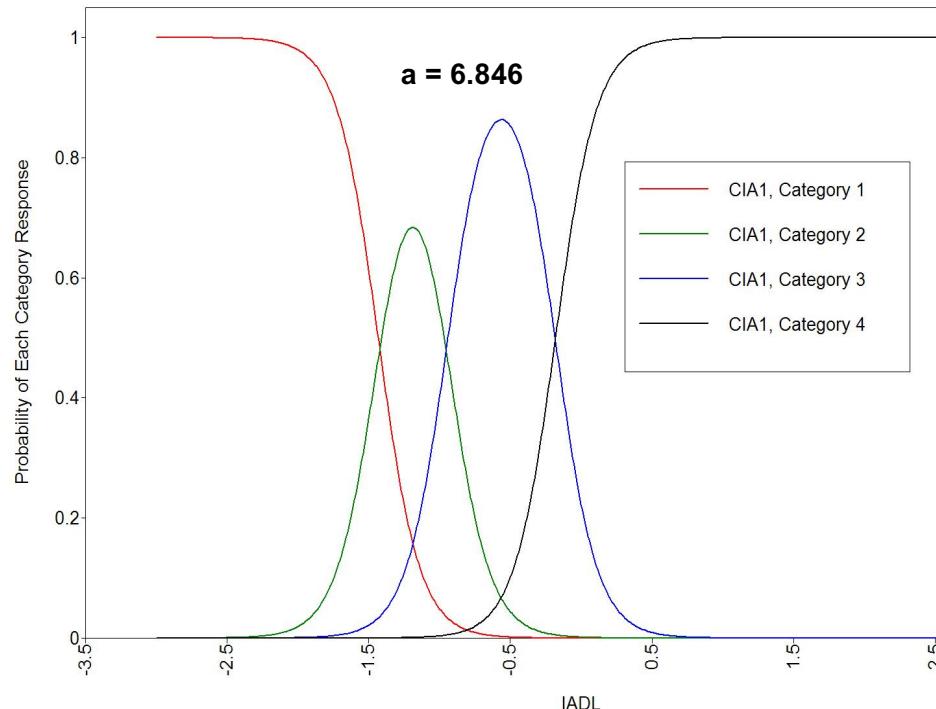
CIA1 loading = 6.846 → a discrimination = 6.846
 CIA1 threshold\$1 = $-9.808/6.846 \rightarrow b \text{ difficulty}_1 = -1.433$
 CIA1 threshold\$2 = $-6.460/6.846 \rightarrow b \text{ difficulty}_2 = -0.944$
 CIA1 threshold\$3 = $-1.238/6.846 \rightarrow b \text{ difficulty}_3 = -0.181$

For 4-category responses, the sub-models look like this:

\$1 Logit(y=0 vs 123) = $a(\theta - \text{difficulty}_1)$
 \$2 Logit(y=01 vs 23) = $a(\theta - \text{difficulty}_2)$
 \$3 Logit(y=012 vs 3) = $a(\theta - \text{difficulty}_3)$

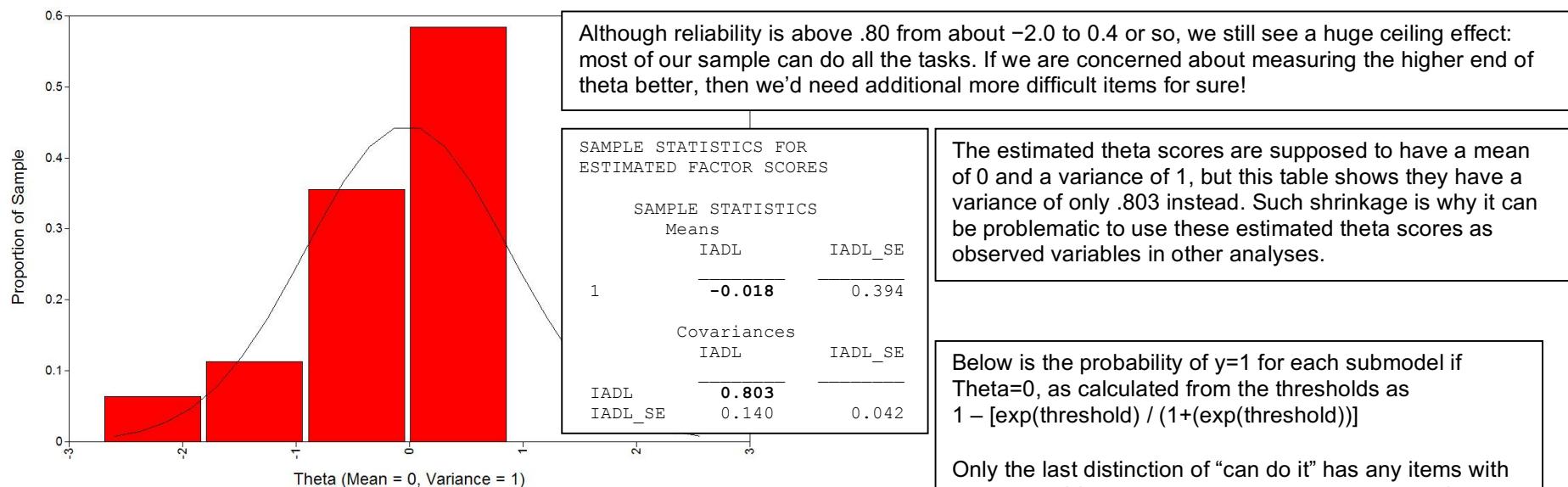
IRT Models:

