Questions and Sample Answers for Chapter 10

Section 1 – Questions

Question 1

Compare and contrast the processes and outcomes of expected a posteriori (EAP) and maximum a posteriori (MAP) estimation.

Question 2

A colleague excitedly shows you what he believes to be study results worth publishing: Using only five items which appear on a well-known standardized test routinely administered to incoming first-year students, he was able to estimate incoming students' critical thinking skills using a DCM to categorize students according to their critical thinking profile, indicating students' mastery or non-mastery status on two distinct attributes. The model categorizes nearly all students who completed the assessment with a high degree of certainty: the median MAP estimate for the incoming class (N=151) is 85%. The diagnostic assessment is efficient and effective!

Do you share in your colleague's excitement about the results? Explain why or why not, taking into account what information is used to estimate each respondent's probability of latent class membership. What additional analyses, if any, might you recommend your colleague pursue prior to publication?

Question 3

Which of the following is true of respondent parameter estimation in DCMs?

a. MAP estimation is the computation of the latent class membership probabilities using data and prior information.

b. EAP estimation yields information about attribute mastery by incorporating information across all latent classes.

c. The impact of the prior distribution on the respondent classification is stronger for shorter diagnostic assessments than for longer assessments.

d. EAP and MAP estimation imply a fully Bayesian estimation framework.

e. All of the above.

f. None of the above.

Question 4

The following summary table is obtained from a *Mplus* respondent output file. There are two attributes that yield four latent classes for a set of diagnostic assessment items. In the table, the first column represents the respondent ID while the last four columns in the table are posterior probabilities of latent class membership.

Respondents	C1 [0,0]	C2 [0,1]	C3 [1,0]	C4 [1,1]
Respondent1	.29	.19	.19	.33
Respondent2	.52	.35	.05	.08
Respondent3	.90	.05	.05	.00
Respondent4	.05	.10	.10	.75
Respondent5	.02	.03	.03	.92
Respondent6	.20	.12	.12	.56

Answer the following questions based on the information in this table.

a. What is the latent class to which each respondent does most likely belong to? What is this value known as?

b. What are the EAP estimates for the two attributes for the six respondents?

c. Is the classification of the six respondents into the four latent classes certain?

Question 1

Both the expected a posteriori (EAP) and the maximum a posteriori (MAP) values are estimated by relying on Bayes' rules, which does not necessarily imply a fully Bayesian estimation approach however. Nevertheless, both EAP and MAP estimates rely on a prior distribution for model parameters – the probabilities of latent class membership for respondents – based on theory or previous empirical studies. Using Bayes' rule, the idea is to calculate a posterior version of this distribution, which is proportional to the product of the prior distribution and the likelihood of the data.

Technically speaking, an EAP estimate of a parameter is the mean of the posterior distribution while an MAP estimate is the mode of the posterior distribution. In the context of DCMs, these concepts translate as follows. The MAP estimate for a respondent is simply the latent class for which he or she has the highest likelihood of belonging; this probability should ideally be close to 1 but is sometimes not even above .50 in practice when classification certainty is poor. The EAP estimate is the marginal estimate of mastery for the individual attributes that constitute the attribute profiles for the latent classes. These probabilities are computed by adding the posterior probabilities of latent class membership for all those latent classes in which a particular attribute is mastered. Thus, EAP and MAP estimates both provide information about attribute mastery, but the former provides joint mastery information for all attributes while the latter provides mastery information for individual attributes.

Question 2

Your colleague claims to have evidence of an economical, efficient, and effective assessment of a complex 21st century skill set, but it is important to be cautious of the strength of the results. Even without knowing more details about your colleague's study, the length of the assessment and knowing the potential effects of assessment length on MAP estimates would be cause for some concern. Although the median max MAP is high, suggesting that students are classified with certainty, it is not clear how strong the empirical evidence is for individual students' classification.

Recall that the posterior probability that respondent *r* belongs to latent class *c* with associated attribute vector

Question 3

Correct answers: b & c

a. This is incorrect, because the language simply describes the computation of a posterior distribution, which provides probabilities of membership for each latent class. However, the MAP estimate is the largest value of those – the maximum of this distribution – and, thus, only one specific number.

b. This is correct even though the weighting of posterior probabilities by the attribute profiles for latent classes effectively excludes the posterior probabilities for those latent classes in which a particular attribute is not mastered.

c. This is correct even though in practice the likelihood tends to dominate the statistical information in the posterior information relatively quickly.

d. This is incorrect. Even though Bayes' rule is used for the computation of the posterior distribution, a fully Bayesian estimation approach using sampling techniques such as Markov-chain Monte Carlo (MCMC) estimation is not necessary. However, if a particular DCM were estimated using a fully Bayesian approach then the concepts of EAP and MAP estimates would still have the same meaning.

Question 4

a. The most likely latent class that each respondent belongs to is class 4, 1, 1, 4, 4, and 3, respectively; these values are known as the MAP estimates for respondents.

b. The EAP estimates are the marginal probabilities of attribute mastery; these pairs of values for the six respondents are (.52, .52), (.13, .43), (.05, .05), (.85, .85), (.95, .95), and (.68, .68), respectively.

c. It is certain for respondents 3, 4, and 5 because they have MAP probabilities of .90, .75, and .92, respectively. However, it is very uncertain for respondents 1, 2, and 6, who have MAP probabilities of .33, .52, and .56, respectively. So the overall certainty is mixed at best.