\$1 Logit = 6.846($\Theta - -1.433$)
 \$2 Logit = 6.846($\Theta - -0.944$)
 \$3 Logit = 6.846($\Theta - -0.181$)

Mplus Category Response Curves – Item 1 (good and steep discrimination) and Item 7 (less good because less steep)


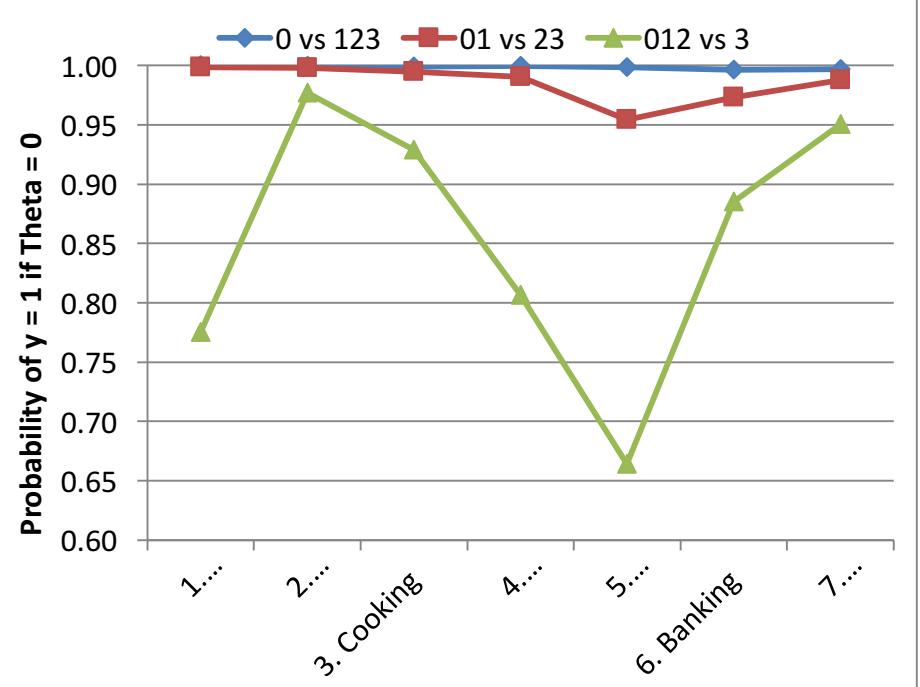
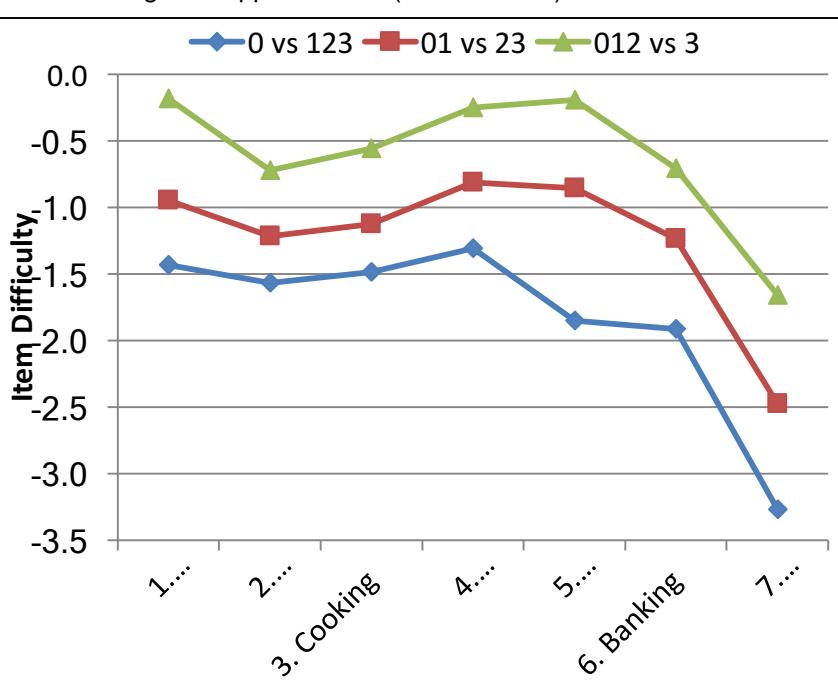
Distribution of Theta under GRM (made in Mplus)

EPSY 906 / CLP 948 Example 6a page 5



Variability in Spread of Item Difficulty (made in excel):

Some items (like 5) have a wider spread of their category thresholds, but all categories appear useful (differentiable).



Here is the graded response model again: a 2PL-ish version vs. a 1PL-ish for Polytomous Responses using WLSMV probit model

```

TITLE: 2PL Graded Response Model under WLSMV
DATA: FILE IS ADL.dat;
VARIABLE: NAMES ARE case dial-dia7 cial-cia7;
USEVARIABLES ARE cial-cia7;
CATEGORICAL ARE cial-cia7;
MISSING ARE .;
IDVARIABLE IS case;

ANALYSIS: ESTIMATOR IS WLSMV; PARAMETERIZATION IS THETA;

MODEL:
! Factor loadings all estimated in 2PL
IADL BY cial-cia7*;
! Item thresholds all estimated
[cial$1-cia7$1*];
[cial$2-cia7$2*];
[cial$3-cia7$3*];
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

OUTPUT: STDYX Residual; ! Standardized solution, local fit
SAVEDATA: DIFFTEST=2PL.dat; ! Save info from bigger model
SAVE = FSCORES; ! Save factor scores (thetas)
FILE = IADL_42Thetas.dat; ! File factor scores saved to

PLOT: TYPE IS PLOT1 PLOT2 PLOT3; ! Get IRT plots

```

MODEL FIT INFORMATION

Number of Free Parameters	28
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Chi-Square Test of Model Fit

Value	96.262*
Degrees of Freedom	14
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.096
90 Percent C.I.	0.079 0.115
Probability RMSEA <= .05	0.000

CFI/TLI

CFI	0.997
TLI	0.995

Chi-Square Test of Model Fit for the Baseline Model

Value	26556.135
Degrees of Freedom	21
P-Value	0.0000

```

TITLE: 1PL Graded Response Model under WLSMV
DATA: FILE IS ADL.dat;
VARIABLE: NAMES ARE case dial-dia7 cial-cia7;
USEVARIABLES ARE cial-cia7;
CATEGORICAL ARE cial-cia7;
MISSING ARE .;
IDVARIABLE IS case;

ANALYSIS: ESTIMATOR IS WLSMV; PARAMETERIZATION IS THETA;
DIFFTEST=2PL.dat; ! Use saved info from bigger model

MODEL:
! Factor loadings all constrained equal in 1PL
IADL BY cial-cia7* (loading);
! Item thresholds all estimated
[cial$1-cia7$1*];
[cial$2-cia7$2*];
[cial$3-cia7$3*];
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

OUTPUT: STDYX Residual; ! Standardized solution, local fit
SAVEDATA:
SAVE = FSCORES; ! Save factor scores (thetas)
FILE = IADL_41Thetas.dat; ! File factor scores saved to

PLOT: TYPE IS PLOT1 PLOT2 PLOT3; ! Get IRT plots

```

MODEL FIT INFORMATION

Number of Free Parameters	22
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Chi-Square Test of Model Fit

Value	202.569*
Degrees of Freedom	20
P-Value	0.0000

Chi-Square Test for Difference Testing

Value	93.833
Degrees of Freedom	6
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.120
90 Percent C.I.	0.105 0.135
Probability RMSEA <= .05	0.000

CFI/TLI

CFI	0.993
TLI	0.993

The Chi-Square for Difference Testing tells us directly that the 2PL version of the polytomous model fits significantly better (now under WLSMV, same as it did under ML).

Here are the parameter estimates under WLSMV Theta Parameterization (Probit) for the 2PL version of polytomous responses

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)				
	Two-Tailed			
	Estimate	S.E.	Est./S.E.	P-Value
FACTOR LOADINGS = CHANGE IN PROBIT(Y=1) PER UNIT CHANGE IN THETA				
IADL BY				
CIA1	3.655	0.330	11.083	0.000
CIA2	3.346	0.388	8.632	0.000
CIA3	2.923	0.269	10.881	0.000
CIA4	3.286	0.299	11.008	0.000
CIA5	2.222	0.159	13.963	0.000
CIA6	1.907	0.169	11.305	0.000
CIA7	1.075	0.130	8.279	0.000
THRESHOLDS = EXPECTED PROBIT(Y=0) WHEN THETA IS 0				
CIA1\$1	-5.151	0.424	-12.137	0.000
CIA1\$2	-3.658	0.347	-10.534	0.000
CIA1\$3	-0.734	0.217	-3.383	0.001
CIA2\$1	-5.096	0.497	-10.254	0.000
CIA2\$2	-4.253	0.445	-9.552	0.000
CIA2\$3	-2.620	0.353	-7.425	0.000
CIA3\$1	-4.193	0.327	-12.825	0.000
CIA3\$2	-3.404	0.296	-11.486	0.000
CIA3\$3	-1.761	0.232	-7.592	0.000
CIA4\$1	-4.379	0.342	-12.794	0.000
CIA4\$2	-2.987	0.269	-11.107	0.000
CIA4\$3	-1.024	0.211	-4.863	0.000
CIA5\$1	-3.866	0.233	-16.616	0.000
CIA5\$2	-1.892	0.160	-11.856	0.000
CIA5\$3	-0.425	0.130	-3.277	0.001
CIA6\$1	-3.450	0.235	-14.697	0.000
CIA6\$2	-2.354	0.184	-12.805	0.000
CIA6\$3	-1.400	0.154	-9.072	0.000
CIA7\$1	-3.282	0.249	-13.169	0.000
CIA7\$2	-2.577	0.181	-14.231	0.000
CIA7\$3	-1.757	0.137	-12.840	0.000

STDYX MODEL RESULTS (STANDARDIZED IFA MODEL SOLUTION)

FACTOR LOADINGS IN STANDARDIZED METRIC = loading*SD(Theta) / SD(Y)				
IADL BY				
CIA1	0.965	0.006	159.169	0.000
CIA2	0.958	0.009	105.293	0.000
CIA3	0.946	0.009	103.821	0.000
CIA4	0.957	0.007	129.875	0.000
CIA5	0.912	0.011	82.875	0.000
CIA6	0.886	0.017	52.429	0.000
CIA7	0.732	0.041	17.844	0.000

Logit = 1.7*probit, or Probit = Logit/1.7

IFA model: Probit(y=1) = -threshold + loading(Theta)
 Threshold = expected probit of (y=0) for someone with Theta=0
 When *-1, threshold → intercept: expected probit for (y=1) instead
 Loading = regression of item probit on Theta

For 4-category responses, the sub-models look like this:
 Probit(y= 0 vs 123) = -threshold\$1 + loading(Theta)
 Probit(y= 01 vs 23) = -threshold\$2 + loading(Theta)
 Probit y= 012 vs 3) = -threshold\$3 + loading(Theta)

**IRT RESULTS ARE NOT GIVEN FOR POLYTOMOUS ITEMS;
 THEY MUST BE CALCULATED BY YOU!**

IRT model: Probit(y) = a(theta - difficulty)
 a = discrimination (rescaled slope) = loading
 b = difficulty (location on latent metric) = threshold/loading

For 4-category responses, the sub-models look like this:
 \$1 Probit(y= 0 vs 123) = a(theta - difficulty\$1)
 \$2 Probit(y= 01 vs 23) = a(theta - difficulty\$2)
 \$3 Probit(y= 012 vs 3) = a(theta - difficulty\$3)

**LOCAL FIT VIA STANDARDIZED RESIDUAL CORRELATIONS
 LEFTOVER POLYCHORIC CORRELATION (HOW FAR OFF FROM DATA)**

Residuals for Covariances/Correlations/Residual Correlations

	CIA1	CIA2	CIA3	CIA4	CIA5	CIA6
CIA1						
CIA2	0.013					
CIA3	0.012	0.017				
CIA4	-0.010	-0.025	-0.036			
CIA5	-0.030	-0.045	-0.067	0.032		
CIA6	-0.040	-0.055	-0.025	0.026	0.035	
CIA7	-0.026	-0.007	0.016	0.022	-0.031	0.025

Bonus material! Here is how to fit the modified graded response model in Mplus using ML. The item location is set as threshold 3, and two distance parameters (c_1 and c_2) are held equal across items, so that the spread of the category thresholds is held equal.

```

TITLE: 2PL MODIFIED Graded Response Model using ML
DATA: FILE IS ADL.dat;
VARIABLE: NAMES ARE case dial-dia7 cial-cia7;
USEVARIBLES ARE cial-cia7;
CATEGORICAL ARE cial-cia7;
MISSING ARE .;
IDVARIABLE IS case;

ANALYSIS: ESTIMATOR IS ML; LINK IS LOGIT;
OUTPUT: STDYX TECH10; ! Standardized solution, local fit
SAVEDATA: SAVE = FSCORES; ! Save factor scores (thetas)
FILE = MGRM_Thetas.dat; ! File factor scores saved to
PLOT: TYPE IS PLOT1 PLOT2 PLOT3; ! Get IRT plots

MODEL:
! Threshold 1 is location - c1 as defined in NEW
[cia1$1] (t11);
[cia2$1] (t12);
[cia3$1] (t13);
[cia4$1] (t14);
[cia5$1] (t15);
[cia6$1] (t16);
[cia7$1] (t17);
! Threshold 2 is location - c2 as defined in NEW
[cia1$2] (t21);
[cia2$2] (t22);
[cia3$2] (t23);
[cia4$2] (t24);
[cia5$2] (t25);
[cia6$2] (t26);
[cia7$2] (t27);
! Threshold 3 defines location per item
[cia1$3] (loc1);
[cia2$3] (loc2);
[cia3$3] (loc3);
[cia4$3] (loc4);
[cia5$3] (loc5);
[cia6$3] (loc6);
[cia7$3] (loc7);

! Factor variance fixed to 1, mean fixed to 0 for identification
IADL@1; [IADL@0];

MODEL CONSTRAINT:
NEW(c1 c2); ! New category spread parameters
! Threshold 1 is location - c1 as defined in NEW
t11 = loc1 - c1;
t12 = loc2 - c1;
t13 = loc3 - c1;
t14 = loc4 - c1;
t15 = loc5 - c1;
t16 = loc6 - c1;
t17 = loc7 - c1;

```

! Threshold 2 is location - c2 as defined in NEW
t21 = loc1 - c2;
t22 = loc2 - c2;
t23 = loc3 - c2;
t24 = loc4 - c2;
t25 = loc5 - c2;
t26 = loc6 - c2;
t27 = loc7 - c2;

Relative to the original graded response model, this modified model fits significantly worse, $-2\text{LL}(12) = 127$, $p < .001$. However, we could examine for which items these constraints do not hold and free just those, resulting a hybrid or "partially modified" graded response model.

MODEL FIT INFORMATION

Number of Free Parameters	16
Loglikelihood	
H0 Value	-2586.984
Information Criteria	
Akaike (AIC)	5205.968
Bayesian (BIC)	5277.201
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	5226.403

		Two-Tailed		
IADL	BY	Estimate	S.E.	Est./S.E.
	IADL	0.000	0.000	999.000
	Thresholds			
	CIA1\$1	-5.971	0.293	-20.350
	CIA1\$2	-3.569	0.255	-13.993
	CIA1\$3	-1.079	0.240	-4.492
	CIA2\$1	-8.729	0.455	-19.197
	CIA2\$2	-6.327	0.431	-14.676
	CIA2\$3	-3.838	0.414	-9.270
	CIA3\$1	-7.615	0.384	-19.816
	CIA3\$2	-5.213	0.354	-14.738
	CIA3\$3	-2.723	0.338	-8.065
	CIA4\$1	-6.235	0.332	-18.801
	CIA4\$2	-3.833	0.296	-12.945
	CIA4\$3	-1.344	0.282	-4.765
	CIA5\$1	-5.463	0.244	-22.411
	CIA5\$2	-3.061	0.202	-15.184
	CIA5\$3	-0.571	0.185	-3.089
	CIA6\$1	-7.084	0.311	-22.756
	CIA6\$2	-4.682	0.277	-16.906
	CIA6\$3	-2.192	0.256	-8.557
	CIA7\$1	-8.004	0.314	-25.509
	CIA7\$2	-5.602	0.285	-19.660
	CIA7\$3	-3.113	0.265	-11.734
	Variances			
	IADL	1.000	0.000	999.000
	New/Additional Parameters			
	C1	4.891	0.167	29.309
	C2	2.489	0.095	26.316